INTERFERENCE OF FORM FACTORS IN LEPTONIC DECAY OF HYPERONS

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It is possible to deduce the energy dependence of the coefficients in front of the products of various form factors in the expression for the probability of leptonic decay of hyperons, and also to predict when part of these coefficients vanish by making use of the invariance of the four-fermion interaction matrix element under some formal transformations, no straightforward calculations being required for this purpose.

1. FORM FACTORS

THE matrix element of the four-fermion interaction H corresponding to the leptonic decay of hyperons

$$Y \to N + l + \bar{\nu}, \qquad (1)$$

have, in the most general case, the form (we shall omit the bar over $\bar{\nu}$ in the indices)

$$\langle N l \bar{\nu} | H | Y \rangle = \sum [(\bar{u}_N \Gamma_j^{(+)} u_Y)(\bar{u}_l O_j u_\nu) + (\bar{u}_N \Gamma_j^{(-)} u_Y)(\bar{u}_l O_j \gamma_5 u_\nu)], \qquad (2)$$

where u_k is a spinor of the k-th particle, and

$$O_j = 1,\ \gamma_5,\ \gamma_\mu,\ i\gamma_\mu\gamma_5,\ \tfrac{1}{2}\sigma_{\mu\nu} \qquad (j = S, P, V, A, T). \qquad (3)$$

Because of the presence of virtual strong interactions, the vertices $\Gamma_j^{(\pm)}$ can have a complicated structure. The most general expression for $[q_\mu \equiv (p_Y - p_N)_\mu]$ follows[1] from relativistic invariance:

$$\Gamma_S^{(\pm)} = C_S^{(\pm)}, \quad \Gamma_P^{(\pm)} = C_P^{(\pm)}\gamma_5;$$

$$\Gamma_V^{(\pm)} = C_V^{(\pm)}\gamma_\mu + B_V^{(\pm)}\sigma_{\mu\nu}q_\nu + iD_V^{(\pm)}q_\mu;$$

$$\Gamma_A^{(\pm)} = iC_A^{(\pm)}\gamma_\mu\gamma_5 + iB_A^{(\pm)}\sigma_{\mu\nu}q_\nu\gamma_5 - D_A^{(\pm)}q_\mu\gamma_5;$$

$$\Gamma_T^{(\pm)} = C_T^{(\pm)}\sigma_{\mu\nu} + B_T^{(\pm)}(\gamma_\mu q_\nu - \gamma_\nu q_\mu) + D_T^{(\pm)}\tfrac{1}{2}(\gamma_\mu \hat{q}\gamma_\nu - \gamma_\nu \hat{q}\gamma_\mu)$$
$$+ iF_T^{(\pm)}(p_{Y\mu}p_{N\nu} - p_{Y\nu}p_{N\mu} - \varepsilon_{\mu\nu\alpha\beta}p_{Y\alpha}p_{N\beta}). \qquad (4)$$

The form factors $C_j^{(\pm)}$, $B_j^{(\pm)}$, $D_j^{(\pm)}$, $F_T^{(\pm)}$ are functions of the invariant

$$Q^2 = -(p_Y - p_N)^2. \qquad (5)$$

In the work of Weinberg,[1] the coefficient of $F_T^{(\pm)}$ has a somewhat different form; however, the transformation properties of the expression described in (4) are much simpler.

Calculation of the energy correlation and the asymmetry of the decay (1) are given in reference 2, and also of the spectrum and polarization of the nucleons, either with $\Gamma_V^{(\pm)}$ and $\Gamma_A^{(\pm)}$ left in (2) (since $C_S^{(\pm)}$ is equivalent to $-D_V^{(\pm)}/m_l$, while $C_P^{(\pm)}$ corresponds to $D_A^{(\pm)}/m_l$, then essentially only the tensor vertex $\Gamma_T^{(\pm)}$ is not considered in this calculation), or with all five variants considered, but without form factors. Calculations with all (including the tensor) vertices would be too cumbersome; however, certain information on the interference of the form factors can be obtained without calculation by studying the behavior of the matrix element (2) — (4) in certain formal transformations which are considered in Sec. 2. In this case, as is shown in Sec. 3, a prediction of the energy dependence of the coefficients in front of the different products of form factors is possible, as is also the vanishing of part of these coefficients. The absence of the interference of certain variants or of form factors, for example, in the ordinary β decay, is a well known fact; its explanation from the general formal point of view has a certain interest. Moreover, the relations that have been obtained can serve as a control of the results of calculations.

2. TRANSFORMATIONS

In calculations with the matrix element (2), it is necessary to calculate an expression having the structure

$$\sum_{r,s} R_F R_G (\bar{u}_{2\lambda}^s F_{\lambda\sigma} u_{1\sigma}^r)(\bar{u}_{1\alpha}^r G_{\alpha\beta}^{+} u_{2\beta}^s), \qquad (6)$$

where $F_{\lambda\sigma}$ and $G_{\alpha\beta}$ are any of the spinor operators (matrices) written down in (4); R_F and R_G are the form factors standing in front of them in (4); u_1 and u_2 are spinors, either Y and N, or $\bar{\nu}$ and l. As a rule, $G^+ = \pm G$. It seems that the expression (6) should be multiplied by $(\bar{u}_4 F' u_3) \times (\bar{u}_3 G'^+ u_4)$; however, inasmuch as the spinors u_3 and u_4 do not take part in the transformations (12) (see below), this factor is not essential for our consideration.

Strictly speaking, one should compute not the expression (6), but

$$R_F R_G \operatorname{Sp}\{F\rho_1 G\rho_2\}, \tag{7}$$

because only in the case of a pure state is the density matrix $\rho_{\sigma\alpha}$ expressed in terms of u_α:

$$\rho_{\sigma\alpha} = \sum_r u^r_\sigma \bar{u}^r_\alpha, \tag{8}$$

and (7) reduces to (6). In the general case, the density matrix of the i-th particle ρ_i, as is well known,[3] has the form

$$\rho_i = (1 + i\gamma_5 \hat{\zeta}'_i)(m_i - i\hat{p}_i)/4E_i, \tag{9}$$

while if we introduce a unit vector $\mathbf{n}_i \equiv \mathbf{p}_i/|\mathbf{p}_i|$ in the direction of the momentum $\mathbf{p}_i$, and the vector $\boldsymbol{\zeta}_i$ of the spin of the i-th particle in its rest system, then

$$\zeta'_i = (\zeta_i \mathbf{n}_i)\mathbf{n}_i E_i/m_i + [\zeta_i - (\zeta_i \mathbf{n}_i)\mathbf{n}_i]; \quad \zeta'_{i\mu} p_{i\mu} = 0, \tag{10}$$

i.e., (the notation is obvious),

$$\zeta'_{i\parallel} = E_i \zeta_{i\parallel}/m_i\ \zeta'_{i\perp} = \zeta_{i\perp},\ \zeta'_{i0} = (\zeta_i, \mathbf{p}_i)/m_i. \tag{11}$$

Let us consider further three types of formal transformations:

$$\begin{aligned} &\text{I. } u_1 \to \gamma_5 u_1, \quad u_2 \to u_2; \\ &\text{II. } u_2 \to \gamma_5 u_2, \quad u_1 \to u_1; \\ &\text{III. } u_1 \to u_2^c = C\bar{u}_2, \quad u_2 \to u_1^c = C\bar{u}_1. \end{aligned} \tag{12}$$

In transformations I and II, we have

$$\rho_i \to -\gamma_5 \rho_i \gamma_5 = (1 - i\gamma_5 \hat{\zeta}'_i)(-m_i - i\hat{p}_i)/4E_i, \quad (i = 1, 2), \tag{13}$$

i.e., $m_i \to -m_i$ (or $p_i \to -p_i$), $\boldsymbol{\zeta}'_i \to -\boldsymbol{\zeta}'_i$.
If we consider (11), then this means

$$m_i \to -m_i \ (\text{or} \quad p_i \to -p_i), \quad \zeta_{i\perp} \to -\zeta_{i\perp}. \tag{14}$$

In the case of transformation III,

$$\rho_1 \to -(C^{-1}\rho_2 C)^T = (1 + i\gamma_5 \hat{\zeta}'_2)(-m_2 - i\hat{p}_2)/4E_2, \quad \rho_2 \to -(C^{-1}\rho_1 C)^T, \tag{15}$$

whence

$$p_1 \rightleftarrows -p_2,\ m_1 \rightleftarrows m_2\ (\text{or} \quad p_1 \rightleftarrows p_2,\ m_1 \rightleftarrows -m_2); \quad \zeta_{1\parallel} \rightleftarrows -\zeta_{2\parallel},\ \zeta_{1\perp} \rightleftarrows \zeta_{2\perp}. \tag{16}$$

If we now return to (7), it is not difficult to see that it is invariant under transformations I or II, i.e., upon replacement of (14) with simultaneous transformation of the spinor operators:

$$\begin{aligned} &\text{I. } F \to F\gamma_5, \ G \to G\gamma_5, \\ &\text{II. } F \to -\gamma_5 F, \ G \to -\gamma_5 G; \end{aligned} \tag{17}$$

and similarly in case III, i.e., in the transformation (16), together with

$$\text{III. } F \to -(C^{-1}FC)^T, \ G \to -(C^{-1}GC)^T \tag{18}$$

(if the particles 1 and 2 are Y and N, while F or G is equal to $p_{Y\mu}p_{N\nu} - p_{Y\nu}p_{N\mu} - \epsilon_{\mu\nu\alpha\beta}p_{Y\alpha}p_{N\beta}$, then an additional change in sign should take place).

The transformation of the operators (17) and (18) is equivalent to a transformation of form factors for unchangeable operators. If the particles 1 and 2 are $\bar{\nu}$ and l, then the transformations I, II and III in (17), (18) correspond to [$R_j^{(\pm)}$ are arbitrary form factors in (4)]

$$\text{I.} \quad R_j^{(\pm)} \to R_j^{(\mp)}; \tag{19}$$

$$\text{II.} \quad \begin{aligned} &R_j^{(\pm)} \to -R_j^{(\mp)} \quad (j = S, P, T), \\ &R_j^{(\pm)} \to R_j^{(\mp)} \quad (j = A, V); \end{aligned} \tag{20}$$

$$\text{III.} \quad \begin{aligned} &\text{change sign} \quad R_S^{(\pm)},\ R_P^{(\pm)},\ R_A^{(+)},\ R_V^{(-)}, \\ &\text{do not change sign} \quad R_T^{(\pm)},\ R_A^{(-)},\ R_V^{(+)}. \end{aligned} \tag{21}$$

If particles 1 and 2 are Y and N, then the form factors transform in the following fashion:

$$\begin{aligned} \text{I.}\ & C_S^{(\pm)} \rightleftarrows C_P^{(\mp)},\quad C_V^{(\pm)} \rightleftarrows -C_A^{(\mp)},\quad B_V^{(\pm)} \rightleftarrows -B_A^{(\mp)}, \\ & D_V^{(\pm)} \rightleftarrows -D_A^{(\mp)},\ C_T^{(\pm)} \rightleftarrows C_T^{(\mp)},\ B_T^{(\pm)} \rightleftarrows D_T^{(\mp)},\ F_T^{(\pm)} \rightleftarrows F_T^{(\mp)}; \end{aligned} \tag{22}$$

$$\begin{aligned} \text{II.}\ & C_S^{(\pm)} \rightleftarrows -C_P^{(\mp)},\quad C_V^{(\pm)} \rightleftarrows -C_A^{(\mp)},\quad B_V^{(\pm)} \rightleftarrows B_A^{(\mp)}, \\ & D_V^{(\pm)} \rightleftarrows D_A^{(\mp)},\ C_T^{(\pm)} \rightleftarrows -C_T^{(\mp)},\ B_T^{(\pm)} \rightleftarrows D_T^{(\mp)},\ F_T^{(\pm)} \rightleftarrows -F_T^{(\mp)}; \end{aligned} \tag{23}$$

$$\text{III.} \quad \begin{aligned} &\text{change sign} \quad C_S^{(\pm)},\ C_P^{(\pm)},\ D_V^{(\pm)},\ C_A^{(\pm)},\ D_A^{(\pm)},\ D_T^{(\pm)}; \\ &\text{do not change sign} \quad C_V^{(\pm)},\ B_V^{(\pm)},\ B_A^{(\pm)},\ C_T^{(\pm)},\ B_T^{(\pm)},\ F_T^{(\pm)}. \end{aligned} \tag{24}$$

The invariance of (7) means that the even (odd) combinations of the quantities m_k, p_k, ζ_k ($k = Y, N, l, \bar{\nu}$) relative to any of the transformations considered should be multiplied in formulas for the probability of decay by a combination of form factors that is even (odd) relative to the same transformations in (19) — (24).

3. INTERFERENCE OF FORM FACTORS

We shall now make clear how the conclusions can be obtained.

1) Inasmuch as $m_\nu = 0$, while summation is carried out over $\boldsymbol{\zeta}_\nu$, the variables $\bar{\nu}$ do not remain in (14), so that the probability of decay can contain only even [relative to (19)] bilinear combinations of form factors, namely,

$$R_j^{(+)}R_h^{(+)} + R_j^{(-)}R_h^{(-)},\ R_j^{(+)}R_h^{(-)} + R_j^{(-)}R_h^{(+)}. \tag{25}$$

2) If we are not interested in $\boldsymbol{\zeta}_{l\perp}$, then it fol-

lows from (14) and (20) that the products of form factors inside each of the groups (20) which must enter into the combinations (25) should not be multiplied by m_l, whereas the coefficient in the interference between the groups certainly contains m_l. In formulas with $\zeta_{l\perp}$ the contrary would be true.

3) If we are not interested in ζ_l and ζ_ν, then the interference of form factors inside each of the lines of (21) should be multiplied by even [relative to (16)] products of the quantities E_l, $\mathbf{p}_l$, m_l and E_ν, $\mathbf{p}_\nu$ $(m_\nu = 0)$, while the interference between the lines must be multiplied by odd products.

In what follows, we limit ourselves to an investigation of the spectrum, the asymmetry of emission and the polarization N. It is then necessary to integrate over the variables l and $\bar{\nu}$. This is most convenient to do in the system of coordinates where

$$\mathbf{p}_Y = \mathbf{p}_N, \quad \mathbf{p}_l = -\mathbf{p}_\nu. \tag{26}$$

Integration over the variables l and $\bar{\nu}$ means here simply integration over the angles $\mathbf{p}_l$, after which all the components which are linear in $\mathbf{p}_l = -\mathbf{p}_\nu$ disappear. Therefore, the expressions that are even relative to (16) are

$$\mathscr{E}_l\mathscr{E}_\nu, \quad -\mathbf{p}_l\mathbf{p}_\nu = \mathbf{p}_l^2 = \mathbf{p}_\nu^2 = \mathscr{E}_\nu^2, \quad m_l\mathscr{E}_\nu - m_\nu\mathscr{E}_l = m_l\mathscr{E}_\nu, \tag{27}$$

while the odd is

$$m_l\mathscr{E}_\nu + m_\nu\mathscr{E}_l = m_l\mathscr{E}_\nu. \tag{27$'$}$$

In order to distinguish the system $\mathbf{p}_Y = \mathbf{p}_N$ from the system $\mathbf{p}_Y = 0$, all the energies in the first are denoted by the letter $\mathscr{E}$, and in the second by E. It is not difficult to see that here

$$|\mathbf{p}_Y| = |\mathbf{p}_N| = m_Y\sqrt{E_N^2 - m_N^2}\,/Q, \quad \mathscr{E}_l = (Q^2 + m_l^2)/2Q,$$
$$\mathscr{E}_Y = m_Y(m_Y - E_N)/Q, \quad |\mathbf{p}_l| = |\mathbf{p}_\nu| = \mathscr{E}_\nu = (Q^2 - m_l^2)/2Q,$$
$$\mathscr{E}_N = (m_YE_N - m_N^2)/Q, \quad Q^2 = m_Y^2 + m_N^2 - 2m_YE_N. \tag{28}$$

If now the form factors in (20) are found in one line, i.e., they should not be multiplied by m_l, but in (21), in different lines:

$$R_S^{(\pm)}R_T^{(\pm)},\ R_P^{(\pm)}R_T^{(\pm)},\ R_S^{(\pm)}R_T^{(\mp)},\ R_P^{(\pm)}R_T^{(\mp)},$$
$$R_V^{(\pm)}R_V^{(\mp)},\ R_A^{(\pm)}R_A^{(\mp)},\ R_V^{(\pm)}R_A^{(\pm)}, \tag{29}$$

then (the signs must be chosen either both upper or both lower), by virtue of (27) such interference is forbidden. Since $C_S^{(\pm)}$ and $C_P^{(\pm)}$ can be reduced to $-D_V^{(\pm)}/m_l$ and $D_A^{(\pm)}/m_l$ and conversely, then the forbiddenness of the interferences

$$C_S^{(\pm)}C_P^{(\pm)},\ C_S^{(\pm)}C_S^{(\mp)},\ C_P^{(\pm)}C_P^{(\mp)},\ D_V^{(\pm)}R_T^{(\pm)},\ D_V^{(\pm)}R_T^{(\mp)},$$
$$D_A^{(\pm)}R_T^{(\pm)},\ D_A^{(\pm)}R_T^{(\mp)} \tag{30}$$

also follows from (29).

From a comparison of (27) and (21) with (20), it is further clear that the product of form factors from one line both in (20) and in (21), i.e., (the expressions $R_V^{(\pm)}R_V^{(\pm)}$, etc, denote products both of identical and also different form factors, in this case of the vector type

$$R_S^{(\pm)}R_S^{(\pm)},\ R_P^{(\pm)}R_P^{(\pm)},\ R_V^{(\pm)}R_V^{(\pm)},\ R_A^{(\pm)}R_A^{(\pm)},$$
$$R_T^{(\pm)}R_T^{(\pm)},\ R_S^{(\pm)}R_P^{(\mp)},\ R_T^{(\pm)}R_T^{(\mp)},\ R_V^{(\pm)}R_A^{(\mp)}, \tag{31}$$

should be multiplied by $\mathscr{E}_l\mathscr{E}_\nu$ or $\mathbf{p}_l^2 = \mathscr{E}_\nu^2$. If the form factors in (20) are found in different lines, then they interfere with the factor $m_l\mathscr{E}_\nu$:

$$R_SR_A,\ R_SR_V,\ R_PR_A,\ R_PR_V,\ R_TR_A,\ R_TR_V. \tag{32}$$

In the case of electron decays, one can neglect m_l. This means an absence of interference both in (29) — (30) and in (32). This result was obtained in somewhat stronger fashion by Weinberg,[1] who used only the invariance relative to the transformation III, and obtained forbiddenness for the interference of two groups in (21). The γ_5 invariance leading to Eq. (20) gives the additional information here.

Thus, in the case of electron decays, the $R_T^{(\pm)}$ interfere only with one another; in the group V, A, only the expressions

$$R_V^{(+)}R_V^{(+)} + R_V^{(-)}R_V^{(-)},\ R_A^{(+)}R_A^{(+)} + R_A^{(-)}R_A^{(-)},$$
$$R_V^{(+)}R_A^{(-)} + R_V^{(-)}R_A^{(+)},$$

can be encountered, while in the group S, P, only

$$R_S^{(+)}R_S^{(+)} + R_S^{(-)}R_S^{(-)},\ R_P^{(+)}R_P^{(+)} + R_P^{(-)}R_P^{(-)},$$
$$R_S^{(+)}R_P^{(-)} + R_S^{(-)}R_P^{(+)}.$$

When only the form factors C_j remain, this conclusion is confirmed by direct calculation.[2,4] Inasmuch as the form of the matrix element $(\bar{u}_N\Gamma_j u_Y)$ has no effect on the considerations given here, all the above results are applicable to the case of ordinary β decay, where they are generally well known.

Application of the transformations (22) — (24) to the baryon part of the matrix element (2) gives new relations. If in this case one calculates the probability of decay with given energy E_n, angle between $\mathbf{n}_N$ and ζ_Y, and polarization ζ_N $(|\zeta_N| = 1)$ then

$$dW \sim dE_N d\Omega_N S_N(E_N)\{1 + \alpha_N(E_N)(\zeta_Y\mathbf{n}_N)\}$$
$$\times\{1 + P_1(E_N)(\zeta_N\mathbf{n}_N) + P_2(E_N)(\zeta_N\mathbf{n}_N)(\zeta_Y\mathbf{n}_N)$$
$$+ P_3(E_N)[(\zeta_Y\zeta_N) - (\zeta_N\mathbf{n}_N)(\zeta_Y\mathbf{n}_N)]\}, \tag{33}$$

where S_N characterizes the spectrum, α_N — the asymmetry of the flight, and P_1, P_2, P_3 — the polarization of N. The quantities α_N and P_1 can be different from zero only in parity nonconservation.

A discussion completely analogous to that given above shows that in S_N and $S'_N P_2$ ($S'_N \equiv S_N [1 + \alpha_N (\boldsymbol{\zeta}_Y \cdot \mathbf{n}_N)]$ the following expressions are multiplied by $\mathscr{E}_Y \mathscr{E}_N$ and $\mathbf{p}_N^2$ ($R_j R_h \equiv R_j^{(+)} R_h^{(+)} + R_j^{(-)} R_h^{(-)}$; the form factors B_j and D_j have the additional factors $\mathscr{E}_Y - \mathscr{E}_N = Q$ while F_T has the factor $\mathscr{E}_Y \mathscr{E}_N$ or $\mathbf{p}_N^2$; C_S and C_P behave as $-D_V$ and D_A):

$$C_V^2 + C_A^2,\ B_V^2 + B_A^2,\ D_V^2 + D_A^2,\ C_T^2,\ B_T^2 + D_T^2,\ F_T^2,\ B_T D_T,\ C_T F_T\quad C_V B_T - C_A D_T,\ (B_V - B_A) C_T,\ (B_V - B_A) F_T. \tag{34}$$

The factor $m_Y m_N$ multiplies

$$C_V^2 - C_A^2,\ B_V^2 - B_A^2,\ D_V^2 - D_A^2,\ B_T^2 - D_T^2,\ C_V B_T + C_A D_T,\ (B_V + B_A) C_T,\ (B_V + B_A) F_T. \tag{35}$$

Moreover, interference expressions of the form

$$(R_i R_j + R_h R_k) m_Y \mathscr{E}_N + (R_i R_j - R_h R_k) m_N \mathscr{E}_Y \tag{36}$$

are possible in S_n and $S'_N P_2$, where

$$R_i R_j + R_h R_k = C_A B_A - C_V B_V,\ C_V D_V - C_A D_A,\ C_A C_T + C_V C_T,\ C_A F_T + C^V F_T,\ B_A D_T + B_V B_T,\ B_V D_T + B_A B_T,\ C_T D_T - C_T B_T,\ D_T F_T - B_T F_T. \tag{37}$$

The components (34) in the coefficient $S'_N P_3$ which determine, in accord with (33), the transverse polarization, are multiplied by $m_Y m_N$, while (35) is multiplied by $\mathscr{E}_Y \mathscr{E}_N$ and p_N^2. The expressions (37) interfere according to the rule

$$(R_i R_j + R_h R_k) m_N \mathscr{E}_Y + (R_i R_j - R_h R_k) m_Y \mathscr{E}_N. \tag{38}$$

Besides (29) and (30), the interference

$$B_V D_V,\ B_A D_A,\ C_V D_T,\ C_A B_T. \tag{39}$$

in S_N, $S'_N P_2$, $S'_N P_3$ is shown to be forbidden.

As far as the components with parity nonconservation are concerned, i.e., $S'_N \alpha_N$ and $S'_N P_1$ in (33), a similar discussion leads to the result that the interference (35) does not enter into it (here $R_j R_h \equiv R_j^{(+)} R_h^{(-)} + R_j^{(-)} R_h^{(+)}$) while the coefficients for the remaining products of form factors in $S_N \alpha_N$ and $S'_N P_1$ are simply related to each other. For example, in the products (34) (with the substitution of $C_V^2 + C_A^2$, $B_V^2 + B_A^2$, $D_V^2 + D_A^2$ by $C_V C_A$, $B_V B_A$, $D_V D_A$) the factors $\mathscr{E}_Y$ and $\mathscr{E}_N$ appear, while the transition from α_N to P_1 is connected with $\mathscr{E}_Y \rightleftarrows -\mathscr{E}_N$. In the interference (37) (with the substitution of $C_A B_A - C_V B_V$, $C_V D_V - C_A D_A$ by $C_A B_V - C_V B_A$, $C_V D_A - C_A D_A$) we get

$$(R_i R_j + R_h R_k) m_Y \rightleftarrows -(R_i R_j - R_h R_k) m_N. \tag{40}$$

According to the general rule, one can also ascertain that the form factors $B_V D_A - B_A D_V$ and $C_A B_T + C_V D_T$ should not make a contribution to α_N and P_1, while $B_V D_A + B_A D_V$ and $C_A B_T - C_V D_T$ should be multiplied by $\mathscr{E}_Y$ and $\mathscr{E}_N$, while in the transition from α_N to P_1, $\mathscr{E}_Y \rightleftarrows \mathscr{E}_N$. Direct calculation[2] shows that the coefficients for $B_V D_A$ and $B_A D_V$ are equal to zero. Thus, if α_N is computed, then the expression for P_1 can be obtained without additional calculation.

The results of the present section can be put in a form suitable for comparison with the formulas of reference 2, where the notation

$$J \equiv Q^2 / (m_Y - m_N)^2,\quad \xi \equiv (m_Y - m_N) / (m_Y + m_N),\quad \eta \equiv m_l^2 / (m_Y - m_N)^2 \tag{41}$$

is used.

According to (41) and (28),

$$\begin{aligned}
\mathscr{E}_Y &= m_Y (1 + \xi J) / \sqrt{J} (1 + \xi),\\
\mathscr{E}_N &= m_N (1 - \xi J) / \sqrt{J} (1 - \xi),\\
\mathbf{p}_N^2 &= m_Y m_N (1 - J)(1 - \xi^2 J) / (1 - \xi^2) J,\\
\mathscr{E}_l &= \tfrac{1}{2} (m_Y - m_N) \sqrt{J} (1 + \eta / J),\\
\mathscr{E}_\nu &= |\mathbf{p}_\nu| = |\mathbf{p}_l| = \tfrac{1}{2} (m_Y - m_N) \sqrt{J} (1 - \eta / J).
\end{aligned} \tag{42}$$

Therefore, in addition to the added factors (Q or $\mathscr{E}_Y \mathscr{E}_N$, $\mathbf{p}_N^2$), for B_j, D_j or F_T, the expressions (34) in S'_N and $S'_N P_2$ [and (35) in $S'_N P_3$] are multiplied by $(1 - J)(1 - \xi^2 J)$ or $(1 - \xi^2 J^2)$, while (35) [(34) in $S'_N P_3$] should be multiplied by $(1 - \xi^2) J$. The interference of (36) or (37) can now be written as

$$\sqrt{J}\,[R_i R_j (1 - \xi^2 J) \pm R_h R_k \xi (1 - J)]. \tag{43}$$

In the transition from α_N to P_1, the rule $\mathscr{E}_Y \rightleftarrows \mp \mathscr{E}_N$ is equivalent to $1 \to \mp 1$, $\xi J \to \mp \xi J$; the rule (40) now means that for $R_i R_j$ we have $1 \to -1$, $\xi \to \xi$, while for $R_h R_k$ we have $1 \to 1$, $\xi \to -\xi$. The factors $\mathscr{E}_l$, $\mathscr{E}_\nu$, $p_l^2 = \mathscr{E}_\nu^2$ and $m_l \mathscr{E}_\nu$ in (31) and (32) are proportional to $1 + \eta/J$, $1 - \eta/J$ and $\sqrt{\eta/J}$.

Direct calculation[2] agrees with the results given here, in particular with the absence of interference of the form factors in (29), (30), and (39). It should be noted that in the expression for the energy correlation, such interference will certainly appear with the factor

$$X \sim E_l - E_\nu [1 + m_l^2 / Q^2] / [1 - m_l^2 / Q^2], \tag{44}$$

the contribution from which vanishes upon integration over the variables l and $\bar{\nu}$.

In conclusion, I express my deep gratitude to S. V. Maleev for stimulating discussions.

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