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ON MESON-NUCLEON SCATTERING IN LARGE ORBITAL ANGULAR MOMENTUM STATES

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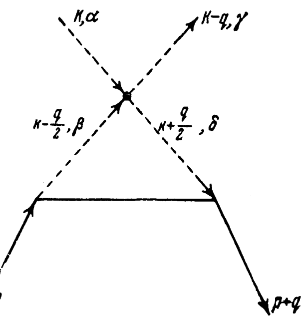
The πn phase shifts for scattering in large orbital angular momentum states are described in terms of the $\pi\pi$-interaction coupling constant. If the results obtained can be extended to the $l = 2$ case and if the assumption is made that the $\pi\pi$ scattering amplitude exhibits no resonance at low energies, then a disagreement with experiment is obtained.

As was shown by Okun' and Pomeranchuk,[1] πn scattering in large orbital angular momentum states is dominated by the diagram shown in the figure. This diagram contains the πn scattering amplitude and the $\pi\pi$ scattering amplitude. Consequently experimental data on πn scattering phase shifts in large orbital angular momentum states may in principle yield information about the magnitude of $\pi\pi$ scattering.



Principal diagram for πn scattering in large orbital angular momentum states.

The scattering phase shifts in large angular momentum states are determined by the value of the amplitude in the vicinity of the nearest singularity in the momentum transfer q^2. In the case under consideration the nearest singularity occurs at $q^2 = 4\mu^2$ and the next one at $q^2 = 16\mu^2$. The large interval between the two nearest singularities encourages the speculation that the indicated diagram for πn scattering will also be dominant for not very large angular momenta.

The general expression for the $\pi\pi$ scattering amplitude is given by

$$\Gamma_{\alpha\beta\gamma\delta}(p_1, p_2, p_3, p_4) = A\delta_{\alpha\beta}\delta_{\gamma\delta} + B\delta_{\alpha\gamma}\delta_{\beta\delta} + C\delta_{\beta\gamma}\delta_{\alpha\delta}, \quad (1)$$

where A, B, C are functions of the momenta p_i. We have here the following symmetry conditions: $p_1 \to p_2$, $B \to C$; $p_2 \to p_3$, $A \to B$; $p_1 \to p_3$, $A \to C$. Using the notation indicated in the figure we get $p_1 = K$, $p_2 = k - q/2$, $p_3 = -K + q$, $p_4 = -k - q/2$ and to the substitution $k \to -k$ corresponds $A \to C$.

The general expression for the πn scattering amplitude is given by

$$f_{\alpha\beta}(p_1, p_2, p_3, p_4) = (\bar{u}_{-p_4}|\delta_{\alpha\beta}[L^{(1)} + \tfrac{1}{2}(\hat{p}_3 - \hat{p}_1)M^{(1)}] - i\varepsilon_{\alpha\beta\gamma}\tau_\gamma[L^{(2)} + \tfrac{1}{2}(\hat{p}_3 - \hat{p}_1)M^{(2)}]|u_{p_2}) \quad (2)$$

and we have the symmetry conditions (p_1, p_3 — meson momenta, p_2, p_4 — nucleon momenta) $p_1 \to p_3$, $L^{(2)} \to -L^{(2)}$, $M^{(1)} \to -M^{(1)}$. In the notation of the figure: $p_1 = -k + q/2$, $p_2 = p$, $p_3 = k + q/2$, $p_4 = -p - q$ and for $k \to -k$ we have

$$L^{(2)} + \hat{k}M^{(2)} \to -(L^{(2)} + \hat{k}M^{(2)}).$$

The amplitude corresponding to the diagram in the figure may be written as follows:

$$M_{\alpha\gamma}(q^2) = \frac{1}{2\pi i}\int \Gamma_{\alpha\beta\gamma\delta}\left(K, k - \frac{q}{2}, -K + q, -k - \frac{q}{2}\right) \times f_{\beta\delta}\left(-k + \frac{q}{2}, p, k + \frac{q}{2}, -p - q\right)\Delta(k, q)\, d^4k,$$

$$\Delta(k, q) = [(k - q/2)^2 - \mu^2]^{-1}[(k + q/2)^2 - \mu^2]^{-1} \quad (3)$$

or, after substitution of (1) and (2), as follows:

$$M_{\alpha\gamma}(q^2) = \frac{1}{2\pi i}\int [\delta_{\alpha\gamma}(A + C + 3B)(\bar{u}_{p+q}|L^{(1)} + \hat{k}M^{(1)}|u_p) - i\varepsilon_{\alpha\gamma\sigma}\tau_\sigma(A - C)(\bar{u}_{p+q}|L^{(2)} + \hat{k}M^{(2)}|u_p)]\Delta(k, q)\, d^4k. \quad (4)$$

When calculating scattering phase shifts in large angular momentum states values of q^2 near $4\mu^2$ and small values of the variable of integration k in (4) are of importance. An estimate of the effective values of k^2 may be obtained by considering the integral ($q^2 = -2\mathbf{p}^2(1 - \cos\theta)$ with θ the scattering angle and $\mathbf{p}$ the momentum in the c.m.s.)

$$I_l = \int_{-1}^{+1} M(\vartheta)P_l(\cos\vartheta)\, d(\cos\vartheta),$$

in the same way as was done previously[2] for the nn scattering amplitude. Such an estimate leads to

$$|k^2_{\text{eff}}| \approx \mu^2/L, \quad (5)$$

$$L = (l + 1)\xi/\sqrt{1 + \xi^2}, \quad \xi = \mu/|\mathbf{p}|; \quad (6)$$

it is assumed that the inequality $L \gg 1$ holds.

Since $A + C + 3B$ and $L^{(1)} + \hat{k}M^{(1)}$ are even functions of k, and $A - C$ and $L^{(2)} + \hat{k}M^{(2)}$ are odd functions of k one may, accurate to first order terms in 1/L, omit in (4) the term proportional to $\epsilon_{\alpha\gamma\sigma}\tau_\sigma$. Thus, for $L \gg 1$, the πn scattering amplitude is independent of isotopic spin indices:

$$M_{\alpha\gamma}(q^2) = M(q^2)\,\delta_{\alpha\gamma},$$

$$M(q^2) = \frac{1}{2\pi i}\int (A + C + 3B) \times (\bar{u}_{p+q} \mid L^{(1)} + \hat{k}M^{(1)} \mid u_p)\,\Delta(k, q)\,d^4k. \quad (7)$$

If A + C + 3B is a sufficiently smooth function of k inside the region (5) it can be taken out from under the integral sign in (7):

$$M(q^2) = -\frac{5\lambda}{2\pi i}\int (\bar{u}_{p+q} \mid L^{(1)} + \hat{k}M^{(1)} \mid u_p)\,\Delta(k, q)\,d^4k, \quad (8)$$

where we have set $-5\lambda = (A + C + 3B)_{k=0}$

The dependence of the scattering phase shifts on the mechanical spin variables can be obtained without a knowledge of the explicit form of $L^{(1)}$ and $M^{(1)}$. As a matter of fact $L^{(1)}$ and $M^{(1)}$ are functions of the vectors p, q and k. In carrying out the k-integration in Eq. (8) we will obtain either $(\bar{u}_{p+q} \mid \hat{q} \mid u_p)$ or $(\bar{u}_{p+q} \mid \hat{p} \mid u_p)$ in place of $(\bar{u}_{p+q} \mid \hat{k} \mid u_p)$. But $(\bar{u}_{p+q} \mid \hat{q} \mid u_p) = 0$ and $(\bar{u}_{p+q} \mid \hat{p} \mid u_p) = m(\bar{u}_{p+q}u_p)$. It then follows that the spin dependence of M is contained in the factor $(\bar{u}_{p+q}u_p)$, i.e.,

$$M(q^2) = (\bar{u}_{p+q}u_p)\,F(q^2), \quad (9)$$

where the function $F(q^2)$ depends on the precise form of $L^{(1)}$ and $M^{(1)}$.

Passing from the four-component spinors u to the two-component v we obtain

$$M(q^2) = (v'^* \mid m^{-1}(E + \omega)(f + i\sigma[\mathbf{K}\times\mathbf{K}']\varphi) \mid v), \quad (10)$$

where

$$f = \frac{m + E}{2(\omega + E)}\left[1 - \frac{\mathbf{p}^2 t}{(E + m)^2}\right]F(q^2),$$

$$\varphi = \frac{F(q^2)}{2(E + \omega)(E + m)} \quad (11)$$

($t = \cos\theta$, ω = meson energy, E = nucleon energy).

We next expand the amplitudes f and φ in Legendre polynomials (see, e.g., reference 2):

$$f = (2i\,|\mathbf{p}|)^{-1}\sum_l [(l+1)\exp(2i\delta_l^+) + l\exp(2i\delta_l^-) - 2l - 1]\,P_l(t) \approx |\mathbf{p}|^{-1}\sum_l [(l + 1)\,\delta_l^+ + l\delta_l^-]\,P_l(t), \quad (12)$$

$$\varphi = (2i\,|\mathbf{p}|^3)^{-1}\sum_l (\exp(2i\delta_l^+) - \exp(2i\delta_l^-))\,dP_l(t)/dt \approx -(|\mathbf{p}|^3\sqrt{1 - t^2})^{-1}\sum_l (\delta_l^+ - \delta_l^-)\,P_l^{(1)}(t), \quad (13)$$

where $\delta_l^\pm$ are the scattering phase shifts in states of orbital angular momentum l and $j = l \pm 1/2$.

Consequently*

$$[(l + 1)\,\delta_l^+ + l\delta_l^-]/(2l+1) = \frac{(m + E)\,|\mathbf{p}|}{4(E + \omega)}\int_{-1}^{+1}\left[1 - \frac{\mathbf{p}^2 t}{(E + m)^2}\right]F(q^2)\,P_l(t)\,dt, \quad (14)$$

$$\delta_l^+ - \delta_l^- = -\frac{|\mathbf{p}|^3}{4(E + m)(E + \omega)}\frac{2l + 1}{l(l + 1)} \times \int_{-1}^{+1}(1 - t^2)\frac{dF(q^2)}{dt}P_l(t)\,dt. \quad (15)$$

For $L \gg 1$ the following formula holds:[2]

$$\int_{-1}^{+1} F(q^2)\,P_l(t)\,dt = \frac{4\xi^2}{\pi i}\,Q_l(1 + 2\xi^2)\int_0^\infty e^{-Ls^2}(\Delta F)\,s\,ds, \quad (16)$$

where Q_l is the Legendre function of the second kind;

$$s = \sqrt{1 - q^2/4\mu^2} = \sqrt{1 + (1 - t)/2\xi^2}, \quad (17)$$

and (ΔF) is the jump in the function $F(q^2)$ across the cut in the complex s-plane running from $s = 0$ to $s = \infty$.

Taking into account the fact that $dF/dt = -(4\xi^2 s)^{-1}dF/ds$, as well as that $(\Delta F) = 0$ at $s = 0$ we obtain [for $s \ll 1: t = 1 + 2\xi^2$, $1 - t^2 = -4\xi^2(1 + \xi^2)$]

$$[(l + 1)\,\delta_l^+ + l\delta_l^-]/(2l + 1) = -c_1 bQ_l(2 + 2\xi^2), \quad (18)$$

$$\delta_l^+ - \delta_l^- = -c_2 bQ_l(1 + 2\xi^2), \quad (19)$$

where

$$c_1 = \frac{\mu\xi}{m}\frac{E + m}{E + \omega}\left(1 - \frac{\mathbf{p}^2 + 2\mu^2}{(E + m)^2}\right),$$

$$c_2 = \frac{2l + 1}{l}\frac{2\mu^3\sqrt{1 + \xi^2}}{m(E + m)(E + \omega)},$$

$$b = -\frac{m}{\pi i}\int_0^\infty e^{-Ls^2}(\Delta F)\,s\,ds. \quad (20)$$

It is obvious that $c_2 \ll c_1$. In the nonrelativistic approximation for the nucleons when $|\mathbf{p}| \ll m$, but $|\mathbf{p}| \sim \mu$, the ratio c_2/c_1 is of the order of $\sqrt{2} \times (\mu/m)^2 \approx 0.03$. It then follows that δ_l^+ and δ_l^- are almost equal. This conclusion is a consequence of the assumption that A + C + 3B is a sufficiently smooth function within the region (5), i.e., that the $\pi\pi$ scattering amplitude has no resonances for low meson energy. If this assumption is not justified then in addition to the matrix element $(\bar{u}_{p+q}u_p)$, M will also contain the matrix element $(\bar{u}_{p+q} \mid \hat{K} \mid u_p)$ and the sign dependence of M may be substantially altered.

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*We make use of

$$\int_{-1}^{+1} P_l^{(1)}(t)\,P_{l'}^{(1)}(t)\,dt = \delta_{ll'}\frac{2l(l + 1)}{2l + 1}; \quad dP_l(t)/dt = -\frac{1}{\sin\theta}P_l^{(1)}(t),$$

$$d[(1 - t^2)\,F(q^2)]/dt \approx (1 - t^2)\,dF/dt \text{ for } L \gg 1.$$

According to the experiments of Mukhin and Pontecorvo[3] $\delta_2^+ \approx -\delta_2^-$, which disagrees with (18) and (19). Thus, if it is true that the two-pion interaction (see figure) is already dominant for $l = 2$, the discrepancy with experiment would indicate the existence of a resonance in the $\pi\pi$ scattering amplitude at low energies (about the $\pi\pi$ resonance see also references 4 and 5).

For a quantitative determination of the πm scattering phase shifts for $L \gg 1$ it is necessary to evaluate $F(q^2)$ and b. We obtain

$$f_{\alpha\gamma} = \frac{\alpha}{m}\delta_{\alpha\gamma} + \tau_\alpha\tau_\gamma \frac{-\hat{k}}{(k+q/2)^2 + 2p(k+q/2)} + \tau_\gamma\tau_\alpha \frac{\hat{k}}{(k-q/2)^2 - 2p(k-q/2)},$$

(which is in agreement with Galanin et al.[2,6]) from where, making use of the results obtained in reference 6, we deduce

$$F(q^2) = \frac{5g^2\lambda}{8m}\left[(\alpha-1)s + \frac{\varepsilon}{2}\ln(\varepsilon + 2s)\right],$$

where $\epsilon = \mu/m$ and $\alpha = 1.2$. Carrying out the s-integration in Eq. (20) we get (see reference 6; it is assumed that $-5\lambda = (A+C+3B)_{k=0}$ is independent of q^2):

$$b = (5g^2\lambda / 16\sqrt{\pi}L^{3/2})\left[\alpha - 1 + \sqrt{\pi}\zeta\left(1 - \frac{2}{\sqrt{\pi}}\zeta\right)\right],$$

where it was assumed that $1 \ll L \ll 4m^2/\mu^2$ and $\zeta = \epsilon\sqrt{L}/2 \ll 1$, and terms quadratic in ζ were taken into account.

In this way the values of πm scattering phase shifts for sufficiently large l provide an opportunity for obtaining λ which determines the $\pi\pi$ interaction. The qualitative discrepancy with the experimental data (indicated above) on the signs of the D-wave phases, due either to a $\pi\pi$ amplitude resonance or to the fact that an orbital angular momentum two is not sufficiently large for the considerations here outlined to be valid even as an order of magnitude estimate,* precludes the use of the Mukhin and Pontecorvo data[3] on the D phase shifts for a determination of the constant λ.

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[1] L. B. Okun' and I. Ya. Pomeranchuk, JETP **36**, 300 (1959), Soviet Phys. JETP **9**, 207 (1959).

[2] Galanin, Grashin, Ioffe, and Pomeranchuk, JETP **37**, 1663 (1959), Soviet Phys. JETP **10**, 1179 (1960).

[3] A. I. Mukhin and B. Pontecorvo, JETP **31**, 550 (1956), Soviet Phys. JETP **4**, 373 (1957).

[4] W. R. Frazer and J. R. Fulco, Phys. Rev. Lett. **2**, 365 (1959).

[5] G. F. Chew and S. Mandelstam, Preprint.

[6] Galanin, Grashin, Ioffe, and Pomeranchuk, JETP **38**, 475 (1960), Soviet Phys. JETP **10**, in press.

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*Lowest order perturbation theory gives D phase shifts of opposite sign and gives a δ^+ phase (which is determined more reliably experimentally) amounting to a significant part of its experimental value. This circumstance indicates that $l = 2$ is apparently too small to apply the theory developed above. The author is grateful to Prof. Chew for indicating this possibility.