ANALYSIS BASED ON DISPERSION RELATIONS OF DATA ON PION PHOTOPRODUCTION NEAR THRESHOLD

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It is found that in the region near the threshold it is possible to make a direct comparison of data on the photoproduction of π mesons with the dispersion relations. This new method of analysis is applied to the available experimental data on the photoproduction of π^0, π^+, and π^- mesons. An investigation is made of the reasons for the significant discrepancy found earlier[4] between conclusions based on the dispersion relations and experimental data. Experiments are suggested which should ascertain whether this discrepancy is due to a large contribution to the dispersion integrals from the domain of very high energies. The solution of this problem may greatly affect the efforts to construct a theory of pion photoproduction on the basis of dispersion relations.

1. INTRODUCTION

UNTIL very recently the analysis of the photoproduction of π mesons founded on dispersion relations was based on the well known paper by Chew et al.,[1] in which an attempt was made to obtain the amplitudes for the photoproduction of π mesons from dispersion relations.

In a number of papers[1,2] it was noted that these amplitudes, generally speaking, agree with experimental data. In a paper by Govorkov and Baldin[4] an analysis of new experimental data[5] was made, and it was shown that the photoproduction amplitudes given in reference 1 contradict these data. In discussing this contradiction one must bear in mind the fact that the amplitudes in reference 1 were obtained on the basis of a number of assumptions of special nature. In this connection, a method of analysis of photoproduction data which would enable us to compare directly the dispersion relations with experiment would be of considerable interest. Until now the dispersion relations have been investigated in this way only for the case of the scattering of π mesons by nucleons. In the case of photoproduction such a comparison, generally speaking, presents great difficulties. The object of the present article is the application of the dispersion relations to the photoproduction of π mesons in the near-threshold region, where, as it turns out, a number of difficulties disappears and the comparison mentioned previously turns out to be possible.

We have made partial use of this method in reference 4. It was shown there that the contradiction between the experimental data and the conclusions reached on the basis of the dispersion relations, persists to a great extent also in the case of this method of analysis. Thus, the contradiction becomes much more acute since the new approach does not depend on the assumptions made by Chew et al.[1]

Earlier[4] we have investigated only the forward part of the photoproduction amplitude. Without any doubt a complete analysis of the available experimental data on the photoproduction of π mesons in the near-threshold region, an analysis of the possible reasons for the observed contradiction, and also a complete exposition of the method mentioned previously and a proposal of further experiments are topics of definite interest.

The present article is devoted to all these problems.

2. THE METHOD OF ANALYSIS

The dispersion relations for the photoproduction of π mesons have been given in references 1 and 3. In the center-of-mass system they have the following structure:

$$\operatorname{Re} F_i(w, \nu_1) = F_i^B + \frac{1}{\pi} \int\limits_{M+1}^{\infty} dw' \operatorname{Im} \sum_j F_j(w', \nu_1)\, Q_{ij}(w', w, \nu_1). \qquad (1)$$

Here the quantities F_i are proportional to the individual amplitudes which make up the total amplitude for the process

$$F = i(\sigma\varepsilon)F_1 + \frac{(\sigma q)(\sigma[k\varepsilon])}{qk}F_2 + i\frac{(\sigma k)(q\varepsilon)}{qk}F_3 + i\frac{(\sigma q)(q\varepsilon)}{q^2}F_4, \quad (2)$$

σ is the Pauli matrix, ϵ is the photon polarization vector, $\mathbf{q}$ and $\mathbf{k}$ are respectively the meson and the photon momenta.

In this case the cross section has the following form

$$\frac{d\sigma}{d\Omega} = (q/k)\{|F_1|^2 + |F_2|^2 - 2\,\mathrm{Re}\,F_1^*F_2\cos\theta + \tfrac{1}{2}\sin^2\theta\,[|F_3|^2 + |F_4|^2 + 2\,\mathrm{Re}\,F_2^*F_3 + 2\,\mathrm{Re}\,F_1^*F_4 + 2\,\mathrm{Re}\,F_3^*F_4\cos\theta]\}, \quad (3)$$

$w = \sqrt{1+q^2} + \sqrt{M^2+q^2}$ is the total energy,* $Q_{ij}(w, w', \nu_1)$ are certain real functions. The explicit expressions for F_i^B and Q_{ij} are given in references 1 and 3.

An attempt to make a direct comparison of relations (1) with experiment encounters great difficulties. These difficulties are first of all associated with the fact that the experimental data on the amplitudes F_i are very meager. Moreover, the integral in (1) contains a non-physical region ($\cos\theta > 1$) which also introduces some indefiniteness into the comparison of (1) with experiment. Further, the range of integration in (1) over w' extends to infinity, and since the experimental data always refer to a limited range of energies, this reduces the value of the conclusions with respect to the verification of the dispersion relations themselves.

Another difficulty in making a comparison of (1) with experiment involves the numerical evaluation of the principal value of the integrals of functions obtained experimentally with very limited accuracy.

The method utilized by us enables us to a large extent to avoid these difficulties.

We undertake the investigation of the near-threshold region, by which we mean the region of energies such that $q < 1$. In this region the problem of comparing (1) with experiment is simplified.

If we expand the amplitude [and correspondingly Eq. (1)] into a power series in q, we shall be dealing with the amplitude and with its derivatives at $q = 0$. In this case the non-physical region reduces to a point. The difficulty associated with the evaluation of the principal value of integrals is also eliminated since at $q = 0$ the poles of the functions Q_{ij} are cancelled by the zeros of the functions F_i.

Moreover, in virtue of the well known connection between scattering and photoproduction and of the fact that the scattering phases vanish as $q \to 0$, we may obtain the left hand side of (1) from the experimental data without making use of polarization experiments. We have already discussed this point in reference 4.

One might think that the principal difficulty associated with estimating the role played by the region of high values of w' will also be considerably reduced in our case since the integrand falls off quite rapidly as w' increases.

We represent the experimental data on the photoproduction of π mesons in the following form:

$$\frac{d\sigma}{d\Omega} = \sum_{mn} b_{mn} q^m (\cos\theta)^n; \quad (4)$$

the parameters b_{mn} may be evaluated from the dispersion relations and compared with the experimental values, as we shall demonstrate later.

We begin with a few general remarks on the coefficients b_{mn}. In the photoproduction of π^0 mesons, the predominant role is played by the $(3/2, 3/2)$ resonance amplitude, for which the coefficients b_{m0} and b_{m2} differ from zero. Moreover, in accordance with the resonance model the amplitude depends on q through the factor

$$\frac{\sin\alpha_{33}}{q^2} \approx \frac{\alpha_{33}}{q^2} = \mathrm{const}\,\frac{q}{\omega\,[1-\omega/\omega_r]}, \qquad \omega_r \approx 2,$$

from which it follows that in the expansion of this amplitude the terms of the second, third and fourth degree in q are absent.

Thus, one might expect that the experimental data on the photoproduction of π^0 mesons are described by a small number of coefficients b_{mn} over quite a large interval (as was demonstrated in reference 2).

In the case of charged mesons an essential complication arises because of the "retarded term"

$$q^2\sin^2\theta/2k^2\,[1-(q/\omega)\cos\theta]^2,$$

which in the region $q \lesssim 1$ considerably exceeds the contribution of the p-waves. Therefore, the lowest terms in the expansion will describe $d\sigma/d\Omega$ over a considerably smaller range of q, than in the case of neutral mesons.*

In such a case it is apparently useful to make an expansion into a series of type (4) not of the differential cross section $d\sigma/d\Omega$, but of the quantity

$$[1-(q/\omega)\cos\theta]^2\,d\sigma/d\Omega,$$

which, nevertheless, will lead to the necessity of

*We utilize the system of units such that $\hbar = \mu = c = 1$.

*However, it should be noted that in the neighborhood of $\theta = 0$ and $\theta = \pi$ the contribution of this term becomes small, and possibly these ranges of angles (particularly the angles close to π) are the most suitable ones for analysis by our method.

analyzing a considerably greater number of coefficients than in the case of photoproduction of π^0 mesons.

In order to obtain the theoretical values of the different quantities we shall need expansions of the amplitudes* F in powers of q:

$$F_1 \approx f_1^{(0)} + i f_1^{(1)} q + f_1^{(2)} q \cos\theta + f_1^{(3)} q^2 + \ldots;$$
$$F_2 \approx f_2^{(1)} q + \ldots;\quad F_3 \approx f_3^{(1)} q + \ldots;\quad F_4 = f_4^{(2)} q^2 + \ldots, \quad (5)$$

$f_i^{(k)}$ is expressed in the form of two terms, one of which is the expansion coefficient of the Born part of the amplitude F_i^B in expression (1) evaluated exactly, while the other term is determined by the dispersion integral. In the case of charged mesons, in order to reduce the necessary number of parameters b_{mn}, it is advantageous to retain the "retarded term" without expanding it. This means that the amplitudes F_2 and F_4 will contain the following terms

$$f_3^{(2)} = \frac{ef}{1 + \omega / M} \frac{qk}{\omega k - qk\cos\theta}, \quad f_4^{(2)} = \frac{ef}{1 + \omega / M} \frac{-q^2}{\omega k - qk\cos\theta}. \quad (6)$$

Here f is the renormalized coupling constant, and e is the charge.

3. NEUTRAL MESONS

The photoproduction of π^0 mesons was partially investigated by the method just outlined in reference 4. It was shown there that if the forward part of the photoproduction amplitude is calculated on the assumption that the principal contribution to the dispersion integrals is made by the (3/2, 3/2) resonance amplitude, then a significant disagreement with experiment exists (approximately three standard deviations). However, this result depends on the uncertainties associated with the calibration of the beam. Moreover, if the errors in the measurement of the renormalized coupling constant and of the total photoproduction cross section for π^0 mesons are taken into account, this may further reduce the disagreement to some extent. In this connection we have carried out a complete analysis of the photoproduction of π^0 mesons (not only for the forward photoproduction amplitude). In the evaluation of errors we have also taken into account some small errors of the type just indicated. It is also necessary to keep in mind that, strictly speaking, the data of reference 5 are insufficient for the determination of the coefficients b_{mn}.

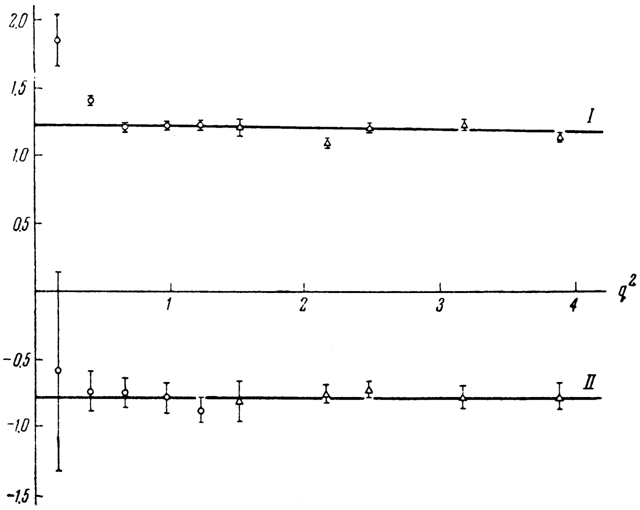
In reference 4, in order to determine the first few coefficients in expansion (4), we have utilized data for $q > 1$, after correcting them for the (3/2, 3/2) resonance. These corrections strongly depend on the value of the "resonance" frequency* ω_r.

In order to be able to draw more definite conclusions with respect to the coefficients b_{mn} we require additional measurements of cross sections in the region $q < 1$. A more rigorous approach to the available experimental data consists of collecting all the available data on the reaction $\gamma + p \to p + \pi^0$, choosing an appropriate ω_r and then extrapolating to $q = 0$. The diagram shows the experimental results of references 2 and 5 on the determination of A and C presented in the form of

$$(A/q\omega)\,[a_{33}q^2/\sin\alpha_{33}]^2$$

and

$$(C/q\omega)\,[a_{33}q^2/\sin\alpha_{33}]^2.$$



The experimental data of reference 2 (triangles) and of reference 5 (circles) presented in the following form: I – $(A/q\omega)[a_{33}q^2/\sin\alpha_{33}]^2$, II – $(C/q\omega)[a_{33}q^2/\sin\alpha_{33}]^2$.

The resonance frequency satisfying all these data has turned out to be equal to $\omega_r = 1.95$. The values of the coefficients b_{mn} obtained as a result are given in the second line of the table. They differ considerably from those obtained by us earlier[4] with $\omega_r = 2.1$.

b_{32}	b_{21}	b_{30}	b_{10}
−0.78 ±0.10	−0.38 ±0.04	1.23 ±0.03	0.10 ±0.03
+0.32 ±0.03	−0.71 ±0.07	1.04 ±0.1	0.12 ±0.01

In order to determine b_{mn} from the dispersion relations we must obtain $f_k^{(i)}$. A numerical evaluation of the dispersion integrals on the basis of the same assumptions as in reference 4 yields†

*For the sake of simplicity we have omitted everywhere the isotopic indices.

*In reference 4 we assumed $\omega_r = 2.1$.

†Here $f_i^{(k)}$ are amplitudes corresponding to the process $\gamma + p \to p + \pi^0$, and not partial amplitudes referring to a state of definite isotopic spin.

$$f_1^{(0)} = -0.79 + 0.46 = -0.33, \quad f_1^{(2)} = 0.85 + 0.77 = 1.62,$$
$$f_2^{(1)} = 0.0 + 0.61 = 0.61, \quad f_3^{(1)} = -0.98 - 0.77 = -1.75,$$
$$f_4^{(2)} = 0.35 + 0.0 = 0.35. \qquad (7)$$

Here $f_1^{(k)}$ is expressed in units of $10^{-2}\hbar/\mu c = 1.4 \times 10^{-15}$ cm.

The first figures indicate the contribution of the Born part of the amplitude, while the second ones give the contribution of the dispersion integrals.* The statistical error in these quantities amounts to approximately 5%.

These values of the amplitudes correspond to the values of the coefficients b_{mn} shown in the third line of the table. In order to eliminate the uncertainties associated with the calibration of the beam we compare the quantities b_{32}/b_{30} and b_{21}/b_{30}.† Experiment yields for these quantities the following values: -0.63 ± 0.08 and -0.31 ± 0.03 respectively, while from the dispersion relations we obtain: $+0.32 \pm 0.03$ and -0.66 ± 0.07.

Thus, the discrepancy amounts to several standard deviations. The reasons for the discrepancy should be sought only in the evaluation of the dispersion integrals, if we disregard the possible inaccuracies in the extrapolation of the experimental data to $q = 0$. (One should also remember that all our conclusions are based on the data of only a single experiment and require additional experimental confirmation.) We note that the data (7) correspond to approximately equal absolute values and opposite signs of the amplitudes of the magnetic dipole transitions M_{1+} and M_{1-}, while in the integrand we took into account only the imaginary part of the amplitude M_{1+}. We could achieve agreement with experiment at $q = 0$ by assuming that the integral of the imaginary part of M_{1-} is close in magnitude to the integral of the imaginary part of M_{1+} and that these integrals have the same sign. However, this is, in turn, in poor agreement with experimental data on photoproduction and on scattering at energies where the integrands are large, $q \sim 1$.

Indeed, from our analysis[4] it follows that in the region of the threshold energies, M_{1-} is equal to zero within experimental error. At higher energies we may utilize the data of McDonald, Peterson and Corson[2] for the evaluation of the amplitude of M_{1-}.

An estimate of the magnitude of the amplitude of M_{1+} may be obtained from the integrated cross section from which all interference terms drop out. We neglect the squares of the other p-wave amplitudes. Then, on taking into account the relation between scattering and photoproduction, and the fact that the scattering phases α_{11}, α_{31} and α_{13} do not exceed $10-15°$, it is not difficult to obtain an estimate for the magnitude of M_{1-} from the general expression for the angular distribution. It turns out to be of the order of magnitude of the experimental error, and certainly less than $0.2\,M_{1+}$. From this we see that the integral of the imaginary part of M_{1-} cannot be close in magnitude to the integral of the imaginary part of M_{1+}. At the present time, because of the inaccuracy of the experimental data, it is difficult to make an estimate of the total value of the discrepancy under discussion taking into account the possible contribution of M_{1-}.

4. CHARGED MESONS

The photoproduction amplitudes for charged mesons differ by the fact that the Born part $f_1^{(0)}$ is much larger than in the case of neutral mesons. This is a reflection of the well known fact that in the near-threshold region the s-wave meson photoproduction plays the dominant role in the case of charged mesons. Moreover, the Born parts of the amplitudes F_2 and F_4, containing the "retarded term," also give a contribution which considerably exceeds the contribution of the dispersion integrals. The value of $f_1^{(0)}$ expressed in terms of our units amounts to 2.56 for the photoproduction of π^+ mesons, the contribution of the "retarded term" in the region $q \sim 1$ in the expressions for F_2 and F_4 amounts to $\sim 0.7\, f_1^{(1)}$, so that it cannot be neglected. The dispersion integrals, obtained under the same assumptions as the dispersion integrals for the photoproduction amplitudes for π^+ mesons, are close to these integrals in magnitude. From this it is clear that only in the region of $q \sim 1$ will the contribution of the dispersion integrals to the cross section be as high as $20-25\%$. As q decreases this contribution is reduced significantly and falls outside the limits of the available experimental accuracy. On the basis of this one should expect that the experimental data on the photoproduction of charged mesons in the region of interest to us may be described with a sufficient degree of accuracy by formulas which agree with the first nonvanishing approximation of perturbation theory. The contribution of the dispersion integral to $f_1^{(1)}$ which does not diminish as q decreases requires

*As can be seen from these figures, the overwhelmingly large contribution of the Born part of the amplitude noted in reference 4 is a peculiarity of the forward photoproduction, and not of near-threshold photoproduction in general.

†It is necessary to emphasize that the uncertainty associated with the value of ω_r does not affect these quantities within experimental error, while changes in the values of b_{mn} when ω_r is changed from 2.1 to 1.95 lie far outside the limits of statistical errors.

a separate comment. This contribution, in spite of being small, nevertheless introduces some uncertainty into the determination of the coupling constant from the photoproduction cross section for charged π mesons extrapolated to the threshold.

A comparison of the b_{mn}, obtained by neglecting the contribution of the dispersion integrals, with the experimental data on the photoproduction of π^+ mesons shows good agreement between them. The essential role played by the "retarded term" is demonstrated by the recent results of references 6 and 7 in which an increase in the square of the amplitude in the threshold region was discovered.

Data on the photoproduction of π^- mesons can at present be obtained only by analyzing the data on photoproduction in deuterium. It appears to us that in the near-threshold region this may be done only by analyzing experiments carried out in such a way that all the product particles of the reaction $\gamma + d \rightarrow 2p + \pi^-$ are recorded.[8] The squares of the amplitudes of the process $\gamma + n \rightarrow p + \pi^-$ were obtained by us[9] on the basis of the experimental data of Adamovich et al.[8]

In the paper by Beneventano et al.[10] it was noted that from these data and from the new data[6] on π^+ the near-threshold ratio $\sigma^-/\sigma^+ \lesssim 1$ is obtained. We do not agree with this statement, since on the basis of the data given in our paper[9] it is quite possible to draw the curve corresponding to $\sigma^-/\sigma^+ \sim 1.35$ where σ^+ is taken from the most recent publications.[6,11] Moreover, the small corrections* introduced recently by Adamovich, Kharlamov, and Larionova[11] show that the curve corresponding to the value of the square of the π^- photoproduction amplitude obtained by neglecting the contribution of the dispersion integrals agrees very well with the improved experimental data. The contribution of the "retarded term" to σ^- lies outside the limits of experimental accuracy.

In connection with the conclusions reached by Beneventano et al.[10] we have analyzed the only doubtful point in our theory of the $\gamma + d \rightarrow 2p + \pi^-$ reaction and have obtained a better founded method of dealing with the two-nucleon wave function in the region where the nuclear forces are effective, and we consider it appropriate to describe it here.

It is necessary to emphasize at the outset that the application of this method does not alter our quantitative results, but merely makes them more convincing.

In reference 9, in the evaluation of the cross section for the photoproduction on deuterium, it turned out to be necessary to make estimates of integrals of the form

$$\delta I_s = \int \psi_{fs}^{*} e^{i\mathbf{q}\mathbf{r}} \psi_d d\mathbf{r} - \int \varphi_{fs} e^{i\mathbf{q}\mathbf{r}} \varphi_d d\mathbf{r}.$$

Here ψ_{fs} and ψ_d are the true wave function for the relative motion of the two nucleons in the final s-state, and the true deuteron wave function. φ_{fs} and φ_d are model wave functions; these functions differ only within the region where the nuclear forces are effective. As was noted in reference 9, δI_s gives a significant correction only in the region of small relative momenta of the nucleons. On integrating over the solid angle and on taking into account the fact that $q \lesssim 1/r_0$, we may easily show that δI_s may be expressed in terms of the integral:

$$\int (u_s u_d - v_s v_d)\, dr. \tag{8}$$

Here u_s is the true radial wave function, while v_s is the model radial wave function; u_s and v_s coincide outside the range of the forces and are normalized in accordance with the condition $v(0) = 1$. This also applies to u_d and v_d.

We note that the integral (8) coincides with $-\frac{1}{2}\bar{\rho}$, where $\bar{\rho}$ is the "mixed effective radius" defined by Feshbach and Schwinger.[12] On the basis of an investigation of the photomagnetic disintegration of the deuteron, they have obtained

$$\bar{\rho} = (2.18 \pm 0.3) \cdot 10^{-13} \text{ cm}.$$

If we make use of this value for estimating δI_s, then the difference between the value of $A(p, q)$ obtained by the present author[9] and of $A(p, q)$ obtained on the basis of such an estimate lies outside the limits of accuracy of the calculations over the whole range of variation of the arguments p, q.

In summarizing the analysis of data on the photoproduction of charged mesons near the threshold we can say that both σ^- and σ^+ are well described by the Born part of the amplitude alone, if we choose the renormalized coupling constant equal to $f^2 = 0.08$. Within experimental error this value of the coupling constant agrees well with the values estimated on the basis of other effects.

5. DISCUSSION

The method of analyzing experimental data on photoproduction utilized by us has turned out to be quite effective even in the present state of the experimental data. However, if we set as our aim the accumulation of experimental data to which our method is applicable, then as the result of a relatively small effort we can reach quite important conclusions on the properties of the π meson photoproduction amplitude.

*These corrections lie within the 10% accuracy limit previously ensured.

The effective use of the long wavelength approximation in the near-threshold energy region enables us to carry out a generalized phase analysis. Thus, in reference 4 we have obtained the amplitudes for the s-photoproduction in the case of both protons and neutrons, which turned out to be significantly different;* an estimate was given of the value of the amplitude for the magnetic dipole transition M_{1-}. However, the most important results are obtained by applying the dispersion relations in this region. The application of the dispersion relation here turns out to be, on the one hand, particularly simple (a number of difficulties enumerated earlier is avoided), and on the other hand, most productive of results from the point of view of checking the predictions of field theory. The latter is due to the fact that the sharpest predictions of field theory consist of the singularities in the energy and in the angular dependences associated with the existence of a known pole in the nonphysical region. These singularities are expressed by the presence of the Born part in (1). The threshold is that point in the physical region of variation of the variable photoproduction amplitudes, which lies very close to this pole. Therefore the pole exerts a decisive influence on the near-threshold region. Thus, for example, in the case of charged mesons the Born part of the amplitude describes the experimental data satisfactorily, while the contribution of the dispersion integral lies outside the limits of experimental accuracy.

Moreover, it is clear from the foregoing that in the near-threshold region one may measure the "retarded term" sufficiently reliably since its contribution turns out to be considerably greater than the contribution of the dispersion integrals.

The application of the method to the photoproduction of neutral mesons has led to essential difficulties. From the analysis just carried out it is clear that the Born part of the amplitude by no means determines the cross section for the process.

The evaluation of the dispersion integrals on the assumption that the principal contribution to them is made by the $(\frac{3}{2}, \frac{3}{2})$ resonance amplitude of the magnetic dipole transition leads to a large discrepancy with the experimental results. The inclusion of the second resonance in the photoproduction changes the situation very little. We do not have much basis for suspecting that the dispersion relations are not valid, since, as we have seen, in the case of charged mesons, for which the contribution of the dispersion integrals is small, good agreement with experiment is obtained even in such details as the "retarded term." From this it is clear that a solution must be sought in a more detailed evaluation of the dispersion integrals. First of all, an analysis should be made as to whether this contradiction might not be an indication of the presence of other partial amplitudes — of the magnetic dipole M_{1-} and the electric quadrupole transitions.

An exact answer to this question could be given by an experiment on the measurement of the polarization of the recoil protons in the reactions $\gamma + p \rightarrow p + \pi^0$ for γ-quanta energies lying in the range 240-270 Mev. Experimental data on the angular distribution enable us to obtain only upper limits on these amplitudes. A preliminary investigation shows that these upper limits give too low values of the amplitudes to eliminate the discrepancy. The discrepancy is considerably reduced if the contribution of the integral of M_{1-} is positive, while the quantity M_{1-} itself is negative in the near-threshold region, while the data of Pontecorvo's group (cf. reference 14) show clearly that the scattering phase α_{11} is positive over a wide range. If we assume that M_{1-} changes sign at an energy in the neighborhood of 200 Mev, then the contribution of the integral, which is already too small, is significantly reduced still further.

In summarizing we can say that the possibility of removing the difficulty should first of all be sought in the evaluation of the dispersion integrals. In our opinion it is of the greatest interest to obtain an answer to the question whether this discrepancy can be removed by considering the contribution to the dispersion integrals made by the small photoproduction amplitudes in the energy range in the neighborhood of the $(\frac{3}{2}, \frac{3}{2})$ resonance, or whether in order to remove the discrepancy it is necessary to invoke the contribution of the photoproduction amplitudes in the energy range above 1 Bev.

Additional investigations are required to give an answer to this question. First of all, more accurate measurements than those made in reference 5 are required of the angular distribution for the π^0-meson photoproduction in the near-threshold region. Such measurements would, firstly, give more accurate information on the magnitude of the above discrepancy, and secondly, an estimate could be made of the coefficients b_{mn} in (4) for higher values of m. It may be easily

*We note that this is an interesting example of an exception to the rule[13] on the relationship between the matrix elements of the isotopic scalar and isotopic vector parts of the S-matrix for processes involving photon absorption: $\langle S\rangle \sim (\mu/M)\langle V_3\rangle$, where μ and M are respectively the meson and nucleon masses.

shown* that the high-energy region has a considerably smaller effect on these coefficients.

In order to determine the magnitude of the small amplitudes it is necessary to obtain more precisely the angular distributions in the $\gamma + p \rightarrow p + \pi^0$ reaction for energies in the 220 — 450 Mev range. Experiments on the measurement of the polarization of the recoil nucleons could yield much information.

In order to evaluate the role played by the dispersion integrals in the amplitudes for charged mesons it is necessary to increase significantly the accuracy of the experimental data in the near-threshold region.

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