due to the fact that the probability of registering showers whose axes pass outside the center group of the counters of the array is considerably reduced, because of the conditions of anticoincidence of the discharges in the central group of counters with the discharges in any peripheral group.

Numerical calculations of the number of registered showers were performed under the assumption that the integral spectrum of extensive atmospheric showers has the form $f(>N) = A/N^{1.45}$, and the function of lateral distribution of the charged particles does not depend on the number of particles in the shower and corresponds to the experimental data of Abrosimov, Goryunov, et al.[6] The results of calculations for the counter areas $\sigma = 0.4\,m^2$ are given in Fig. 2. To reconcile the calculated number of showers with the experimentally observed one (H = 200 m above sea level) it is necessary to assume in the foregoing spectrum a value $A = 9 \times 10^4\,m^{-2}hr^{-1}$, which differs somewhat from the corresponding value given by Norman[7] ($1.15 \times 10^5\,m^{-2}hr^{-1}$), but is in better agreement with the data obtained by individual investigations of extensive atmospheric showers.[3]

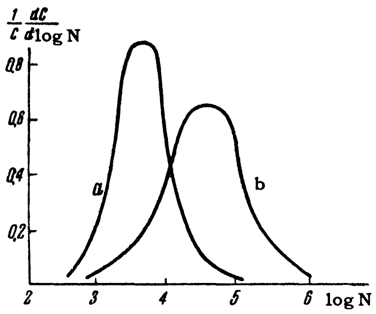

FIG. 2. a – shower spectrum by number of particles N, registered by the array at $\sigma = 0.4$ m²; b – number of showers causing coincidence of discharges in counters of the same area.

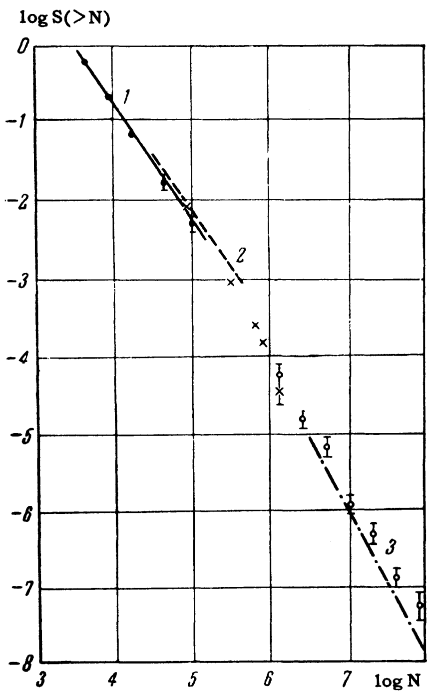

FIG. 3. Intensity of extensive atmospheric showers S (in $m^{-2}hr^{-1}$) with different number of particles N: 1 – data of present investigation (H = 200 m), 2 – data of reference 7 (H = 60 m), 3 – data of reference 5 (H = 60 m), × – data of reference 3 (H = 200 m), o – data of reference 4 (H = 180 m).

Comparison of the shower spectrum by number of particles which we obtained with data of other authors is given in Fig. 3.

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INVERSE DISPERSION RELATIONS FOR PHOTOPRODUCTION OF PIONS ON NUCLEONS

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It has been emphasized by Blank and Shirkov[1] that it is inconvenient to use conventional ("direct") dispersion relations when the imaginary part of the amplitude for some process becomes larger than the real part since one then deals with a small integral of a large (and generally speaking, alternating in sign) quantity. To overcome this difficulty Blank and Shirkov proposed "inverse" dispersion relations and derived them explicitly for the pion-nucleon scattering process. In contrast to direct dispersion relations the inverse relations express the imaginary part of the amplitude in terms of a Cauchy integral over the real part of the same amplitude.

The aim of this note is the construction of inverse dispersion relations for pion photoproduction on nucleons.* Direct dispersion relations for pion photoproduction on nucleons were obtained by several authors.[4-8] In the following we make use of the notation and the results of Logunov, Tavkhelidze and Solov'ev.[8]

Using the same method as Blank and Shirkov did, we obtain inverse dispersion relations in the following form (for the case when the nonphysical region is contiguous with the continuous spectrum region):

$$A_j^i(E, \mathbf{p}^2) = \frac{1}{\pi} \ln \frac{E+E_n}{E-E_n} D_j^i(E, \mathbf{p}^2) - \frac{2}{\pi} \mathrm{P} \int_{E_n}^{\infty} \frac{dE' \eta'_{ij}}{E'^2 - E^2} \left\{ D_j^i(E', \mathbf{p}^2) + \frac{1}{\pi} A_j^i(E', \mathbf{p}^2) \ln \frac{E'+E_n}{E'-E_n} \right\} - \frac{1}{E^2 - E_p^2} (\eta'_{ij})_p \ln \frac{E_n+E_p}{E_n-E_p} \frac{M}{p_0} \frac{2}{\pi} \varepsilon_{ij}. \tag{1}$$

Here

$$(\eta'_{ij})_p = \begin{vmatrix} E & E & E_p & E \\ E & E & E_p & E \\ E_p & E_p & E & E_p \end{vmatrix},$$

$$\|\varepsilon_{ij}\| = \begin{vmatrix} \frac{f}{\mu}\frac{e}{2p_0E_p} & \frac{f}{\mu}(\mu'_p - \mu_n) & -\frac{f}{\mu}(\mu'_p - \mu_n) & -\frac{ef}{4\mu} - \frac{Mf}{2\mu}(\mu'_p - \mu_n) \\ \frac{f}{\mu}\frac{e}{2p_0E_p} & \frac{f}{\mu}(\mu'_p + \mu_n) & -\frac{f}{\mu}(\mu'_p + \mu_n) & -\frac{ef}{4\mu} - \frac{Mf}{2\mu}(\mu'_p + \mu_n) \\ \frac{f}{\mu}\frac{e}{2p_0E_p} & -\frac{f}{\mu}(\mu'_p - \mu_n) & \frac{f}{\mu}(\mu'_p - \mu_n) & \frac{ef}{4\mu} + \frac{Mf}{2\mu}(\mu'_p - \mu_n) \end{vmatrix},$$

$$p_0 = \sqrt{\mathbf{p}^2 + M^2}, \qquad \mathbf{p}^2 = \mu^2 M / 4(M+\mu), \qquad E_n = (\mathbf{p}^2 + \mu^2/4)/p,$$

e, f are the electromagnetic and mesonic coupling constants; μ'_p, μ_n are the anomalous magnetic moments of the proton and neutron; M, μ are the nucleon and meson masses. Although the large function A appears under the integral sign in the right hand side of the expression (1) its contribution to the integral (for high energy photons) should generally speaking be small because it is multiplied by the logarithm.† Unfortunately at this time the experimental data needed to carry out a detailed estimate are not available.

For practical purposes it is convenient to have the inverse dispersion relations expressed in the barycentric frame of reference. We shall not write them out since they can easily be obtained from the direct dispersion relations.

Inverse dispersion relations for pion photoproduction may be written in another form[9] which differs from the preceding in that the imaginary part does not appear under the integral sign.

For the case when the nonphysical region is contiguous with the continuous spectrum region they are given by

$$\mathrm{Im}\left[\frac{t_j^i(E, \mathbf{p}^2)}{\sqrt{E^2 - E_n^2}}\right] = -\frac{2}{\pi} \mathrm{P} \int_{E_n}^{\infty} \frac{\mathrm{Re}\, t_j^i(E', \mathbf{p}^2)\, \eta_{ij}\, dE'}{(E'^2 - E^2)\sqrt{E^2 - E_p^2}} - \frac{2}{E_p^2 - E^2} (\eta_{ij})_p \varepsilon_{ij} \frac{M}{p_0} \frac{1}{\sqrt{E_n^2 - E_p^2}}. \tag{2}$$

In order to obtain these relations in the barycentric frame[8] from the direct dispersion relations it is necessary to multiply the one-nucleon term by

$$[(2M+1)(2M+1-4M\nu_1)]^{-1/2}$$

and replace in the left hand side $\mathrm{Re}\, U_j^i(W, \nu_1)$ by

$$\frac{-\{\mathrm{Im}\, U_j^i(W, \nu_1)\}}{\sqrt{W^2 - (M+1)^2}\sqrt{W^2 - M^2 + 2M + 1 - 4M\nu_1}}.$$

and under the integral signs $\mathrm{Im}\, U_j^i(W', \nu_1)$ by

$$\frac{\{\mathrm{Re}\, U_j^i(W', \nu_1)\}}{\sqrt{W'^2 - (M+1)^2}\sqrt{W'^2 - M^2 + 2M + 1 - 4M\nu_1}}.$$

Lastly we observe that in expression (2), in contrast to expression (1), there appears under the integral sign a factor $\sim 1/E$. This circumstance may turn out to be useful if it is necessary to discard the high energy part of the integral.

*For the analysis of experimental data on pion photoproduction using direct dispersion relations, see references 2 and 3.

†An analogous situation also occurs for inverse dispersion relations for pion-nucleon scattering[1] where estimates indicate that the contribution of A to the integral does not exceed 20%.

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GALVANOMAGNETIC PHENOMENA IN INDIUM AND ALUMINUM

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THE existing data on galvanomagnetic phenomena in indium and aluminum[1-7] refer to specimens in which the resistivity decreases less than a thousand-fold between room temperature and 4.2° K. We had at our disposal metals of appreciably greater purity which enabled us to reach a region of larger effective fields[8] than in previous investigations. Besides increasing the range, a study of aluminum in large effective fields is of interest in connection with the results of Lüthi and Olsen.[7]

The indium specimen was a single crystal, 1.84 mm in diameter and 12.55 mm between the potential leads. The aluminum specimen was a polycrystalline strip, 0.27 mm thick, 3 mm broad, and 36 mm between the leads. The temperature dependence of resistivity is shown in the table.

The measurements were made with the current normal to the magnetic field. A rotation diagram was taken for indium of the resistance change and Hall effect, and the measurements showed that the Hall constant is isotropic and is independent of field in the range $10 - 28 \times 10^3$ oe; its value is

T, °K	$[r_0(T)/r_0(273)] \cdot 10^4$	
	In	Al
20.4		9.6
4.2	0.8	4.3
2.2		4.3

$R = 1.5 \times 10^{-3}$ cgs magnetic units. The ratio of Hall field, E_y, to the field in the current direction, E_x, increases linearly with magnetic field, as is shown in Fig. 1 (curve 1). The Hall field is nearly 20 times greater than the field in the current direction at the largest magnetic fields.

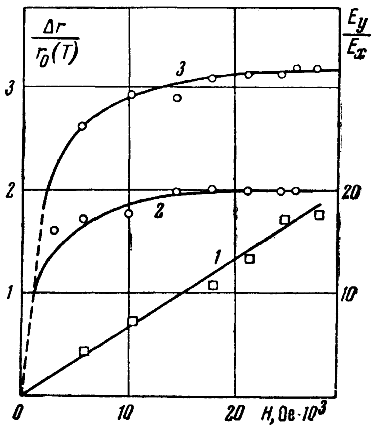

FIG. 1. Hall effect and magnetoresistance in In, T = 4.2°K. Curve 1 – E_y/E_x; Curve 2 – $\Delta r/r$ for the direction corresponding to the minimum effect; Curve 3 – $\Delta r/r$ for the direction for maximum effect.

The anisotropy of the relative change of resistance was found to be small, the greatest difference from the mean value not exceeding 25%. Curves 2 and 3 of Fig. 1 show the resistance change in a magnetic field for directions which show the maximum and minimum effects. It can be seen that in high fields the resistance tends to a limiting value. The results agree with those of other authors.[2,3,4]

The results for aluminum are shown in Fig. 2.

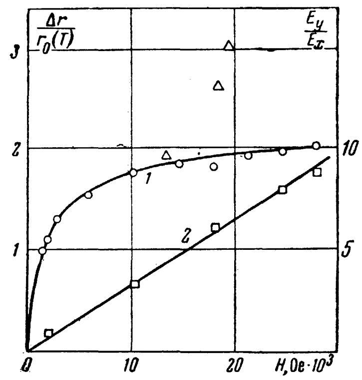

FIG. 2. Hall effect and magnetoresistance in Al, T = 4.2°K. Curve 1 – $\Delta r/r$; Curve 2 – E_y/E_x; Δ – data of Lüthi and Olsen.

Here again we have a linear increase of E_y/E_x (curve 2) with increasing magnetic field and the resistance (curve 1) tends to a limiting value.