# *ENERGY DISTRIBUTIONS OF PRODUCTS OF REACTIONS IN WHICH SEVERAL PARTICLES ARE EMITTED*

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IN studying the energy distributions of products of reactions in which several particles emerge, which occur via a direct interaction with subsequent decay of the residual nucleus, it is necessary to take into account the interaction of the decay products in the final state. The question of the effect of final-state interaction on the shape of the energy spectra of reaction products was first considered by Migdal,[1] and later by various other authors.[2] The effect of interaction of the reaction products in the final state is that, independently of the mechanism of the decay itself, the interaction of the products from the decay of a system changes the magnitude and shape of the effective cross section, and the energy and angular distributions of the decay products. The final state interaction is assumed to be strong, of short range, and to affect the wave functions of the reaction products until they go beyond the range of the nuclear forces. For this treatment it is essential that the energies of the particles interacting in the final state be small.

In the present note it is shown that the spectra of products of a reaction from which several particles emerge can be explained quite well by computing in the Born approximation and taking account of the interaction of the particles in the final state. The wave function describing the relative motion of the particles interacting in the final state depends on their separation $\rho$, and must have a different form inside ($\rho < \rho_0$) and outside ($\rho > \rho_0$) the range of the nuclear forces. The parameters determining the nuclear interaction of the particles in the final state are found by matching the wave functions for $\rho < \rho_0$ and $\rho > \rho_0$ at the boundary of the region of action of the nuclear forces.[3]

As an example, we calculated the energy distributions of $He^3$ nuclei from the reaction T + d $\rightarrow$ $He^3$ + n + n for various angles, for a deuteron energy of 12 Mev, taking into account the interaction of the neutrons in the final state. This reaction can be regarded as a direct interaction accompanied by the decay of the residual system, since two neutrons cannot form a bound state. In writing the matrix element it was assumed that the reaction proceeds via a direct process of temporary capture, i.e., the deuteron pulls a proton out of the triton and there is a $\delta$-function interaction between this proton and each of the nucleons in the deuteron. Evidence that the reaction proceeds via a direct process comes from the anisotropic angular distribution of the $He^3$ nuclei, which are ejected preferentially forward.[4]

The computation of the energy distributions of the $He^3$ nuclei for different angles of emergence was made using the formula

$$d\sigma/dEd\vartheta \sim |H_{ba}|^2 f(E),$$

where f(E) is the density of states of $He^3$, while $H_{ba}$ is the matrix element for the transition including the interaction of the two neutrons in the final state, which was taken to have the form of a square well with radius $\rho_0 = 2.8 \times 10^{-13}$ cm.

The wave function describing the internal state of the potentially scattering neutrons, has the form

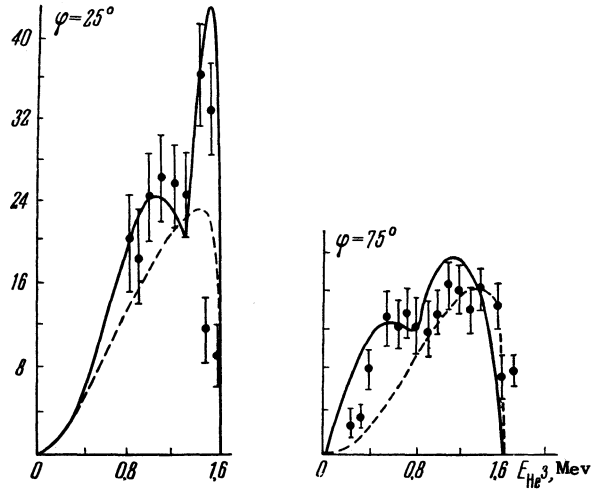
$$\psi(\rho) = e^{iq\rho} + f(\theta)\, e^{-iq\rho}/\rho,$$

for $\rho > \rho_0$, where q is the wave vector of the relative motion of the two neutrons and $f(\theta)$ is the amplitude of the scattered wave. For an incident deuteron energy of 12 Mev, the energy of relative motion of the two neutrons does not exceed 1.6 Mev, so that the only contribution to the differential cross section comes from the S-wave part of the partial wave expansion of the wave function for the relative motion of the two neutrons. The spin function for two neutrons must be antisymmetric (s = 0) because of the Pauli principle. For the S-state, the radial wave function of the relative motion of the two neutrons for $\rho > \rho_0$ has the form

$$\psi^{(1)}(\rho) = \frac{i\sqrt{\pi}}{q\rho}(e^{-iq\rho} - e^{iq\rho}) + \sqrt{4\pi}a\rho^{-1}e^{-iq\rho},$$

where $a = -(\alpha - iq)^{-1}$ is the neutron-neutron scattering length in an S-state. The interaction energy of the neutrons in our case is 70 kev. Inside the region of interaction, $\rho < \rho_0$, the radial part of the wave function for the relative motion of the two neutrons has the form $\psi^{(2)}(\rho) = A \sin k'\rho$, where A is an arbitrary constant and k' is the wave vector of the relative motion of the two neutrons within the region of action of the nuclear forces. A and k' are found by matching the functions $\psi^{(1)}$ and $\psi^{(2)}$ at the boundary of the region of action of the nuclear forces.

The figure shows the energy distributions of $He^3$ nuclei at angles of 25 and 75° in the center-of-




mass system, computed including (solid curve) and omitting (dashed curve) the interaction of the neutrons in the final state, and the experimental energy distribution[4] of the $He^3$ nuclei (the points on the figure). As one sees from the figure, the energy distributions of $He^3$ nuclei, computed including the potential scattering of the neutrons in the final state, give a good description of the experimental results.

In conclusion it should be mentioned that, on the basis of these considerations, one can not only explain the energy distributions of products of reactions which lead to emission of several particles, but one also gets a value for the parameters of the interaction of these particles in the final state.

[1] A. B. Migdal, JETP **28**, 3 (1955), Soviet Phys. JETP **1**, 2 (1955).

[2] K. Brueckner, Phys. Rev. **82**, 598 (1951); K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[3] V. V. Komarov and A. M. Popova, JETP **36**, 1574 (1959), Soviet Phys. JETP **9**, 1118 (1959).

[4] Brolley, Hall, Rosen, and Stewart, Phys. Rev. **109**, 1277 (1958).

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## *ISOTHERMAL DISCONTINUITIES IN MAGNETOHYDRODYNAMICS*

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IN magnetohydrodynamics, under the condition that

$$\beta_m/\beta_T \ll \gamma M_m^2 u_2 [(1/\gamma M_1^2 + 1/2 M_m^2 + 1) u_2 + 3/2 M_m^2]/(\gamma+1), \quad (1)$$

the principal role in the diffusion of the shock front is played by thermal conductivity, while magnetic viscosity can be neglected. Here and below we use the dimensionless quantities

$$u = v/v_1, \quad M_1^2 = v_1^2/a_1^2, \quad M_m^2 = v_1^2/a_m^2;$$

$$\beta_m = \nu_m/v_1 l_1, \quad \beta_T = \chi/v_1 l_1$$

where

$$a_m^2 = H_1^2/4\pi\rho_1, \quad a_1^2 = \gamma p_1/\rho_1.$$

Here, $\nu_m = c^2/4\pi\sigma$ is the magnetic viscosity, $\chi$ the coefficient of temperature conductivity, and $l$ the mean free path of the ion. The system of coordinates moves with the velocity of the wave in the direction of its propagation (for example, from right to left). The remaining notation is universal in magnetohydrodynamics.

Account of thermal conductivity alone leads to the appearance of an isothermal discontinuity. It is well known that in the absence of a magnetic field, a gradual change in the hydrodynamic quantities takes place only at gas velocities smaller than $u_2 = (\gamma+1)/(3\gamma-1)$ by $+\infty$, whereas in this case $M_1^2 = (3\gamma-1)/\gamma(3-\gamma)$ (see, for example, reference 1). We call the velocity $u_2 = u_l$, for which the isothermal discontinuity appears, the limiting velocity. If the magnetic field is not equal to zero, then the limiting velocity can vary from $u_l = (\gamma+1)/(3\gamma-1)$ to 1, depending on the values of $M_1^2$ and $M_m^2$. These three quantities are related in the following fashion:

$$M_1^2 = 2[\gamma - (2-\gamma)u_l]/\gamma[(5\gamma-7)u_l^2 + (5-\gamma)u_l - (3\gamma-1)u_l^3 - (\gamma-1)],$$

$$M_m^2 = [\gamma - (2-\gamma)u_l]/u_l^2[(3\gamma-1)u_l - \gamma - 1]. \quad (2)$$

The region in front of the isothermal discontinuity is especially clearly seen in the diagram of $M_1^2$, $M_m^2$. The connection between $u_2$, $M_1^2$ and $M_m^2$ has the form

$$M_m^2 = (\gamma u_2 + 2 - \gamma)/[(\gamma+1)u_2^2 - (\gamma-1)u_2 - 2u_2/M_1^2]. \quad (3)$$

In shock waves with parameters taken from the shaded region, the changes of all variables take