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RADIATIVE CORRECTIONS TO PHOTOPRODUCTION AND SINGLE-PHOTON ANNIHILATION OF PAIRS

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General formulas have been obtained for radiative corrections to photoproduction and single photon annihilation of electron-positron pairs. Some limiting cases are considered.

1. An expression for radiative corrections to the differential cross section of pair production by a photon in a Coulomb field (photoproduction) can be obtained from the general formula for corrections to bremsstrahlung,[1,2] by making use of the well-known "substitution rule" (see, for example, reference 3, p. 162) which reduces in the given case to the substitution

$$p_1 \to -p_+, \qquad p_2 \to p_-, \qquad k \to -k, \tag{1}$$

where $p_1 = (\mathbf{p}_1, i\epsilon_1)$, $p_2 = (\mathbf{p}_2, i\epsilon_2)$ and $k = (\mathbf{k}, i\omega)$ are the initial and final momenta of the electron and the momentum of the photon in the bremsstrahlung; $p_+ = (\mathbf{p}_+, i\epsilon_+)$, $p_- = (\mathbf{p}_-, i\epsilon_-)$ and $k = (\mathbf{k}, i\omega)$ are the momenta of the positron, electron, and photon in photoproduction.

In addition to the substitution (1), it is necessary to take the following into account. The expression for the radiative corrections to the bremsstrahlung contains the parameter y, which is determined by the equation

$$4\sinh^2 y = \rho - \varkappa - \tau \tag{2}$$

and, in particular, the transcendental function

$$h(y) = y^{-1}\int_0^y u \coth u\, du. \tag{3}$$

A similar parameter will enter into the correction for photoproduction. In the case of bremsstrahlung, y is real, since

$$\rho - \varkappa - \tau = 2(\varepsilon_1\varepsilon_2 - \mathbf{p}_1\mathbf{p}_2) - 2 \geqslant 0. \tag{4}$$

For photoproduction we have, with account of (1),

$$\rho - \varkappa - \tau = -2(\varepsilon_+\varepsilon_- - \mathbf{p}_+\mathbf{p}_-) - 2 \leqslant -4. \tag{5}$$

This means that y becomes complex, and the integration in (3) is carried out in the complex plane.

In the case of photoproduction, we define a new parameter z by the relation

$$y = z - i\pi/2. \tag{6}$$

On the basis of (2), we obtain

$$4\cosh^2 z = \varkappa + \tau - \rho \geqslant 4, \tag{7}$$

i.e., z is real.

Carrying out the integration in (3) over the appropriate path (in this connection, see the paper by Harris and Brown[4]) and taking it into account that only the real parts of the corresponding functions enter into the cross section, we obtain as a result the following substitution rule:

$$yh(y) \to zg(z) \equiv \int_0^z u \tanh u\, du,$$

$$2yh(2y) \to 2zh(2z) - \pi^2/2 \equiv \int_0^{2z} u \coth u\, du - \pi^2/2,$$

$$(y \pm x)\, h(y \pm x) \to (z \pm x)\, g(z \pm x) \equiv \int_0^{z \pm x} u \tanh u\, du. \tag{8}$$

As a result of the changes, which are connected with (1), (6) and (8), the following result is obtained for the cross section of photoproduction:*

$$d\sigma = d\sigma_0 [1 - (e/2\pi)^2 \delta_R], \tag{9}$$

where $d\sigma_0$ is given by the Bethe-Heitler formula[5,6] and

$$\delta_R = 2W(x) + [U(\varkappa, \tau, \rho; -\varepsilon_+, \varepsilon_-) + U(\tau, \varkappa, \rho; \varepsilon_-, -\varepsilon_+)]/U_0, \tag{10}$$

$$W(x) = (1 - x\coth x)\left(1 - \frac{1}{3}\coth^2 x\right) - \frac{1}{9},$$

$$4\sinh^2 x = \rho, \tag{11}$$

*We shall use units in which $c = \hbar = m = 1$, $e^2/4\pi = 1/137$.

$$2U(\varkappa, \tau, \rho; -\varepsilon_+, \varepsilon_-) = -T^2 2z/\sinh 2z$$
$$+ U_0\{2(1-2z\coth 2z)\ln\lambda$$
$$+ 4z\coth 2z[h(2z) - g(z) - \pi^2/4z] + 2 - z\tanh z$$
$$+ (4+\rho)z/\sinh 2z + 2\tau M\} + (-\tau U_0 - S_1\partial/\partial\varkappa$$
$$+ S_5\partial/\partial\varkappa + S_6\partial/\partial\alpha + S_7\partial/\partial\rho)J(u^2) + (S_2\partial/\partial\varkappa$$
$$+ S_3\partial/\partial\alpha + S_4\partial/\partial\rho)J(u) + (\tau U_0 + S_1\partial/\partial\varkappa + S_8\partial/\partial\alpha$$
$$+ S_9\partial/\partial\rho)J(u^2V) + (S_{10} - S_{11}\varkappa\partial/\partial\varkappa$$
$$+ {}^1/_2 S_{12}\varkappa^2\partial^2/\partial\varkappa^2)J(isu) + (S_{13} - S_{14}\varkappa\partial/\partial\varkappa$$
$$+ {}^1/_2 S_{15}\varkappa^2\partial^2/\partial\varkappa^2)J(isuv_1). \qquad (12)$$

The expressions J(...) appearing here are expressed by Eqs. (A 5) and (37)—(40) of reference 1, with the substitution $y\coth y \rightarrow z\tanh z$, and

$$L = 2\left(z^2 - x^2 - \frac{\pi^2}{4}\right) + F(\varkappa - 1) - F(-1),$$
$$M = 2\sinh^{-1}2z\{z\ln\varkappa + zg(z) - (z+x)g(z+x) - (z-x)g(z-x)\},$$
$$N = \int_0^1 \frac{dv}{1+(1-v)v\rho - \varkappa v}\ln\left|\frac{1+(1-v)v\rho}{\varkappa v}\right|,$$
$$F(x) = \int_0^1 \frac{\ln|1+v|}{v}dv \qquad (13)$$

with the new values of the parameters:

$$\varkappa = 2\omega(\varepsilon_- - p_-\cos\theta_-),\quad \theta_- = \widehat{\mathbf{kp}_-},$$
$$\tau = 2\omega(\varepsilon_+ - p_+\cos\theta_+),\quad \theta_+ = \widehat{\mathbf{kp}_+},$$
$$\alpha = \varkappa + \tau,\quad \omega = \varepsilon_+ + \varepsilon_-,$$
$$\rho = -2 + \varkappa + \tau - 2(\varepsilon_+\varepsilon_- - p_+p_-\cos\theta),$$
$$\theta = \widehat{\mathbf{p}_+\mathbf{p}_-},\quad p = |\mathbf{p}|, \qquad (14)$$

U_0, T^2 and S_i are obtained from the corresponding expressions in the case of bremsstrahlung [Eqs. (14) and (41) of reference 1] by direct substitution of (1).

It is necessary to add the cross section of pair photoproduction, which is accompanied by radiation of a soft photon, to the cross section (9); the energy of this photon does not exceed a certain $\Delta E \ll m$, which is determined by the accuracy of the measurements. This cross section, integrated over the momentum of the soft photon, is equal to[3]

$$d\sigma_D = d\sigma_0(e/2\pi)^2\delta_D, \qquad (15)$$
$$\delta_D = \frac{1}{4\pi}\int_0^{\Delta E}\frac{dk_1}{\sqrt{k_1^2+\lambda^2}}\left(\frac{p_-}{p_-k_1} - \frac{p_+}{p_+k_1}\right)^2 = 2(1-2z\coth 2z)\ln\frac{\lambda}{2\Delta E}$$
$$+ \frac{1}{2}\left[\frac{1}{v_+}\ln\frac{1+v_+}{1-v_+} + \frac{1}{v_-}\ln\frac{1+v_-}{1-v_-} - Y(z, \varepsilon_+, \varepsilon_-)\cosh 2z\right],$$
$$v_+ = p_+/\varepsilon_+,\qquad v_- = p_-/\varepsilon_-,$$
$$Y(z, \varepsilon_+, \varepsilon_-) = \int_{-1}^{1}\frac{dv}{\cosh^2 z - v^2\sinh^2 z}\frac{\varepsilon_v}{p_v}\ln\left|\frac{\varepsilon_v + p_v}{\varepsilon_v - p_v}\right|,$$
$$2\varepsilon_v = \varepsilon_+ + \varepsilon_- + (\varepsilon_+ - \varepsilon_-)v,\quad p_v^2 = \varepsilon_v^2 + v^2\sinh^2 z - \cosh^2 z.$$

In the total cross section, which has the form

$$d\sigma = d\sigma_0\{1 - (e/2\pi)^2(\delta_R - \delta_D)\}, \qquad (17)$$

the term containing the "mass" of the photon λ disappears.

2. From the principle of detailed balance, it is easy to obtain the result that the radiative corrections to the single photon pair annihilation in a Coulomb field are described by the same expressions as the corrections to the photoproduction, if we mean by p_+, p_-, k the momenta of the annihilating pair and the radiated photon.

3. Let us consider some limiting cases of radiative correction to photoproduction:

1) Photoproduction at threshold ($p_+, p_- \ll 1$)

$$\delta_R = -\frac{\pi^2}{|v_+ - v_-|} + a + b\frac{\cos\theta - \cos\theta_+\cos\theta_-}{\frac{p_+}{p_-}\sin^2\theta_+ + \frac{p_-}{p_+}\sin^2\theta_-} + O(p_+, p_-), \qquad (18)$$
$$\delta_D = O(p_+, p_-),$$
$$a = \frac{65}{18} - \frac{13}{48}\pi^2 + \frac{5}{8}F(3) + \frac{7}{4}\Delta + \frac{29}{9}\ln 2 - \frac{53}{12}\sqrt{2}\ln(1+\sqrt{2}) - \frac{3}{2}\ln^2(1+\sqrt{2}) = 2,4;$$
$$b = 3 - \frac{5}{24}\pi^2 + \frac{1}{4}F(3) - \Delta - 2\ln 2 + 2\sqrt{2}\ln(1+\sqrt{2}) - \ln^2(1+\sqrt{2}) = -1,0;$$
$$\Delta = F\left(\frac{-3}{2+\sqrt{2}}\right) + F\left(\frac{-3}{2-\sqrt{2}}\right) - F\left(\frac{-1}{2+\sqrt{2}}\right) - F\left(\frac{-1}{2-\sqrt{2}}\right) - F(-3) + F(1) = 2.8. \qquad (19)$$

The first term in (18), which diverges as the relative velocity approaches zero, arises as a result of the fact that account of the interaction of the produced electron and positron is carried out automatically in radiative corrections in the first Born approximation. For a more detailed discussion, see reference 4.

2) Relativistic case of equal energies ($\epsilon_+ = \epsilon_- \gg 1$) at small angles ($\theta^2, \theta_+^2, \theta_-^2 \ll 1/\epsilon^2$). In this case, the momentum transferred to the external field is small ($\rho \ll 1$).

$$\delta_R = -\pi^2/2z + \frac{13}{2} - \pi^2/4 + O(\varepsilon^2\theta^2), \qquad (20)$$
$$\delta_D = O(\varepsilon^2\theta^2\ln\varepsilon). \qquad (21)$$

The small invariant quantity 2z in the center-of-mass system of the pair has the meaning of relative velocity of the electron and positron. The term $-\pi^2/2z$ is completely analogous to the first term of Eq. (18) in the threshold case.

3) Ultrarelativistic case. Assuming the angles to be sufficiently large, we write

$$\ln(\alpha - \rho),\quad \ln\varkappa,\quad \ln\tau,\quad \ln\rho \gg 1,$$

However,

$$\ln\frac{\alpha-\rho}{\varkappa},\quad \ln\frac{\alpha-\rho}{\tau},\quad \ln\frac{\rho}{\varkappa},\quad \ln\frac{\rho}{\tau} \sim 1.$$

It is not difficult to prove that in this case δ_R and δ_D can be obtained directly from the asymptotic expressions (5) and (6) of reference 2* by the substitution

$$y = \frac{1}{2}\ln(\rho - \alpha) \to z = \frac{1}{2}\ln(\alpha - \rho), \ \varepsilon_1 \to \varepsilon_+, \ \varepsilon_2 \to \varepsilon_-.$$

The equations thus obtained can be written in the form:

$$\delta_R = 2(1 - \ln 2p_+p_-)\ln\lambda + \ln 2p_+p_-\left(\frac{1}{2}\ln 2p_+p_- - \frac{13}{6}\right) + O(1), \quad (22)$$

$$\delta_D = (1 - \ln 2p_+p_-)(2\ln\lambda + \ln[\varepsilon_+\varepsilon_-/(\Delta E)^2]) + \frac{1}{2}\ln^2 2p_+p_- + O(1), \qquad p_+p_- = \varepsilon_+\varepsilon_- - \mathbf{p}_+\mathbf{p}_-. \quad (23)$$

In the limiting case under consideration, the radiative corrections to photoproduction were calculated earlier by Drell et al.[7] Our result (22) coincides with their formula (27), while (23) differs somewhat from their Eq. (28) which is brought about by a different determination of ΔE in the reference mentioned.[7]†

*We note that an error in sign occurred in (5) of reference 2 and in (56) of reference 1 in front of the term 4x/3.

†We note that the expression obtained in reference 8 for the ultrarelativistic case of radiative corrections to the bremsstrahlung does not coincide with the corresponding formula (5) of reference 2 [or (56) of reference 1] and, consequently, does not agree (for corresponding substitution of parameters) with the result (27) of reference 7.

4. Analogous limiting cases for single photon pair annihilation are described by the same equations (18) – (23) with corresponding change in the meaning of the parameters.

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