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PRODUCTION OF PIONS IN p-d COLLISIONS AND MOTION OF NUCLEONS INSIDE THE NUCLEUS

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It is shown that the energy dependence of the cross section for production of π mesons in nucleon-deuteron collisions and the energy spectra of the π mesons can be calculated rather accurately from the data on free nucleon-nucleon collisions. In the energy range considered from the threshold for meson production to ~ 700 Mev, the effect of nucleon binding results mainly in a change in the magnitude of the cross sections as a result of motion of the nucleons inside the deuteron. The effective momentum distribution of nucleons in the deuteron is found.

1. INTRODUCTION

THE study of reactions following from collisions of protons with deuterons is a convenient method of studying the proton-neutron interaction. In the energy region $\gtrsim$ 1000 Mev, lying substantially above the meson production threshold (280 Mev), this method has been successfully employed both for measuring the total p-n interaction cross section[1] and for obtaining information about π-meson production.[2] The effect of binding of nucleons in the deuteron at such high energies is not large and can easily be taken into account by introducing a small correction into the measured cross sections.[1,2]

The closer the energy is to threshold, the more important is the effect of binding of the nucleons in the deuteron on the π-meson production processes. The main effect of the binding in this energy region is the change in magnitude of the cross section because of the motion of the nucleons inside the nucleus. Among other effects, connected with the presence of the spectator nucleon, one should note: the mutual screening of the nucleons (which is small[1-3] on account of the large radius of the deuteron), reabsorption of the produced π meson by the nucleon pair, exclusion of a number of final states because of the Pauli principle, possible effects from interference of nucleon states, and contributions from reactions proceeding without breakup of the deuteron (the cross sections of these processes are very small[4]).

In order to obtain information in this energy region about π-meson production in p-n collisions from the data on p-d collisions, it is necessary, if only approximately, to take into account the effect of binding of the nucleons in the deuteron and, first of all, to try to evaluate the magnitude of the change in cross section coming from motion inside the nucleus (this variation is especially important near threshold, where the corresponding enhancement factor for the cross section goes to infinity). This problem will be considered below in the impulse approximation, using as example the production of π^0 mesons in p-d collisions:

$$p + d \rightarrow \pi^0 + \text{nucleons} \qquad (1)$$

which was studied in detail earlier.[5]

2. MOMENTUM DISTRIBUTION OF NUCLEONS IN THE DEUTERON

If we consider the deuteron to be two nucleons moving relative to each other, and neglect interference, then the total cross section σ_{pd} for reaction (1) can be written as

$$\sigma_{pd} = \int \{\sigma_{pn}[\eta_m(p_1, \mathbf{p}_2), \mathbf{p}_2] + \sigma_{pp}[\eta_m(p_1, \mathbf{p}_2), \mathbf{p}_2]\} F(\mathbf{p}_2)\, d\mathbf{p}_2. \qquad (2)$$

Here $F(\mathbf{p}_2)$ is the momentum distribution of nucleons in the deuteron, $\mathbf{p}_2$ their momentum in the center-of-mass system, η_m the maximum momentum of the produced π^0 meson, p_1 the momentum of the incident proton, σ_{pn} and σ_{pp} the cross sections for π^0 production in collisions of the incident proton with the neutron and proton of the deuteron. For small values of p_1, the dependence of σ_{pn} and σ_{pp} on $\eta_m(p_1, p_2)$ is most important, making it possible to simplify these functions by setting them equal to $k\sigma_{pn}[\eta_m(p_1, p_2)]$ (and analogously for σ_{pp}). The factor k so introduced takes into account all effects of binding other than the motion of the nucleons inside the deuteron. It is assumed that k changes slowly with energy.

Carrying out the integration in Eq. (2) over the unit vector $\mathbf{p}_2/p_2$, we obtain

$$\sigma_{pd} = \int k\,\{\sigma_{pn}(p_1, p_2) + \sigma_{pp}(p_1, p_2)\}\, F(p_2)\, p_2^2 dp_2. \qquad (3)$$

Here the cross sections $\sigma_{pn}(p_1, p_2)$ and $\sigma_{pp}(p_1, p_2)$ correspond to the reactions

$$pn \rightarrow pn\pi^0, \qquad (4)$$

$$pp \rightarrow pp\pi^0, \qquad (5)$$

occurring with the moving nucleons of the deuteron. Since the cross section for the reaction (5) is comparatively small,[6] the contribution from the second term in the sum (3) is small. The functions $\sigma_{pn}(p_1, p_2)$ and $\sigma_{pp}(p_1, p_2)$ entering into the relation (3) were calculated for a wide range of values of p_1 and p_2 on the "Ural" electronic computer. The experimental data of reference 6 was used to determine the functions $\sigma_{pp}(p_1, p_2)$. According to the phenomenological theory,[7] the cross section $\sigma_{pn}(\eta_m)$ should go as η_m^δ near threshold, where $3 < \delta < 4$. Calculations of $\sigma_{pn}(p_1, p_2)$ were carried out for $\delta = 3$ and $\delta = 4$. The integration in Eq. (3) was carried out for several types of momentum distribution $F(p_2)$ (several of these distributions are shown in Fig. 1). The dependence of σ_{pd} on the energy of the incident proton then found is shown on Fig. 2, where the various curves are compared with the energy dependence for the cross section of reaction (1) found experimentally.[5] Curve 4c on this figure was calculated from a momentum distribution of the Chew-Goldberger type (the so-called improved one):

$$F(p_2) \sim (\alpha^2 + p_2^2)^{-2}(\beta^2 + p_2^2)^{-2},$$

$$\beta = 2.5\alpha, \qquad \alpha = 190 \text{ Mev/c} \qquad (6)$$

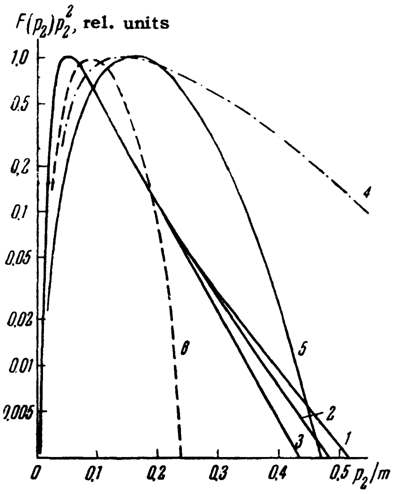

FIG. 1. Momentum distributions. 1, 2, 3 – Salpeter-Goldstein for Yukawa, exponential and Gaussian potentials. 4 – Chew-Goldberger (improved). 5, 6 – Gaussian, with dispersions $(\bar{p}^2/m)^{1/2} = 0.11$ and 0.06 with m the mass of the nucleon.

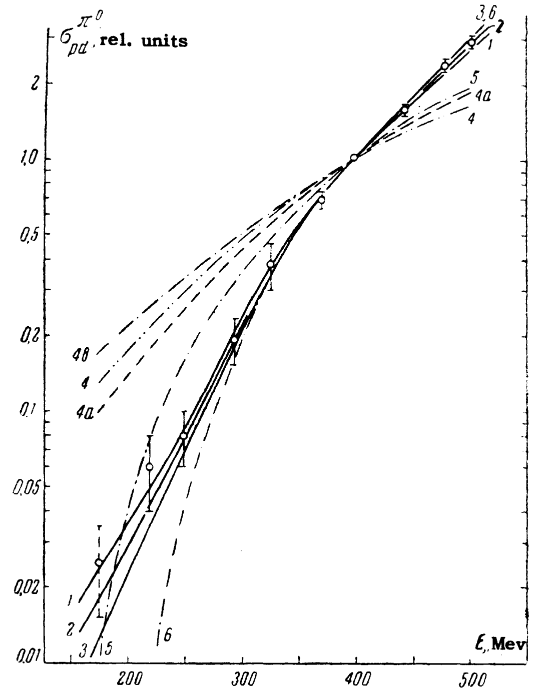

FIG. 2. Energy dependence of the total cross section for reaction (1). o – measured in Ref. 5. 1–6 – energy dependencies, calculated using the momentum distributions given in Fig. 1. 4a and 4c – see text. All data are normalized to unity at incident proton energy E = 400 Mev.

which has a long tail (see Fig. 1).

Together with taking the motion of the nucleons into account, an attempt was also made to estimate in an approximate way the effect of the Pauli principle by excluding from reaction (1) contributions from those collisions in which the secondary nucleons remain inside the Fermi sphere. The change in the size of the cross section because of the Pauli Principle was negligible in this energy range, as can be seen by comparing curve 4c in Fig. 2 with curve 4 which, in contradistinction to 4c, was calculated taking the Paul Principle into account. All of the other curves in Fig. 2 were also calculated with account of the Pauli Principle.

It has already been indicated above that the dependence of the cross section σ_{pd} on energy near threshold is determined mainly by the form of the momentum distribution, and is insensitive to the energy dependence of the cross sections for reactions (4) and (5). One can see this by comparing curves 4 and 4a on Fig. 2, which were calculated for the cases $\sigma_{pn} \sim \eta_m^3$ and η_m^4.

Curves 5 and 6 on Fig. 2 were calculated for a Gaussian distribution

$$F(p_2) \sim \exp(-p_2^2 / 2\bar{p}_2^2), \qquad (7)$$

which describes the momentum distribution in complex nuclei reasonably. A distribution of this type differs from the Chew-Goldgerger distribution in that it contains relatively few high momentum components (see Fig. 1). It can be seen from Fig. 2 that both the Gaussian and Chew-Goldberger distributions are in poor agreement with the experimental data[5] for the reaction (1).

It is possible to obtain good agreement with measured cross sections (see curves 1 — 3 on Fig. 2) if one uses the momentum distributions of Salpeter and Goldstein[8] for the deuteron (see Fig. 1). In the low-momentum region these distributions go approximately as

$$F(p_2) \sim (\gamma^2 + p_2^2)^{-2}, \qquad (8)$$

where $\gamma = 46$ Mev/c. The distributions 1 — 3 (calculated in reference 8 for Yukawa, exponential, and Gaussian potentials) differ, as can be seen from Fig. 1, only in the region of very high momenta. All of these are almost equally good in reproducing the experimental dependence of the cross section for reaction (1) on energy.

Thus, analysis of the energy dependence of the cross section for the reaction $p + d \rightarrow \pi^0 +$ nucleons near threshold shows that the momentum distribution of nucleons in the deuteron is described well by the curves from the Salpeter and Goldstein distributions. It should be remarked that the interpretation of results of measurements on the energy dependence, similar to those of reference 5, in the spirit of the impulse approximation, runs into essential difficulties, since large momenta of the nucleons in the nucleus correspond to small distances between them, where threefold interactions become important. Therefore, the momentum distributions given above should be considered as effective distributions, the knowledge of which makes it possible to take into account the effect of motion inside the nucleus on the magnitude of meson production cross sections of the type of reaction (1), but which may differ markedly from the true momentum distribution.

3. RECONSTRUCTION OF THE TOTAL CROSS SECTION FOR p-n INTERACTION

Using the effective momentum distribution obtained above for the deuteron, the integration in Eq. (3) can be carried out, leading to the following relation between the cross sections of reactions (1), (4) and (5)

$$\sigma_{pd} = k\,(g_{pn}\sigma_{pn} + g_{pp}\sigma_{pp}). \qquad (9)$$

Here $\sigma_{pn} = \sigma_{pn}(p_1, 0)$ and $\sigma_{pp} = \sigma_{pp}(p_1, 0)$ are the usual cross sections, and g_{pn} and g_{pp} are quantities characterizing the change in the cross sections from the motion in the nucleus; they depend only on p_1. In order to carry out such an integration it is necessary to know the energy dependence of the cross sections for reactions (4), (5). In essence, this problem can be solved by the method of successive approximations. However, because the momentum distribution $F(p_2)$ is not broad, the first approximation for the cross section $\sigma_{pn}^{(1)}$ is already sufficient to determine g_{pn}. As first approximation for $\sigma_{pn}^{(1)}$, the dependence η_m^3 was used near threshold and in the high-energy region one can take $\sigma_{pn}^{(1)} = \sigma_{pd} - \sigma_{pp}$. Above 600 Mev the rate of growth of $\sigma_{pn}^{(1)}$ decreases and for energies $\gtrsim 1000$ Mev, $\sigma_{pn}^{(1)} \approx$ const. The functions $\sigma_{pn}^{(1)}(p_1, p_2)$ so obtained and used to determine g_{pn} are given on Fig. 3. The functions $\sigma_{pp}(p_1, p_2)$ have an analogous form. The coefficients g_{pn} and g_{pp} obtained in the way described above are given on Fig. 4.

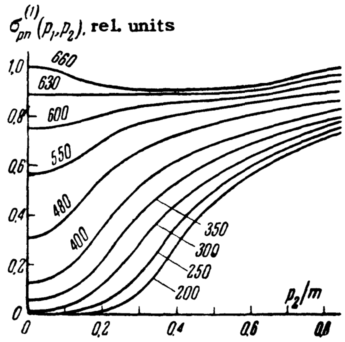

FIG. 3. Functions $\sigma_{pn}^{(1)}(p_1, p_2)$. The numbers near the curves indicate the corresponding values of the kinetic energy $E = \sqrt{p_2^2 + m^2} - m$.

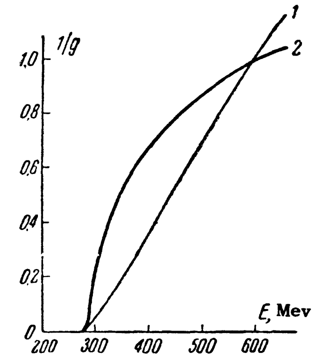

FIG. 4. Dependence of the coefficients g on energy. 1) $1/g_{pp}$; 2) $1/g_{pn}$.

Calculation of the coefficient k, which enters into the expression (9), is practically impossible because of inadequacy of contemporary nuclear theory. One can only say that this coefficient is near to unity at high energies, where $\sigma_{pd} \approx \sigma_{pn} + \sigma_{pp}$. The only factor entering into the coefficient k which one can calculate is the decrease in the cross section because of the mutual screening of the nucleons in the deuteron.[3] The corresponding correction is small (several per cent).

The coefficient k can be found empirically by comparing cross sections measured in proton and neutron beams of the same mean energy. When the incident particle is a neutron, the cross section for the deuteron is

$$\sigma_{nd} = k\,(g_{np}\sigma_{np} + g_{nn}\sigma_{nn}), \qquad (9')$$

which is analogous to Eq. (9), since, because of the charge symmetry of nuclear forces, $\sigma_{np} = \sigma_{pn}$, $\sigma_{nn} = \sigma_{pp}$, $g_{np} = g_{pn}$ and $g_{nn} = g_{pp}$. In order to determine k it is convenient not to use the cross sections, which are not measured with high accuracy, but rather the more accurate measurements of the ratios of cross sections $\alpha_p = \sigma_{pd}/\sigma_{pp}$ and $\alpha_n = \sigma_{nd}/\sigma_{np}$. In this notation

$$1/k = g_{pp}/\alpha_p + g_{pn}/\alpha_n. \qquad (10)$$

The values α_p and α_n were measured at 590 Mev:[5,6,9] $\alpha_p = 3.00 \pm 0.15$, $\alpha_n = 1.30 \pm 0.04$. From these, k (590) = 0.89 ± 0.03.

The quantity k can be determined at one other point, at 380 Mev where the cross sections for the reactions (1), (4) and (5) have been measured:[5,6,10]

$$k\,(380) = 0.72 \pm 0.16.$$

Comparison of the two values obtained gives reason to believe that the coefficient k is constant over the entire energy region from threshold to 600 Mev.

Using the values obtained for g_{pn}, g_{pp} and k it is possible to reconstruct the cross section for production of π^0 mesons in p-n collisions from the experimental data on the σ_{pd} and σ_{pp} cross sections:

$$\sigma_{pn} = \sigma_{pd}/kg_{pn} - \sigma_{pp}g_{pp}/g_{pn}. \qquad (11)$$

4. π-MESON SPECTRA

The energy spectra of π mesons produced in reactions of type 1 depend even more on the binding between the nucleons than does the magnitude of the total cross section σ. Even at high energies, they differ substantially from the spectra of π mesons produced in collisions of free nucleons[11,12] (see Figs. 5 and 6). In particular, the absence of a peak, which is so characteristic of collisions of free protons, is striking. Assuming as previously that the effect of binding leads, in the main, to a change in the magnitude of the differential cross section $d^2\sigma/d\Omega\,dE$ because of motion of the nucleons in the deuteron, it is possible to calculate the change in the form of the spectra by the same method as applied in the preceding paragraphs for obtaining the total cross sections. This calculation was carried out for the spectra of π^+ mesons produced in p-d collisions at 655 Mev, with the

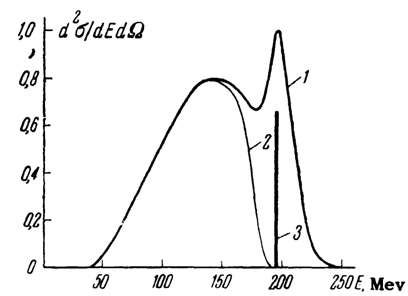

FIG. 5. Energy spectrum of π^+ mesons, produced in p-p collisions (in relative units). 1) measured by Helfer et al.[12] at angle ≈ 90° in the c.m.s. for proton energy 655 Mev. 2) π^+-meson spectrum from the reaction $p + p \rightarrow p + n + \pi^+$. 3) position of the peak corresponding to the reaction $p + p \rightarrow d + \pi^+$.

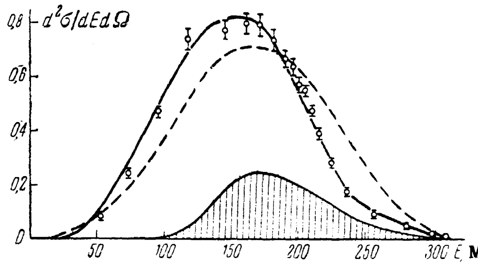

FIG. 6. Spectrum of π^+ mesons, produced in p-d collisions (in relative units). o – measured spectrum of π^+ mesons produced in p-p collisions in the deuteron[12] (obtained as the difference of π^+ and π^- spectra). The thick solid curve is this spectrum, calculated from the Salpeter-Goldstein momentum distribution. The dashed curve shows the spectrum calculated for a Gaussian distribution with dispersion $\sqrt{\overline{p_2^2}}/m = 0.06$. The thin curve is the spectrum of π^+ mesons from the reaction $p + p \rightarrow d + \pi^+$, transformed as a result of motion inside the deuteron. The resolution of the spectrometer[12] was taken into account in constructing these spectra; this only slightly changed the form of the spectra because of their large width.

object of comparing the results of the calculation with the experimental data* on the spectrum of π^+ mesons in the reaction $p + d \rightarrow \pi^+ +$ nucleons.[12] Calculations were carried out separately for the reactions $p + p \rightarrow d + \pi^+$ and $p + p \rightarrow p + n + \pi^+$ (plots of the spectra corresponding to these reactions are shown in Fig. 5). Functions $d^2\sigma/d\Omega\,dE\,(p_1, p_2)$, analogous to those entering into Eq. (3), were calculated using the experimental results for the energy dependence of the π^+-meson production cross section[13] and the spectra obtained in references 12 and 13. Integration over the momenta of the nucleons in the deuteron was carried out for the Salpeter-Goldstein momentum distribution.

The calculated spectra differ substantially from the spectra of π^+ mesons produced in p-p colli-

*I would like to take this opportunity to thank M. G. Meshchyeryakov and collaborators who kindly furnished me with the results of their measurements before publication.

sions. This is especially true of the peak (shaded region in Fig. 6) relating to the reaction $p + p \rightarrow d + \pi^+$, the width of which is increased as a result of motion of the nucleons up to 50% of the whole, making it practically unobservable in the spectrum of π^+ mesons produced in p-d collisions. At the same time, the relative contribution of the peak is somewhat reduced (by 15%) because the energy dependence of the cross section for the reaction $p + p \rightarrow d + \pi^+$ has a resonance character.

Comparison of the spectra of π^+ mesons produced in p-p collisions in the deuteron[12] with the calculated spectra shows (Fig. 6) that the form of the spectra for p-d collisions can be rather accurately predicted from the data on free p-p collisions. The form of the calculated spectrum depends strongly on the momentum distribution employed in the calculation. A π^+ spectrum, calculated in the same way as previously, but for the case of a Gaussian distribution with $\sqrt{\overline{p_2^2}}/m = 0.06$ is shown on Fig. 6. In spite of the fact that this distribution is rather close to the Salpeter-Goldstein ones (see Fig. 1), the corresponding spectra differ markedly. From this it follows that study of π-meson spectra in p-d collisions also makes it possible to obtain quantitative knowledge about the momentum distribution of nucleons in the deuteron.

The considerations above about the π-meson spectra can be turned around, i.e., from the spectrum of π mesons produced in p-d collisions, if measured with high accuracy, one can reconstruct the spectrum of π mesons produced in collisions of free nucleons. This is of particular interest in cases where direct investigation of the corresponding reaction by collisions between free nucleons would involve substantial experimental difficulties (such as, for example, the reaction $p + n \rightarrow p + p + \pi^-$).

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