SPECTRA OF GAMMA RAYS PRODUCED IN THE CAPTURE OF THERMAL NEUTRONS BY HEAVY NUCLEI. II

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Submitted to JETP editor August 28, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 598-611 (February, 1960)

The spectra of γ rays accompanying the capture of thermal neutrons are calculated. The calculations are performed for dipole γ radiation and two types of dependence of the nuclear level density on energy. The results for the level density are compared with data derived from other experiments.

THE capture of a thermal neutron by an atomic nucleus leads to the formation of a relatively strongly excited nucleus with excitation energy 6 — 8 Mev. In the case of heavy nuclei with atomic weight A > 100, owing to the great density of the levels, the transition from the excited state that results from the capture of a thermal neutron may proceed via a large number of different paths. Accordingly, the overwhelming portion of the γ transitions in such elements occurs in a portion of the spectrum that is still unresolved by modern measurement procedures. In the case of light nuclei, and in the case of heavy close-to-magic nuclei, the γ-ray spectrum is almost a line spectrum owing to the small level density.[1] In this paper we consider only continuous spectra. Experimental data have shown that the spectra of γ quanta in heavy nuclei change relatively little from nucleus to nucleus. One can therefore assume that the spectrum of the emitted quanta is determined essentially by the general laws of the distribution of the density of the nuclear levels and by the multipolarity of the radiation transition, and depends little on the specific properties of the matrix elements of the transitions.

In accordance with the statistical theory, we assume that the relative probability of emission of a γ quantum with energy E by a nucleus, located in a certain state with energy u, is given by the following formula:[2]

$$w(u, E) = E^{\varkappa}\rho(u - E)/n(u), \tag{1}$$

where $\rho(u-E)$ is the density of the final states of the nucleus. It is assumed that the distribution of the γ quanta is determined only by the energies of the initial and final states of the nucleus. One can see certain justification for this in the fact that the probability $w(u, E)$ has the meaning of a relative transition probability, averaged over a sufficiently large number of initial and final nuclear states. The distribution (1) is normalized to a total decay probability of unity, from which we determine the normalizing factor $n(u)$

$$n(u) = \int_0^u E^{\varkappa}\rho(u - E)\,dE. \tag{2}$$

The average transition matrix element, which we consider to be a constant, is not contained in Eq. (1). The constant κ has a value $2l+1$, where 2^l is the multipolarity of the radiation.

To calculate the total γ-ray spectrum of the nucleus it is necessary to sum the spectra of all the stages of the cascade. This problem is solved most simply when the energy in the emitted quanta can be considered small compared with the excitation energy of the nucleus. The spectrum was calculated in this approximation by Nosov and one of the authors of the present paper (V.S.).[1] This case is characterized by the emission of a large number of quanta during de-excitation of the nucleus. Considering the number of quanta to be a continuous quantity, it is possible to obtain the following expression for the γ-ray spectrum:

$$\nu(E)\,dE = dE\int_E^{u_0} w(u, E)\frac{du}{\overline{E}(u)}, \tag{3}$$

where $\nu(E)\,dE$ is the number of quanta whose energy is in the interval E, E+dE, emitted by the nucleus, u_0 is the initial excitation energy, and $\overline{E}(u)$ is the average energy of the γ quanta of energy u emitted by the nucleus:

$$\overline{E}(u) = \int_0^u Ew(u, E)\,dE. \tag{4}$$

The total number of quanta $\overline{\nu}$ is in this approximation

$$\overline{\nu} \equiv \int_0^{u_0}\nu(E)\,dE = \int_0^{u_0} du/\overline{E}(u). \tag{5}$$

Putting in (1) $\rho = \rho_0 \exp\{S(u)\}$, where $S(u)$ is the entropy of the nucleus, we obtain

$$\bar{E}(u) \approx (\varkappa + 1) T(u), \tag{6}$$

$$\bar{\nu}(u_0) = \int_0^{u_0} \frac{du}{(\varkappa + 1) T(u)} = \frac{S(u_0)}{\varkappa + 1}. \tag{7}$$

The mean squared fluctuation of the number of quanta is

$$\sqrt{\overline{\nu^2} - \bar{\nu}^2} = \sqrt{S(u_0)} / (\varkappa + 1). \tag{8}$$

In formulas (6) and (7), $T(u)$ is the nuclear temperature, $T(u) = du/dS$.

A comparison has been made previously[1] of the experimental data on the spectra of the γ rays that accompany the capture of thermal neutrons; the calculations there were based on formulas (3) — (7). These formulas yield a fairly good qualitative approximation, but their accuracy when applied to capture of thermal neutrons is insufficient, owing to the small number of emitted quanta ($\sim 3-4$ quanta per capture). The average quantum energy is $\sim 1.5-2$ Mev, i.e., it is far from small compared with the nuclear excitation energy.

A more accurate calculation of the γ-ray spectrum can be made by foregoing one of the premises that underly the derivation of Eqs. (3) — (7), namely that the change in nuclear energy upon emission of the γ quanta is small, and the resultant replacement of the summation of individual decay chains by integration over stages, each of which is characterized by a certain average nuclear excitation energy. Such a calculation can be made on the basis of the kinetic equation that describes a multistage process of γ emission from the nucleus.[3] In our case, when the emission of quanta is the only process, this equation has the following form

$$\partial y_k(u, t) / \partial t = \int_u^{u_0} y_{k-1}(u', t)\, \Gamma(u')\, w(u', u' - u)\, du' - \Gamma(u)\, y_k(u, t), \tag{9}$$

where $y_k(u, t)$ is the probability of finding k quanta and the nucleus in the state with energy u at the instant t in the nucleus-plus-photons system. In Eq. (9) $\Gamma(u)$ is the decay probability per unit time of a nucleus with energy u, multiplied by $\hbar$.

During the initial instant of time, $t = 0$, we have

$$y_k(u_0, 0) = \delta_{k0}\delta(u - u_0) \tag{10}$$

(δ_{ab} is the Kronecker symbol and δ stands for the δ function), since at $t = 0$ there were no quanta in the system, and the nucleus was in the initial state.

One can obtain, on the basis of (9), a simple equation for a certain quantity directly related with the γ-spectrum of interest to us. To derive this equation we introduce the function $N(u, t)$, the probability of having at the instant t a nucleus with energy u with any number of quanta:

$$N(u, t) = \sum_{k=0}^{\infty} y_k(u, t). \tag{11}$$

The function $y_0(u, t)$ satisfies the following differential equation [see (9)]:

$$\partial y_0(u, t) / \partial t = -\Gamma(u)\, y_0(u, t). \tag{12}$$

Taking this into account, we obtain, summing both halves of (9) over k, the following equation for the function $N(u, t)$

$$\partial N(u, t) / \partial t = \int_u^{u_0} \Gamma(u')\, w(u', u' - u)\, N(u', t)\, du' - \Gamma(u)\, N(u, t). \tag{13}$$

Analogously we obtain from (10) the initial condition

$$N(u, 0) = \delta(u - u_0). \tag{14}$$

We now denote by $\nu(E, t)$ the number of quanta with energy E in the system at the instant t. The function $\nu(E, t)$ satisfies the following equation

$$\partial \nu(E, t) / \partial t = \int_E^{u_0} N(u', t)\, \Gamma(u')\, w(u', E)\, du', \tag{15}$$

a solution of which, satisfying boundary condition $\nu(E, 0) = 0$, is the function

$$\nu(E, t) = \int_E^{u_0} w(u', E)\, du' \int_0^t \Gamma(u')\, N(u', t')\, dt'. \tag{16}$$

The function $\nu(E, t)$ at $t = \infty$ yields the number of quanta of energy E, emitted by the nucleus during the decay process:

$$\nu(E) \equiv \nu(E, \infty) = \int_E^{u_0} w(u', E)\, du' \int_0^{\infty} \Gamma(u')\, N(u', t)\, dt. \tag{17}$$

Let us now find the equation satisfied by the function

$$Z(u) = \int_0^{\infty} \Gamma(u)\, N(u, t)\, dt.$$

For this we integrate both parts of (13) with respect to time:

$$N(u, \infty) - N(u, 0) = \int_u^{u_0} Z(u')\, w(u', u' - u)\, du' - Z(u). \tag{18}$$

This is precisely the sought integral equation for the function $Z(u)$. Actually, using the formula (14)

and noting that $N(u, \infty) = 0$ for all $u > 0$, we find

$$Z(u) = \int_u^{u_0} w(u', u' - u) Z(u') du' + \delta(u - u_0). \quad (19)$$

The function $\nu(E)$ can be written

$$\nu(E) = \int_E^{u_0} w(u', E) Z(u') du'. \quad (20)$$

The function $Z(u)$ gives the probability of the nucleus passing through the state with energy u during the process of emission of the γ quanta. Expressions (3) and (20) for the γ spectrum coincide to the same extent that the function $Z(u)$ is equal to $1/\bar{E}(u)$.

It will be more convenient to transform (19) and (20), introducing a new function $\zeta(u) = Z[u \to \delta(u) - u_0]$. Then (19) and (20) become

$$\zeta(u) = \int_u^{u_0} w(u', u' - u) \zeta(u') du' + w(u_0, u_0 - u), \quad (21)$$

$$\nu(E) = w(u_0, E) + \int_E^{u_0} w(u', E) \zeta(u') du'. \quad (22)$$

The first term in (22) corresponds to the γ quanta emitted from the initial states of the nuclei, while the second represents the spectrum of the succeeding quanta.

By using the method of successive approximation we obtain from integral equation (21) the well known expression for the γ-ray spectrum

$$\nu(E) = w(u_0, E) + \int_0^{u_0 - E} w(u_0, E') w(u_0 - E', E) dE' + \ldots, \quad (23)$$

where each term of the series represents respectively the probability of emission of a quantum with energy E by the first, second, etc term in the chain. The effective number of terms in this series depends on E. When $E \gtrsim 1 - 2$ Mev it is found to be too large and expression (23) cannot be used in practice to calculate the spectrum, owing to the need for evaluating integrals of high multiplicity.

In that important case, when the function $w(u, E)$ is given by expression (1), the integral equation (21) can be transformed into a differential one. For this purpose, after first inserting into (21) the expression (1), for the function $w(u, E)$, we divide both halves of the equation by $\rho(u)$ and differentiate $\kappa + 1$ times with respect to u. As a result we obtain a differential equation of order $\kappa + 1$ for the function $\zeta(u)$:

$$d^{\varkappa+1} f(u) / du^{\varkappa+1} = (-1)^{\varkappa+1} \varkappa! f(u) \rho(u) / n(u), \quad (24)$$

where $f(u) = \zeta(u)/\rho(u)$ and the boundary condition at the point $u = u_0$ is:

$$(d^\lambda f(u) / du^\lambda)_{u=u_0} = \begin{cases} 0 & \text{for } \lambda = 0, \ldots, \varkappa - 1, \\ (-1)^\varkappa \varkappa! / n(u_0) & \text{for } \lambda = \varkappa. \end{cases} \quad (25)$$

In certain simple cases (24) makes a direct determination of $\zeta(u)$ possible. For a qualitative determination of the properties of the function ζ we note that the function $n(u)$ has a value $\kappa! T^{\kappa+1} \rho(u)$ for large values of u, when $u \gg \kappa T$, and has a value $u^{\kappa+1}/(\kappa+1)$ as $u \to 0$ (we assume here that the level density is a continuous function, starting with $u = 0$). Taking this into account, we can find with the aid of (24) that $\zeta(u)$ behaves like

$$w(u_0, u_0 - u) \approx (u_0 - u)^\varkappa \rho(u) / \varkappa! T(u_0)$$

at values of u close to u_0, which satisfy the inequality

$$u_0 - \bar{E}(u_0) \approx u_0 - (\varkappa + 1) T(u_0) \leqslant u \leqslant u_0,$$

like c/u (where c is a constant) as $u \to 0$, and like $1/\bar{E} \approx 1/\{(\kappa+1) T(u)\}$ in the intermediate region, where $u \gg T(u)$. These properties of $\zeta(u)$ are obvious from the point of view of the aforementioned physical meaning of the function $Z(u)$ [see Eqs. (3) and (6)]. In practice the region of energies in which $\zeta(u)$ can be represented as $[(\kappa+1) T(u)]^{-1}$ is small, and this causes (3) to be inaccurate. A plot of the function $\zeta(u)$ is shown in Fig. 6 (curve 1; the variables u and v are identical for curve 1).

The divergence of the function $\zeta(u)$ as $u \to 0$ signifies that nuclei with small excitation energies accumulate during the process of emission of quanta. As a consequence, the spectrum of the γ quanta emitted by a nucleus will also diverge as $1/E$ as $E \to 0$. This circumstance is common to all cases in which the energy density can be considered continuous, starting with small values of the energy. One can expect such a situation to correspond most closely to the case of odd-odd heavy nuclei, where the presence of two odd unpaired particles causes the level density to increase more or less uniformly, starting directly with the ground state of the nucleus. In even-even nuclei the pairing of the particles leads to the occurrence of a gap between the ground and the first excited state. For nuclei of medium atomic weights this quantity (Δ) is equal to 1.0 or 1.2 Mev. For simplicity we shall assume that in an even-even nucleus (ρ_e) there are no levels at all below an energy Δ, and that above this energy the level density is determined by the same law as in the case of odd-odd nuclei (ρ_o), i.e., we take ρ_e in the form

$$\rho_e(u) = \begin{cases} 0 & \text{for } u = \Delta, \\ \rho_0(u-\Delta) = \rho_0\rho(v) & \text{for } u \geqslant \Delta, \end{cases} \quad (26)$$

where v is the excitation energy of the even nucleus, measured from the gap: $v = u - \Delta$. The function $\rho(v)$ is normalized such that $\rho(0) = 1$.

In the presence of a gap in the spectrum of the nuclear levels, radiative transitions in even nuclei can be divided into two groups, since a high-energy transition into the ground state ($E = u$) is also possible from each excited state of the even-even nucleus, along with transitions to excited states with energies $u > \Delta$ ($E < u - \Delta$). Transitions of the former type will be called type A transitions, while those of the latter type will be called type B (see Fig. 1).

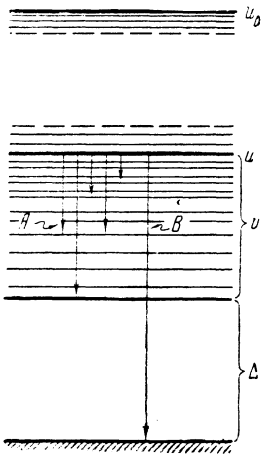

FIG. 1. Qualitative transition scheme in an even-even nucleus.

The function w(u, E) in the case of an even-even nucleus can be written in the following form

$$w_e(u, E) = w_A + w_B, \quad (27)$$

$$w_A = E^3\rho(u - E - \Delta)/n^*(u), \quad (28)$$

$$w_B = \alpha E^3\delta(u - E)/n^*(u). \quad (29)$$

The normalizing factor

$$n^*(u) = \int_0^u \{E^3\rho(u - E - \Delta) + \alpha\delta(u - E)\}\, dE$$
$$= \alpha(\Delta + v)^3 + \int_0^v E^3\rho(v - E)\, dE. \quad (30)$$

The constant $\alpha = (\overline{M_B^2}/\overline{M_A^2})\rho_0^{-1}$, where M_B and M_A are the matrix elements for type B and type A transitions. We assume here that both type A and type B transitions are dipole transitions. In addition, it is assumed, as above, that the mean square of the matrix element for type A transitions is independent of the energy.

If we now introduce instead of the function $\zeta(u)$ the function

$$\zeta(v) = (\zeta(u))_{u=\Delta+v}, \quad (31)$$

it is easy to show that both Eq. (21) and Eqs. (24) – (25) remain valid for the function $\zeta(v)$, provided the function w(u, E) is replaced by $w_A(u, E)$, and we put in the latter $u = \Delta + v$, while the function n(u) is replaced by a function n*(v), of the form

$$n^*(v) = \alpha(\Delta + v)^3 + n(v), \quad (32)$$

where n(v) is given by Eq. (2). We obtain

$$\zeta(v) = \int_0^{v_0} w_A(v', v' - v)\zeta(v')\, dv' + w_A(v_0, v_0 - v),$$
$$w_A(v, E) = (w_A(u, E))_{u=\Delta+v}. \quad (33)$$

The γ-quantum spectrum of an even-even nucleus is written in the form

$$\nu_e(E) = w_A(u_0, E) + \nu_A(E) + \nu_B(E), \quad (34)$$

where $w_A(u_0, E)$ is the spectrum of γ quanta emitted by a nucleus in the initial state (in this term we can neglect direct transitions from the initial state of the nucleus to the ground state);

$$\nu_B(E) = \alpha E^3[\zeta(v)/n^*(v)]_{v=E-\Delta} \quad (35)$$

is the γ spectrum of type B transitions, and

$$\nu_A(E) = \int_E^{v_0} (E^3\rho(v - E)/n^*(v))\zeta(v)\, dv \qquad (v_0 = u_0 - \Delta) \quad (36)$$

is the γ spectrum for type A transitions. The term $\nu_B(E)$ in (34) is taken into account when $E \geq \Delta$. Inasmuch as it is assumed that in an even-even nucleus at energy $u > \Delta$ the energy-level density coincides, accurate to a non-essential constant factor, with the level density in the odd nucleus, and the initial energy is greater than that of an odd-odd nucleus by exactly the amount Δ, then* the solution of the Eq. (21), the function $\zeta(u)$, and the photon spectrum (22) is obtained for the odd-odd nucleus as a particular case of Eqs. (34) and (36) with $\alpha = 0$. Here the variable v should be considered only as the excitation energy of the odd-odd nucleus, measured from the ground state $\Delta = 0$.

The presence of type B transitions in a nucleus with a gap can be readily analyzed qualitatively. Actually, the presence of type B transitions leads first to a weakening of the accumulation of nuclei with energy close to Δ: the function $\zeta(v)$ re-

*Actually the difference in the neutron binding energies in even-even and odd-odd nuclei somewhat exceeds Δ, which is defined as the size of the energy gap in the level spectrum of the even-even nucleus. This is not significant, since the γ-ray spectrum depends relatively little on the initial excitation energy.

mains finite at $\alpha \neq 0$ as $v \to 0$. Accordingly, the function $\nu_A(E)$ vanishes as $E \to 0$. This reduction in the number of low-energy photons is compensated by a corresponding increase in the number of transitions with energy $E > \Delta$ (see below, Fig. 6).

The parameter α can be roughly estimated by specifying the width of the energy interval δ, $0 < v < \delta$, in which type B transitions are most probable. This obviously occurs when the first term in the expression for $n^*(v)$ [formula (32)] exceeds the second. By equating the two terms, we obtain a relation between the parameters δ and α, from which we find

$$\alpha = n(\delta)/(\Delta + \delta)^3. \quad (37)$$

To obtain tentative values of the parameters we indicate, that, according to Eq. (37), the value $\delta \sim 1$ Mev corresponds to $\alpha \sim 1$ Mev, while $\delta \sim 0.2$ Mev corresponds to $\alpha \sim 0.001$ Mev.

The function $n(v)$ is a rapidly increasing function of v. Therefore when $v \gtrsim \delta$ the first term in (32) is negligibly small compared with the second. As a consequence, the spectrum of an even-even nucleus will differ substantially from that of an odd-odd nucleus only in the interval $0 < E < \delta$, where the spectrum of the even-even nucleus is characterized by a smaller number of quanta, as well as in the interval $\Delta < E \lesssim \Delta + \delta$, where the spectrum of the even-even nucleus will contain an additional contribution due to type B transitions.

Let us proceed now to a report of the results of the calculations and compare them with the experimental data. To calculate the γ-ray spectra we integrated Eqs. (24) and (33) numerically, using the Moscow State University "Strela" electronic computer. Two expressions were used for the level densities of the nucleus [the function $\rho(v)$]

$$\rho(v) = \exp(v/\tau), \quad \tau = \text{const}, \quad (38)$$

$$\rho(v) = \exp\sqrt{av}, \quad a = \text{const}. \quad (39)$$

In the first calculations we used (24) and (25) to find the function $\zeta(v)$. This differential equation was solved with automatic interval selection, insuring accuracy not less than 0.1%, using a standard program based on the Runge-Kutta method. After compiling a program for solving integral equation (33) by the Euler broken-line method, all the calculations were performed with the integral equation. An interval 0.2 Mev was chosen, since a comparison of the solutions of the integral equation with the previously obtained solutions of the differential equation had shown that such an interval insures the stipulated accuracy of 0.1%. Simultaneously with solving an integral equation, we evaluated the integral (36), and the functions $w(v_0, E)$ and $\nu_B(E)$. These functions, together with the function $\zeta(v)$ and $n^*(v)$, were obtained from the computer for points from 0.2 to 6.4 Mev every 0.2 Mev. The parameter v_0 was taken in all cases the same, 6.4 Mev, corresponding approximately to the average neutron binding energy in odd-odd nuclei. In most calculations a value of 1.2 Mev was used for Δ, corresponding approximately to the position of the peak in the experimental spectra of even-even nuclei (see below). Values in the interval from 0.4 to 2.5 Mev were taken for τ in (38), and values in the interval from 2 to 30 Mev^{-1} were used for the parameter a [Eq. (39)]. The constant α was taken to be 0, 1/1000, 1/400, 1/32, and 1/2 Mev.

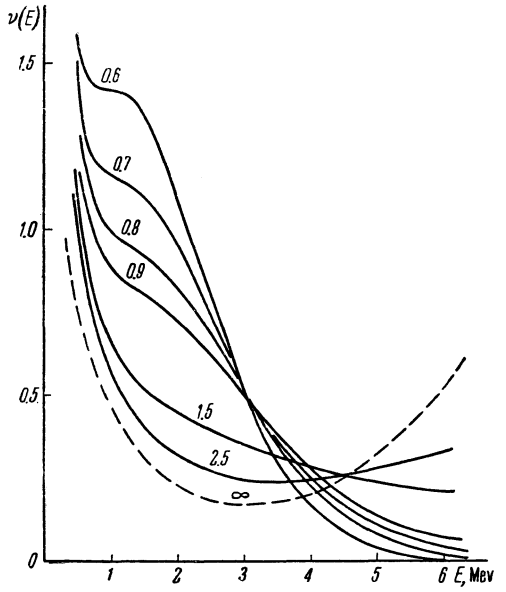

FIG. 2. Spectra of γ quanta, calculated for a level density as given by (38). The values of the parameter τ (in Mev) are indicated in the diagram ($\alpha = 0$).

Figure 2 shows several spectra of odd nuclei ($\alpha = 0$), calculated by means of (38) for level densities at certain values of the parameter τ. Figure 3 also shows several theoretical spectra of odd nuclei, calculated for a level density as given by Eq. (39).

The dotted curve of Fig. 2 shows the photon spectrum for $\tau = \infty$ (or $a = 0$). This spectrum was obtained from an analytic solution of the differential equation (24) and from an evaluation of the integral $\nu_A(E)$, [Eq. (36)], which are possible when $\rho = \text{const}$. As shown in the paper by Groshev et al.,[4] the best agreement with experiment is obtained at $\tau = 0.8$ Mev in the case of (38) or $a = 15$ Mev^{-1} for the case of (39) [corresponding

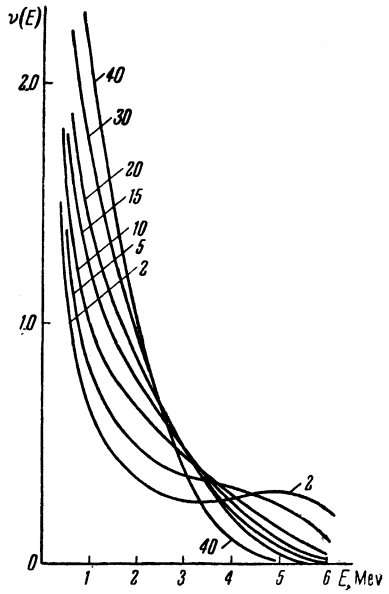

FIG. 3. Gamma spectra calculated for a level density as given by Eq. (39). The curves are labeled with values of the parameter a (in Mev^{-1}) ($\alpha = 0$).

to this value of a is an initial nuclear temperature $T_0 = (4u_0/a)^{1/2} \approx 1.3$ Mev]. Such a ratio between the "equivalent" values of the parameters τ and a is quite natural, if one considers that the average nuclear temperatures should be close.

In exactly the same manner it is possible, for other values of τ, to make the curves $\nu(E)$ obtained in the cases (38) and (39) close to each other by suitable choice of the parameter a. There is, however, a slight qualitative difference between the spectra shown in Figs. 2 and 3. The spectra calculated for an exponential density have a characteristic inflection in the region of $E \sim 1$ Mev. There is no such inflection in curves shown in Fig. 3 [where the density is taken from (39)]. The inflection is due to the fact that the first term in (34) is a bell-shaped curve (see Fig. 4), and the second term $\nu_A(E)$ is a monotonically decreasing function. In the case of version (39), owing to the greater dependence of the level density on the energy at $u \lesssim 1$ Mev, the spectrum $\nu_A(E)$ is greatly enriched by soft quanta, and the presence of the maximum in the function $w(u_0, E)$ does not appear explicitly in the form of the summary spectrum.

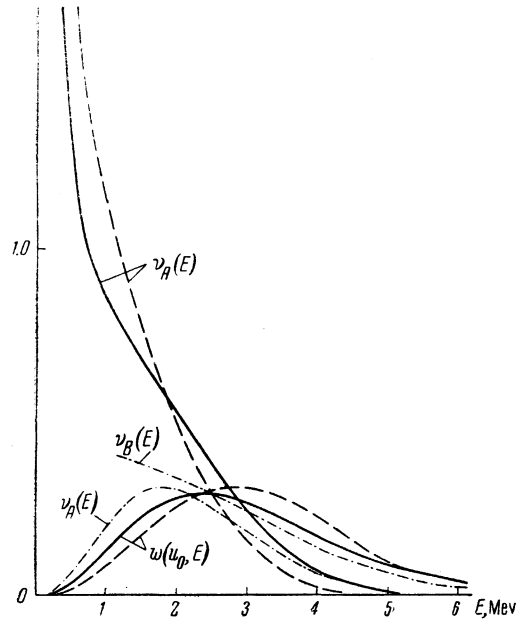

FIG. 4. Component parts of the complete γ spectrum [see (35)]. The solid curves correspond to $\alpha = 0$ for the level density given by (38) ($\tau = 0.8$ Mev); the dotted lines are the same for the case of (39) ($a = 15$ Mev^{-1}), while the dash-dot curve corresponds to $\alpha = 0.5$ Mev and $\tau = 0.8$ Mev.

The presence of an inflection at $E \sim 1$ Mev in the spectra of all the investigated deformed odd-odd nuclei[4] is an indication that in these nuclei the level density increases at low energy ($\lesssim 1$ Mev) more smoothly than $\exp\sqrt{au}$. It is interesting to note that no such inflection is observed in the spectra of spherical nuclei.

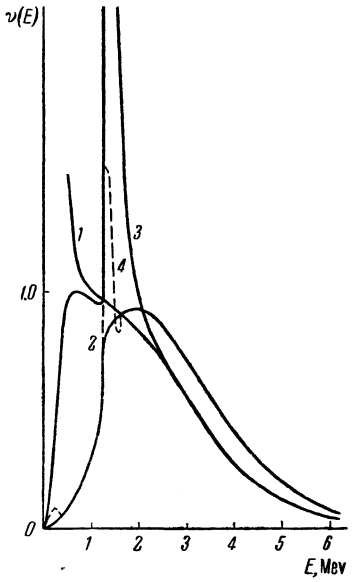

FIG. 5. Spectra of even-even nuclei, calculated for level densities of (38) ($\tau = 0.8$ Mev): curve 1 – at $\alpha = 0$, 2 – $\alpha = 0.5$ Mev, 3 – $\alpha = 1/400$ Mev, 4 – α taken in the form of a step (curve 1 of Fig. 7), $\alpha_1 = 1/400$ Mev, $\alpha_2 = 0.5$ Mev, $v^* = 0.4$ Mev.

We see by comparing the curves shown in Fig. 5 that in the quantum energy region $E \gtrsim 5$ Mev the contribution of the function $\nu_A(E)$ to the total spectrum is quite small (< 10%). Radiative transitions from such an energy are almost exclusively transitions from the initial state. Their average intensity is approximately

$$w(u_0, u_0) \approx u_0^3 \rho(0) / n(u_0) \approx u_0^3 / 6\tau^4 \rho(u_0) \approx 0.5\%, \quad (40)$$

and differs little from the experimental value (~ 0.5 – 3%).

Figure 5 shows the γ spectra calculated with the aid of (32) – (36) for $\alpha \neq 0$ (even-even nuclei) and $\tau = 0.8$ Mev (exponential energy dependence of the level density). Curves of ζ functions for these values of the parameters are given in Fig. 6.

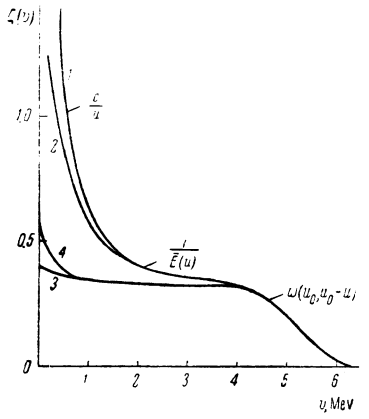

FIG. 6. Plot of the function $\zeta(v)$, calculated for a level density as given by (38) for the following values of the parameters: $1-\alpha=0$ [odd-odd nucleus, $v=u$, cf. (31)]; $2-\alpha=1/400$ Mev, $3-\alpha=0.5$ Mev; $4-\alpha$ as a function of the energy (curve 1 of Fig. 7), $\alpha_1=1/400$ Mev; $\alpha_2=0.5$ Mev, $v^*=0.4$ Mev, $\tau=0.8$ Mev.

When compared with the odd-odd nuclei, the experimental spectra of deformed even-even nuclei are characterized by a large number of quanta of relatively large energies ($\sim 2-4$ Mev), while the spectra contained relatively fewer soft quanta ($E \lesssim 1.5$ Mev) and, in addition, a sharp peak exists with a width $\lesssim 0.5$ Mev at $E \sim 1$ Mev.[4] These first two distinctions suggest the choice of a relatively large value of α. For an exponential density [Eq. (38)] the best agreement from experiment was obtained at $\alpha = 0.5$ Mev and $\tau = 0.8$ Mev,[4] i.e., for the same value of τ as for the odd nuclei located in the same region. The aforementioned similarity in the density as given by (39) leads in the case of even-even nuclei to a considerable difference in the spectra calculated by means of (38) and (39). In the case of (39) the γ spectrum is characterized by much greater values of $\nu(E)$ in the region $E \lesssim 1$ Mev, and diminishes more rapidly with increasing E than the spectrum calculated for an exponential level density. Therefore, in order to make the theoretical spectrum calculated for the density given in (39) closer to the experimental spectrum of the even-even deformed nucleus, it becomes necessary to take $a = 5$ Mev^{-1}. So small a value of this parameter ($T_0 \approx 2.3$ Mev) is very unlikely, particularly if it is considered that the value obtained for a of odd nuclei is 15 Mev^{-1}. It is therefore necessary to conclude that the form of the spectrum of the even-even nuclei is evidence of a weak dependence of the level density of deformed even-even nuclei on the energy above the gap, at least compared with (39).

Let us return now to an examination of the third singularity of the spectra of the even-even deformed nuclei, namely the presence of a sharp peak at $E \sim 1$ Mev. In principle, such a peak is present at a small value of α, on the order of $1/400 - 1/1000$ (see Fig. 5); however, at such a value of α the theoretical spectrum does not differ in practice from the spectrum for $\alpha = 0$ (i.e., the spectrum of the odd-odd nucleus) and all the remaining region, with the exception of the region of the peak and, accordingly, in the region $E \gtrsim 0.5$ Mev (see curves 1 and 3 in Fig. 5). On the other hand, at α on the order of unity, at which the general form of the theoretical spectrum is in good agreement with the experimental spectrum of deformed even-even nuclei, there is no sharp maximum at all at $E \sim \Delta$; the population of the levels of the nucleus with energy $u \gtrsim \Delta$ is found to be small, owing to the competition of type B transitions, which in this case is substantial even for large nuclear excitation energy.

It is natural to attempt to resolve this contradiction by considering the dependence of α on the excitation energy of the nucleus. In fact, all three singularities of the spectrum of deformed even-even nuclei can be explained qualitatively by assuming that the parameter depends on the energy as shown in Fig. 7 (curve 1), taking $v^* \approx 0.4$ Mev.

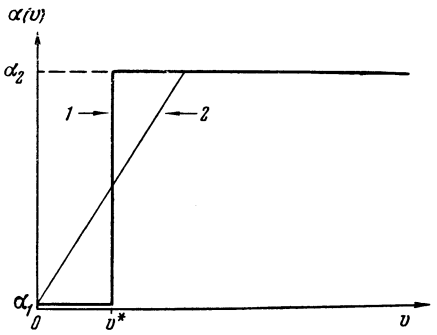

FIG. 7. Two versions of the function $\alpha(v)$ for which the spectra were calculated.

At such a value of α the γ spectrum will coincide with the spectrum calculated for $\alpha = 0.5$ in the quantum energy region $v^* \lesssim E < \Delta$ and $E > \Delta + v^*$, while in the region $\Delta < E \lesssim \Delta + v^*$ the spectrum will have a sharp maximum, as in the case of small α. This can be readily established by considering the dependence of $\nu_B(E)$ on α [$\nu_B(\Delta) = \{\zeta(v)\}_{v=0}$ is independent of α]. The γ spectrum was calculated also for variable α, in the form of a step (curve 1 of Fig. 7), and a ramp function (Fig. 2) for the following values of the parameters: $\alpha_2 = \alpha_1 + 0.5$ Mev, $\alpha_1 = 1/400$ Mev, $v^* = 0.4$ and 0.8, and $\tau = 0.7$, 0.8, and 0.9 Mev. The parameter Δ was taken to be 0.8 and 1.2 Mev. The density of the levels is specified in the form (38).

For illustration, Fig. 5 shows also one of the spectra calculated for the function $\alpha(v)$, taken in the form of a step. The discontinuities there are connected with the presence of a sharp jump in α and, naturally, will be smooth if the region of the sharp variation $\alpha(v)$ is somewhat smeared out. The spectra calculated for the ramp curve 2

of Fig. 7 do not differ in practice from the spectra at constant $\alpha = \alpha_1$. The spectrum shown in Fig. 5 for variable α has all the features of the experimental spectra of even-even nuclei, on the basis of which one can conclude that in the region ~ 0.5 Mev above the gap, the value of the parameter α is very small compared with the entire remaining energy region. The specific value of the parameter α near the gap (α_1) is not important here for the essential singularities of the form of the spectrum.

At the present time the structure of the levels of deformed even-even nuclei in the energy region above the gap has not yet been sufficiently investigated, and one can only guess at the reasons for the forbiddenness of the transitions through the gap for not too high a nuclear excitation. Such a forbiddenness could occur were the emission of the γ quanta to result in production of nuclei with large spins. This, however, can hardly be the cause of the forbiddenness, since near the ground state of even-even deformed nuclei there are always closely-located rotational levels, with sufficiently large spins, to which a radiative transition would be possible. A selection of transitions based on parity can also not lead to a sharp change in α, for the selection of transitions to the ground state by parity takes place also for transitions from strongly excited states.

A possible explanation of an energy dependence of α similar to that shown in Fig. 7 may be the presence of an additional forbiddenness based on the projection of the momentum on the nuclear axis in deformed nuclei (the quantum number K). The ground state is characterized by $K = 0$. A selection rule based on K, $\Delta K = 0$, and 1, would pick out in the case of dipole transitions only those excited states for which $K = 0$ or 1. If it is assumed that states with such K are missing in the vicinity of the gap, with the exception possibly of the lowest state, the dipole transitions would be impossible, and the radiative transitions through the gap would be less probable. Such a picture is confirmed by the well-studied decay schemes of W^{182} (reference 6) and many other deformed even-even nuclei.[7] On the other hand, the quantum number K is only an approximate adiabatic integral of the motion, and one might think that the forbiddenness connected with K does not play a substantial role, if one speaks of a transition into the ground state from strongly excited states of the nucleus: the wave function of such states is apparently a superposition of states with different values of K, including also such for which the transition to $K = 0$ is allowed.

This allows us to assume that the ratio of the matrix elements $\overline{M_A^2}/\overline{M_B^2}$ is close to unity. The factor ρ_0 in front of the exponent in formula (38) then coincides with α^{-1}, whence, using the values of the parameters α and τ, determined from γ spectra of even nuclei, we can estimate the average distance between the nuclear levels at the binding energy. This value should coincide in order of magnitude with the average distance between the neutron resonances (change in the nuclear spin upon emission of quanta is small and can be neglected). One can verify that this actually takes place. Reversing this argument, we could conclude, on the basis of the resultant value of α (~ 0.5 Mev), that the matrix elements are approximately equal. We emphasize, however, that we can speak only of a rough estimate.

For certain values of the parameters we also calculated the spectrum, for constant α, under the assumption that the radiative transitions through the gap (type B) are quadrupole transitions. In all cases the form of the computed spectrum differs greatly from the experimental one.

It is of interest to compare the data on the density distribution of the nuclear levels with results of other methods of determining this quantity. The most direct method is a direct calculation of the number of levels below a given energy. This method was recently used by Erikson[8] to determine the energy dependence of the level density below the neutron binding energy for certain nuclei with $A < 60$. The level distribution agrees with (38), and the parameter τ is found to be $1.0 - 1.2$ Mev. This result agrees with the determination of the level density from the spectrum of secondary particles in the (α, α') reaction in the same region of nuclei.[9] For the (p, p') reaction, a somewhat better agreement with formula (39) is indicated.[10] All these results pertain, unfortunately, to light nuclei, which perhaps explains the discrepancy between the value of the parameter τ, determined from the γ spectra (0.8 Mev), and that determined directly from the level density ($1.0 - 1.2$ Mev). We note incidentally that the results of the analysis of the γ spectra indicate the absence of a strong variation on the parameters that characterize the energy dependence of the level density as A varies from 100 to 200.[4] The absence of a noticeable dependence of the temperature on the atomic weight has been also noted in reference 10. On the other hand, the spectra of nuclei close to magic are characterized by relatively large values of τ (or, respectively, by small values of a). Thus, for gold the best agreement with the experimental

spectrum is obtained when $a = 3 - 5$ Mev^{-1}. We note also that the average distribution of the intensity in the spectra of light odd-odd nuclei[5] is in qualitative agreement with the theoretical values, calculated under the assumption $\rho = \text{const}$ (Fig. 3), which does not contradict other data on the level densities of these nuclei.

The temperature of the nucleus at an energy equal to the binding energy of a neutron can be determined also from the density of the neutron resonances. The value of the initial temperature of nuclei in the rare-earth region is found to be $0.6 - 0.7$ Mev, corresponding to a value of the parameter $a = 40 - 60$ Mev^{-1}. This value differs substantially from the value indicated above, $a = 15$ Mev^{-1}, obtained from the γ spectra. The discrepancy is considerably reduced it it is taken into account that the expression for the density of levels in the given momentum for a Fermi gas contains a factor in front of the exponent, which diminishes as u^{-2} with energy. If we attempt, however, to approximate the function $u^{-2}\exp\sqrt{au}$ by expressions of the form $\exp[(a^*u)^{1/2} + c]$, choosing the constants a^* and c in such a way as to make these expressions coincide at the points, say, $u = 2$ and 6.5 Mev, we obtain for the constant a^* a somewhat smaller value. Thus, at $a = 30$ or 60 Mev^{-1} the constant a^* becomes respectively 20 and 35 Mev^{-1}. It is precisely these values of the "effective" constant a^* that should be compared with the value obtained from the γ spectra or in other similar cases, if the level density is given by an expression of form (39).

If analogously we approximate the function $u^{-2}\exp\sqrt{au}$ by the function $\exp(u/\tau + c)$, we obtain for the constant τ a value ~ 0.7 Mev at $T_0 = 0.65$ Mev, determined from the neutron resonances, i.e., a value which does not greatly differ from the 0.8 Mev obtained from the γ spectra.*

As regards the choice between the distributions (38) and (39), as already noted, both laws lead to quite similar results. The singularities in the form of spectra of odd-odd deformed nuclei, and a comparison of the theoretical spectra with those of even-even deformed nuclei, nevertheless make it preferable to use for these nuclei the exponential law (38), or at least to conclude that the level density depends less on energy near the gap than called for by Eqs. (26) and (39).

This result is in agreement with what is obtained when nucleon pairing is considered. For deformed nuclei the presence of pairing between particles leads to a difference in the law of dispersion of the elementary excitations from the dispersion of free particles. The excitation energy is given by the following expression[11]

$$E_\nu \approx \sqrt{\varepsilon_\nu^2 + (\Delta/2)^2}, \qquad (41)$$

where ϵ_ν is the Fermi energy of the free particle, measured from the boundary. The level density of the elementary excitation $d\nu/dE_\nu$ diminishes with energy. If we speak of an odd-odd nucleus or an even-even nucleus with one broken pair, i.e., at $\Delta < u \lesssim 2\Delta$, where there are two free quasiparticles, the level density remains almost constant. As the excitation energy is increased, the number of broken pairs of particles increases and the level density begins to increase rapidly, and on the average this increase is closer to exponential.[12] In other words, in the presence of the finite energy to be spent on "liberating" the particle, the temperature of the system depends less on the energy than for free particles. Finally, at a certain sufficiently large energy the effect of pairing vanishes, and the level density is given by an expression of the type (39).

The results of the analysis of γ spectra of deformed nuclei are in full agreement with the picture drawn. Unfortunately, in spherical nuclei the grouping of levels in shells and subshells makes it very difficult to describe even qualitatively the dependence of the level density on the energy. The fact that the level density of odd-odd spherical nuclei in the energy region $\lesssim 1$ Mev increases more rapidly than $\exp(u/\tau)$ can be attributed either to the smallness of the pairing energy, or in general to a particle-dispersion law differing from (41). For two free odd particles the level density increases as u^2.

The authors express their gratitude to Professor A. N. Tikhonov and A. V. Luk'yanov for participating in the calculations on the Moscow State University electronic computer.

*These results were obtained jointly with Yu. V. Adamchuk.

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Translated by J. G. Adashko
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