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CIRCULAR POLARIZATION OF GAMMA QUANTA ACCOMPANYING NUCLEAR CAPTURE OF SLOW NEUTRONS

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An estimate is made of the order of magnitude of the circular polarization and the anisotropy of the angular distribution of the cascade quanta emitted by a previously polarized compound nucleus. From a determination of the sign of the mean circular polarization for the cascade it is possible to establish the spin of the original state of the compound nucleus, and a study of the spectral distribution of the polarization and the anisotropy of the angular distribution of the quanta can provide information about the way in which the level density of the compound nucleus depends on the spin.

INTRODUCTION

UPON capturing a neutron, the nucleus emits a number of quanta in the process of transition to the ground state. Spectra of the capture γ quanta have recently been studied intensively by Groshev's group[1] and the following relationships have been established: (1) In the region of atomic number A < 100, and in the neighborhood of the magic-number nuclei, the capture γ spectra possess a line structure, each isotope having individual peculiarities in the distribution of the lines over the energy interval; (2) In the region 100 < A < 200, i.e., sufficiently far away from the magic numbers, the spectrum of the n-capture quanta appears to be continuous (or unresolved) with superimposed individual intense lines whose contribution, however, is small compared with that of the continuum.

In the case of the discrete spectra, the problem of studying the nuclear level structure reduces to an investigation of the characteristic lines of the spectrum of each isotope.

Experience has shown that the continuous spectrum of a heavy nucleus, 100 < A < 200, has a number of properties that are typical of the entire interval of A:

1. The distribution of quanta, $\hbar\omega\nu(\hbar\omega)$, where $\nu(\hbar\omega)$ is the number of quanta with energy $\hbar\omega$, is bell-shaped with parameters varying little from one isotope to its neighbor. The maximum of the distribution occurs in the interval $\hbar\omega \simeq 1.5$ to 2.5 Mev.[1]

2. On the average, $\overline{N} \approx 4 \pm 1$ quanta with energies $\hbar\omega \geq 0.3$ Mev and approximately one quantum with an energy $\hbar\omega < 0.3$ are emitted in a cascade.[1]

3. The mean radiation width for n capture in this region of A varies little over the entire interval from A = 100 to 200.[2]

From the general properties of the n-capture process enumerated above, it can be assumed that within the given range of A the spectra of n-capture quanta can be described by the simple statistical model used in a number of papers[2-4] to calculate the mean radiation widths of neutron resonances. In this model the spectrum $\nu(\hbar\omega)$ is determined by the density of nuclear levels $\rho(\epsilon)$ and on the dependence $W_L(\hbar\omega)$ of the emission probability for a quantum of multipolarity L upon the energy of the quantum. It is usually assumed that all the quanta have the same multipolarity.

The shape of the spectrum $\nu(\hbar\omega)$ of the n-capture quanta for the statistical model was first studied by Nosov and Strutinskiĭ, and the results of their calculations are recorded in a review paper by Groshev.[1] In that paper the form

$$W_L(\hbar\omega) = \text{const}\cdot(\hbar\omega)^{2L+1}$$

is used for the emission probability of a quantum with momentum L and energy $\hbar\omega$, and the nuclear level density $\rho(\epsilon)$ is found in the usual way from the entropy S and the temperature T corresponding to the given excitation energy ϵ of the nucleus:

$$\rho(\varepsilon) = e^{S(T)}/\Delta\varepsilon; \quad T = d\varepsilon/dS,$$

while for the nuclear specific heat $C(T) = d\epsilon/dT$ a power-law dependence $C(T) = aT^{\lambda}$ is assumed.

It has been shown that the observed shape of the spectrum $\nu(\hbar\omega)$ can be explained within the framework of the statistical theory of cascades, and according to Groshev[1] the best agreement with experiment is attained when $L = 1$ and $\lambda = 1$. In particular, for the probability of emission of N quanta in cascade[1] we have

$$W(N) = \sqrt{(L+1)/2\pi\bar{N}} \exp[-(L+1)(N-\bar{N})^2/2\bar{N}].$$

Comparison of the calculated spectrum with the experimentally observed spectrum $\nu(\hbar\omega)$ permits the parameters a and λ of the nuclear level density $\rho(\epsilon)$ to be determined. However, $\nu(\hbar\omega)$ contains no information on the density $\rho_I(\epsilon)$ of nuclear levels with a particular spin I. Information about $\rho_I(\epsilon)$ can be obtained from data on the angular characteristics of the cascade quanta emitted from a previously polarized compound nucleus, such as: (1) the circular polarization $e(\hbar\omega, \theta)$ of the cascade γ quanta; (2) the angular distribution $F(\hbar\omega, \theta)$ of the cascade γ quanta with respect to the spin orientation direction of the compound nucleus in its initial state, $\mathbf{I}_0$; (3) the residual spin polarization of the compound nucleus in one of the final states taken up by the nucleus after the emission of a cascade of γ quanta; (4) the angular correlation $W_{12}(\theta_{12})$ of any pair of quanta emitted successively in a cascade sequence.

It should be mentioned here that a beam of polarized neutrons and an unpolarized target are sufficient for a study of the circular polarization $e_{I_0}(\hbar\omega, \theta)$ of cascade quanta and the residual spin polarization of the compound nucleus, whereas the investigation of the angular distribution $F_{I_0}(\hbar\omega, \theta)$ of the quanta requires a polarized target.

For the study of the angular correlation between cascade quanta, neither a polarized beam nor a polarized target are required. However, as a result of the averaging of the anisotropy over all the cascade quanta, the angular correlation turns out to be beyond the limits of experiment. To illustrate the relationships of the quantities $e_{I_0}(\theta)$, $F_{I_0}(\theta)$, and $W_{12}(\theta_{12})$, let us consider the simple case of a four-quantum cascade $I_0 \rightarrow I_1 \rightarrow I_2 \rightarrow I_3 \rightarrow I_4$ with the fixed values $I_0 = 4$ and $I_4 = 1$. The spins I_1, I_2, and I_3 take on all possible values with equal probability. We shall assume that all the quanta of the cascade are dipole quanta ($L = 1$). In this case there are four distinct cascade chains, 16 quanta are emitted, and a correlation is possible between 24 pairs of quanta. A simple calculation leads to the following values for the quantities $e_{I_0}(\theta)$, $F_{I_0}(\theta)$, and $W_{12}(\theta_{12})$:

$$e_{I_0}(\theta) = 1.34\,\zeta_1(I_0)\cos\theta/(1 + 0.27\zeta_2(I_0)P_2(\cos\theta)),$$
$$F_{I_0}(\theta) = 1 + 0.27\,\zeta_2(I_0)P_2(\cos\theta),$$
$$W_{12}(\theta_{12}) = 1 - 0.025\,P_2(\cos\theta_{12}).$$

Here $\zeta_\kappa(I_0) = \sum_{\mu_0} f_{\mu_0} C^{I_0\mu_0}_{I_0\mu_0\kappa 0}$, where f_{μ_0} is the distribution of the compound nucleus in its initial state according to magnetic quantum number ($\sum_{\mu_0} f_{\mu_0} = 1$), and $C^{a\alpha}_{b\beta c\gamma}$ is a Clebsch-Gordan series coefficient.

If the polarization $\mathbf{I}_0$ of the nuclear spin is accomplished by the capture of a polarized neutron by an unpolarized target nucleus with a spin of $J = 7/2$, then $\zeta_2(I_0) = 0$, and $\zeta_1(I_0) = 3\sqrt{5}\,\eta_n/32$ (η_n being the degree of polarization of the neutron). In this case $e(\theta) = 0.45\,\eta_n \cos\theta$. Thus in this example $e_{I_0}(\theta)$ is approximately twenty times larger than the anisotropy $W_{12}(\theta_{12})$; when the average is taken over a large number of different chains, the ratio will increase even more.

The quantities $e_{I_0}(\hbar\omega, \theta)$ and $F_{I_0}(\hbar\omega, \theta)$ will henceforth be considered only for odd target nuclei, so that the compound nucleus will be even-even. The results of the calculation given below depend in a fundamental way on the cascade scheme which is assumed, and numerical values for $e_{I_0}(\theta)$ and $F_{I_0}(\theta)$ can be obtained only for specific models of the distribution $\rho_I(\epsilon)$. Consequently the calculation is of the nature of a guess at the scale of the quantities e_{I_0} and F_{I_0} from a model.

In what follows we shall make use of the following γ-cascade scheme for an even-even compound nucleus (Fig. 1). Upon resonance capture of a neutron by the target nucleus, the compound nucleus (by the emission of N quanta) drops into one of the levels in an interval of width Δ lying just above the energy gap of the nuclear spectrum (the gap width being 2Δ).

The subsequent transition from this interval to one of the states close to the ground state occurs by the emission of a characteristic quantum which appears as an individual intense line in the spectrum $\nu(\hbar\omega)$.

In the interval Δ above the gap of 2Δ, the density of nuclear levels is relatively low, and the γ transitions between levels in this interval depend on the structural peculiarities of the given nucleus. In this case there may be a violation of the basic assumption of a statistical model for the cascade, which has been assumed to be valid for all the γ transitions, in the neighborhood of the spectral region indicated above. In order to exclude these transitions, it is necessary to limit the continuous spectrum to the portion $\hbar\omega > \Delta$ and to ignore the characteristic lines of the spectrum.

For the cascade scheme chosen above, the final state of the nucleus after the emission of N quanta is not the ground state, but one of the states in the

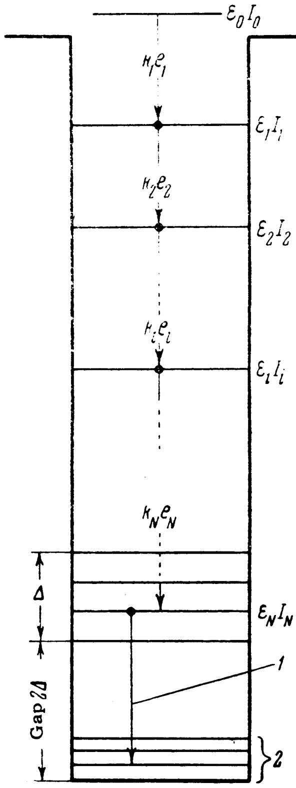

FIG. 1. The cascade scheme used in the estimation of the circular polarization and angular distribution of quanta from (nγ) capture. 1 is the characteristic quantum in the γ spectrum of the n-capture; 2 represents the collective excitation of the nucleus.

interval Δ above the gap. Similarly, the nuclear spin I_N after the cascade emission may take on any value which is permitted in that interval.

At the present time there is no evidence available about the energy dependence $\rho_I(\epsilon)$, either for the interval Δ above the gap or for the remaining portion of the nuclear spectrum. Therefore a calculation of the magnitudes of $e_{I_0}(\hbar\omega, \theta)$ and $F_{I_0}(\hbar\omega, \theta)$ can be made only for specific models of the distribution $\rho_I(\epsilon)$. We shall assume hereafter that the level density for a nucleus with a given spin I can be approximated in the factorable form $\rho_I(\epsilon) \approx \rho(I, \alpha)\rho(\epsilon)$, where $\rho(I, \alpha)$ does not depend on the nuclear excitation energy.

Numerical estimates have been made for two model distributions:

a) In the free nucleon Fermi-gas model, according to Bethe[5] we have for $\rho_I(\epsilon)$

$$\rho_I(\varepsilon) = \text{const}\cdot\rho(\varepsilon)(2I+1)\exp[-I(I+1)/\alpha^2].$$

Here $\alpha^2 = 2J_0\tau/\hbar^2$; $J_0 = \frac{2}{5}MAR_0^2$ is the moment of inertia of the nucleus considered as a solid body; and τ is the nuclear temperature in Mev corresponding to the nuclear excitation energy ϵ. In this model our assumption leads to a value of α^2 which depends weakly on ϵ, instead of being a constant. If the temperature of the initial nuclear state is $\tau = 0.5$ and $A = 150$, we obtain the value $\alpha = 10$, and so the numerical calculation below has been made for the interval $\alpha = 4$ to 10.

b) As a second example, consider $\rho_I(\epsilon)$ in the form

$$\rho_I(\varepsilon) = \text{const}\cdot\rho(\varepsilon)(2I+1)\exp\{-I(I+1)/\alpha^2\},$$

in the interval $\epsilon > 3\Delta$;

$$\rho_I(\varepsilon) = \begin{cases} \text{const for } I_N \leqslant I_{N\,max} \\ 0 \quad \text{for } I_N > I_{N\,max} \end{cases}$$

in the interval $3\Delta > \epsilon > 2\Delta$. Let us assume in addition that the probability of a radiative transition between two levels $I_i \to I_{i+1}$, averaged over the group of nuclear levels with moments I_i and I_{i+1}, may be expressed in the form

$$W_{LI_iI_{i+1}}(\hbar\omega) = W_L(\varepsilon_i, \varepsilon_{i+1}, \hbar\omega)\,\mathfrak{M}_L(I_i\ I_{i+1}),$$

where the factor $\mathfrak{M}_L(I_iI_{i+1})$ does not depend on the excitation energy of the nucleus. In calculations of the radiation widths of neutron resonances (references 2 — 4 and 1) it is assumed that $\mathfrak{M}_L = 1$ and $W_L = \text{const}\cdot(\hbar\omega)^{2L+1}$. As in reference 1, we consider a pure L-cascade, and numerical estimates of $e_{I_0}(\theta)$ and $F_{I_0}(\theta)$ have been made below only for $L = 1$ (M1 and E1 cascades) with spectra $\rho_I(\epsilon)$ of types a and b.

1. RELATIVE PROBABILITY OF AN N-QUANTUM CHAIN

Suppose that, as a result of neutron capture by a target nucleus with spin J, a compound nucleus has been formed in a state with momentum I_0 and energy ϵ_0. The spin $\mathbf{I}_0$ is so oriented in space that the probability of its projection μ_0 on some direction singled out by the experimental conditions is equal to f_{μ_0}; $\sum_{\mu_0} f_{\mu_0} = 1$.

During the emission of N quanta the nucleus passes successively through $N-1$ intermediate states with energies ϵ_i and spins I_i and μ_i ($i = 1, \ldots, N-1$), and appears in one of the states ϵ_N, I_N, μ_N in the interval Δ of the nuclear spectrum above the gap. Let us find the relative probability of emission of a chain of N quanta, in which the nucleus passes through a definite sequence of states $I_0 \to I_1 \to \ldots I_i \to \ldots I_N$. Let us also fix the wave vector $\mathbf{k}_i$ and polarization $\mathbf{e}_i$ of each quantum. For brevity we shall denote the chain of nuclear states by $I_0 \to c \to I_N$.

For the probability of the nuclear transition

$$\varepsilon_{i-1},\ I_{i-1},\ \mu_{i-1} \xrightarrow{L,\ k_i,\ e_i} \varepsilon_i,\ I_i,\ \mu_i$$

with the emission of a quantum $\hbar\omega_i$ in the direction $\mathbf{n}_i = \mathbf{k}_i/k_i$ we have:

$$W^{I_{i-1}I_iL}_{\mu_{i-1}\mu_i}(\hbar\omega_i\mathbf{n}_ie_i) = W_{LI_{i-1}I_i}(\hbar\omega_i)\, S_{\mu_{i-1}\mu_i}(\mathbf{n}_i e_i),$$

$$S_{\mu_{i-1}}(\mathbf{n}_ie_i) = 0.5\left|C^{L+\Lambda_i 0}_{1-1L1}\right|^{-2}\left|\sum_{M_i} C^{I_{i-1}\mu_{i-1}}_{I_i\mu_iLM_i}(e_i\mathbf{Y}^{\Lambda_i}_{LM_i}(\mathbf{n}_i))\right|^2,$$

$$\sum_{\mu_i e_i}\int d\Omega_i S_{\mu_{i-1}\mu_i}(\mathbf{n}_ie_i) = 1.$$

Let us now proceed to the relative probability of the γ transition

$$\gamma_{I_{i-1}I_i;\,\mu_{i-1}\mu_i}(\hbar\omega_i,\mathbf{n}_i,e_i) = \gamma_{I_{i-1}I_iL}(\hbar\omega_i)\, S_{\mu_{i-1}\mu_i}(\mathbf{n}_i,e_i),$$

where

$$\gamma_{I_{i-1}I_iL}(\hbar\omega_i) = \frac{W_{LI_{i-1}I_i}(\hbar\omega_i)}{\sum_{I_i}\int_0^{\varepsilon_{i-1}} W_{LI_{i-1}I_i}(\hbar\omega_i)\,\rho_{I_i}(\varepsilon_i)\,d\hbar\omega_i};$$

and for the relative probability of the emission of N quanta we find, in the cascade $(I_0, \mu_0, k_1, e_1) \rightarrow \ldots (I_i, \mu_i, k_i, e_i) \ldots \rightarrow (I_N, \mu_N, k_N, e_N)$:

$$P_N = \Big(\prod_{i=1}^N \gamma_{I_{i-1}I_iL}(\hbar\omega_i)\Big)\Big(\prod_{i=1}^N S_{\mu_{i-1}\mu_i}(\mathbf{n}_i,e_i)\Big).$$

Experimentally, the detector determines the quantum energy $\hbar\omega$, the direction of travel of the quantum $\mathbf{n}$ with respect to the direction of the nuclear spin polarization $\mathbf{I}_0$, and the circular polarization e of the quantum, but not the position of the quantum emission in the sequence, or the form of the cascade chain.

We shall therefore find the relative probability of emission of a quantum with energy $\hbar\omega$ and $\mathbf{n}$ and e at any position in the cascade chain $I_0 - c - I_N$

$$P_{I_0cI_N}(\hbar\omega;\mathbf{n},e) = \sum_{i=1}^N\Big\{\underbrace{\int\cdots\int}_N \delta(\hbar\omega-\hbar\omega_j)$$

$$\times\prod_{i=1}^N d(\hbar\omega_i)\,\gamma_{I_{i-1}I_iL}(\hbar\omega_i)\,\rho_{Ii}(\varepsilon_i)\,\delta\Big(E_{0N}-\sum_{i=1}^N\hbar\omega_i\Big)\Big\}$$

$$\times\Big\{\sum_{\mu_0} f_{\mu_0}\sum_{e_i\mu_i}\delta_{ee_j}\underbrace{\int\cdots\int}_N\delta(\mathbf{n}-\mathbf{n}_j)\prod_{i=1}^N d\Omega_i\, S_{\mu_{i-1}\mu_i}(\mathbf{n}_i,e_i)\Big\},$$

where $E_{0N} = \epsilon_0 - \epsilon_N$ is the energy of the nuclear transition. Taking $\rho_{iI}(\epsilon_i) = \rho(I_i,\alpha)\rho(\epsilon_i)$ and $W_{LI_{i-1}I_i} = W_L\mathfrak{M}_L(I_{i-1}, I_i)$, we obtain for $P_{I_0cI_N}(\hbar\omega;\mathbf{n},e)$

$$P_{I_0cI_N}(\hbar\omega;\ \mathbf{n},\ e) = \sum_{j=1}^N W_{jN}(\hbar\omega)\, G_{jI_0cI_N}(\mathbf{n}e)\cdot R_{I_0cI_N};$$

$$W_{jN}(\hbar\omega) = \underbrace{\int\cdots\int}_N\delta(\hbar\omega-\hbar\omega_j)\,\delta\Big(E_{0N}-\sum_{i=1}^N\hbar\omega_i\Big)$$

$$\times\prod_{i=1}^N d(\hbar\omega_i)\rho(\varepsilon_i)\,\gamma_L(\hbar\omega_i,\ \varepsilon_{i-1},\ \varepsilon_i),$$

$$\gamma_L(\hbar\omega_i,\varepsilon_{i-1},\varepsilon_i) = W_L(\hbar\omega_i,\varepsilon_{i-1},\varepsilon_i)\Big/\int_0^{\varepsilon_{i-1}} W_L(\hbar\omega_i,\varepsilon_{i-1},\varepsilon_i)\rho(\varepsilon_i)d(\hbar\omega_i);$$

$$R_{I_0cI_N} = \prod_{i=1}^N\Big(\mathfrak{M}_L(I_{i-1},I_i)\,\rho(I_i,\alpha)\Big/\sum_{I_i}\mathfrak{M}_L(I_{i-1},I_i)\,\rho(I_i,\alpha)\Big);$$

$$G_{jI_0cI_N}(\mathbf{n}e) = \sum_{\mu_0} f_{\mu_0}\sum_{\mu_i e_i}\delta_{ee_j}\underbrace{\int\cdots\int}_N\delta(\mathbf{n}-\mathbf{n}_j)\prod_{i=1}^N d\Omega_i S_{\mu_{i-1}\mu_i}(\mathbf{n}_ie_i).$$

Here $W_{jN}(\hbar\omega)$ is the probability of emission of quanta with energies $\hbar\omega$ in the j-th position of an N-quantum chain, and $G_{jI_0cI_N}(\mathbf{n}e)$ is proportional to the emission probability of quanta in the direction $\mathbf{n}$ with polarization e after $j-1$ quanta have already been emitted. These quantities have been considered in other specific applications by Dolginov.[6]

Considering the relationship

$$\left|C^{I_{j-1}\mu_{j-1}}_{I_j\mu_jLM_j}\right|^2 = \sum_\varkappa\frac{2\varkappa+1}{2L+1}U(\varkappa LI_{j-1}I_j;\ LI_{j-1})\,C^{I_{j-1}\mu_{j-1}}_{I_{j-1}\mu_{j-1}\varkappa 0}\,C^{LM_j}_{LM_j\varkappa 0},$$

we find for $G_{jI_0cI_N}(\mathbf{n}e)$

$$G_{jI_0cI_N} = (8\pi)^{-1}\sum_\varkappa P_\varkappa(\theta)\,[2\varkappa+1]\,C^{Le}_{Le\varkappa 0}\,\zeta_\varkappa(I_0)\Big(\prod_{i=1}^{j-1}U_i(\varkappa)\Big)U_j(\varkappa),$$

where $P_\kappa(\theta)$ is the κ-th Legendre polynomial and θ is the angle between the vector $\mathbf{n}$ and the direction of polarization $\mathbf{I}_0$,

$$U_i(\varkappa) = U(LI_{i-1}I_i\varkappa;\ I_iI_{i-1});\ \ U_j(\varkappa) = U(\varkappa LI_{j-1}I_j;\ LI_{j-1}),$$

$$U(abcd;\ gf) = \sqrt{(2g+1)(2f+1)}\,W(abcd;\ gf),$$

where W(abcd; gf) is the Racah function (see references 7 and 8).

For $P_{I_0cI_N}(\hbar\omega;\mathbf{n},e)$, using the expression already obtained for $G_{jI_0cI_N}$, we have

$$P_{I_0cI_N}(\hbar\omega;\ \mathbf{n},\ e) = (8\pi)^{-1}\sum_\varkappa P_\varkappa(\theta)\,[2\varkappa+1]\,C^{Le}_{Le\varkappa 0}\,\zeta_\varkappa(I_0)$$

$$\times\Big[R_{I_0cI_N}\sum_{j=1}^N W_{jN}\Big(\prod_{i=1}^{j-1}U_i(\varkappa)\Big)U_j(\varkappa)\Big].$$

In order to obtain the probability of emission of a quantum $(\hbar\omega;\mathbf{n},e)$ in a cascade, we sum $P_{I_0cI_N}$ over all the types of chains $I_0 \rightarrow c \rightarrow I_N$ and over all attainable values of I_N:

$$P_{I_0}(\hbar\omega;\ \mathbf{n},\ e) = (8\pi)^{-1}\sum_\varkappa P_\varkappa(\theta)\,[2\varkappa+1]\,C^{Le}_{Le\varkappa 0}\,\zeta_\varkappa(I_0)\,\{\varkappa\},$$

$$\{\varkappa\} = \sum_{N=1}^\infty\sum_{I_N}\sum_{j=1}^N\sum_c W_{jN}(\hbar\omega)\,R_{I_0cI_N}\Big[\Big(\prod_{i=1}^{j-1}u_i(\varkappa)\Big)U_j(\varkappa)\Big]_{cI_N}.$$

Hereafter we shall denote by $\{\kappa\}'$ the value of $\{\kappa\}$ for odd κ, and $\{\kappa\}''$ for even κ; the sum $\sum_\kappa{}'$ is taken over odd κ and $\sum_\kappa{}''$ over even κ. By definition, the polarization of a quantum is equal to

$$e_{I_0}(\hbar\omega;\ \theta) = [P_{I_0}(\mathbf{n},\ \hbar\omega,\ e=+1)$$

$$-P_{I_0}(\mathbf{n},\ \hbar\omega;\ e=-1)]/[P_{I_0}(\mathbf{n},\ \hbar\omega;\ e=+1)$$

$$+P_{I_0}(\mathbf{n},\ \hbar\omega,\ e=-1)],$$

and the unnormalized angular distribution is

$$F_{I_0}(\hbar\omega,\ \theta)\backsim\sum_{e=\pm1}P_{I_0}(\mathbf{n},\ \hbar\omega;\ e).$$

Noting that $U_i(0) = 1$, $U_j(0) = 1$, $\sum_{cI_N} R_{I_0cI_N} = 1$, and $\{0\} = \sum_{N=1}^{\infty} \sum_{j=1}^{N} W_{jN}(\hbar\omega)$, we obtain for $e_{I_0}(\hbar\omega, \theta)$ and $F_{I_0}(\hbar\omega, \theta)$:

$$e_{I_0}(\hbar\omega, \theta) = \frac{\sum_{\varkappa=1}' P_\varkappa(\theta)[2\varkappa+1] C^{L1}_{L1\varkappa 0} \zeta_\varkappa(I_0)\{\varkappa\}'}{\sum_{\varkappa=0}'' P_\varkappa(\theta)[2\varkappa+1] C^{L1}_{L1\varkappa 0} \zeta_\varkappa(I_0)\{\varkappa\}''};$$

$$F_{I_0}(\hbar\omega, \theta) = \frac{1 + \sum_{\varkappa=2}'' P_\varkappa(\theta)[2\varkappa+1] C^{L1}_{L1\varkappa 0} \zeta_\varkappa(I_0)\{\varkappa\}''}{\sum_{N=1}^{\infty} \sum_{j=1}^{N} W_{jN}(\hbar\omega)}.$$

Computations have shown that, as a result of the summation over all the cascade chains with all attainable values of I_i, the values of $\{\kappa\}$ are found to become smaller as the value of κ grows larger. In particular, if $\{0\} = 1$, then $\{1\} \approx 0.1$, and $\{2\} \approx 0.01$. Therefore in the sums $e_{I_0}(\hbar\omega, \theta)$ and $F_{I_0}(\hbar\omega, \theta)$ it is sufficient to limit oneself to the first values, $\kappa = 0$, 1, and 2:

$$e_{I_0}(\hbar\omega, \theta) \approx P_1(\theta)\, 3C^{L1}_{L110}\, \zeta_1(I_0)\{1\} \Big/ \sum_{N=1}^{\infty} \sum_{j=1}^{N} W_{jN}(\hbar\omega);$$

$$F_{I_0}(\hbar\omega, \theta) = 1 + P_2(\theta)\, 5C^{L1}_{L120}\, \zeta_2(I_0)\{2\} \Big/ \sum_{N=1}^{\infty} \sum_{j=1}^{N} W_{jN}(\hbar\omega).$$

For the case of a dipole cascade $L = 1$ (an E1 or M1 cascade) this formula is exact.

2. THE AVERAGES $\bar{e}_{I_0}(\theta)$ AND $\bar{F}_{I_0}(\theta)$ TAKEN OVER THE CASCADE

The spectral distributions $e_{I_0}(\hbar\omega, \theta)$ and $F_{I_0}(\hbar\omega, \theta)$ depend on the value of $W_{jN}(\hbar\omega)$. This quantity depends on $\rho(\epsilon)$, and even for a relatively simple form of $\rho(\epsilon)$ the calculation of W_{jN} and $\sum_{j=1}^{N} W_{jN}$ entails great difficulties. We shall therefore consider the averages $\bar{e}_{I_0}(\theta)$ and $\bar{F}_{I_0}(\theta)$, taken over the spectrum, which are simpler for purposes of numerical estimation. Integrating $P_{I_0cI_N}(\hbar\omega, \mathbf{ne})$ with respect to the quantum energy $\hbar\omega$ between the limits $\hbar\omega_{min} = \Delta$ and $\hbar\omega_{max} = \epsilon_0 - 2\Delta$, we obtain

$$\bar{P}_{I_0cI_N}(\mathbf{ne}) \approx (8\pi)^{-1} \sum_\varkappa P_\varkappa(\theta)[2\varkappa+1] C^{Le}_{Le\varkappa 0} \zeta_\varkappa(I_0) \times \sum_{N=1}^{\infty} W_N \sum_{cI_N} R_{I_0cI_N} \Big[\sum_{j=1}^{N} \Big(\prod_{i=1}^{j-1} U_i(\varkappa)\Big) U_j(\varkappa)\Big]_{cI_N}.$$

Here we have taken

$$\int_\Delta^{\epsilon_0 - 2\Delta} W_{jN}(\hbar\omega)\, d(\hbar\omega) \approx W_N,$$

since for chains with $N \le 5$ the probability of emitting a single soft quantum is relatively small, and the contribution of chains with $N > 5$ is negligible.[1]

Taking into account the fact that, for $\kappa = 0$,

$$\sum_{j=1}^{N} \Big(\prod_{i=1}^{j-1} U_i(\varkappa)\Big) U_j(\varkappa) = N$$

and introducing

$$Q_N(\varkappa) = \sum_{I_N} \sum_c \mathfrak{N}_{NI_0cI_N} R_{I_0cI_N},$$

$$\mathfrak{N}_{NI_0cI_N} = N^{-1} \sum_{j=1}^{N} \Big(\prod_{i=1}^{j-1} U_i(\varkappa)\Big) U_j(\varkappa),$$

we find for the polarization and angular distribution

$$\bar{e}_{I_0}(\theta) \approx 3P_1(\theta) C^{L1}_{L110} \zeta_1(I_0) \sum_{N=1}^{\infty} NW_N Q_N(1) \Big/ \sum_{N=1}^{\infty} NW_N,$$

$$\bar{F}_{I_0}(\theta) \approx 1 + 5P_2(\theta) C^{L1}_{L120} \zeta_2(I_0) \sum_{N=1}^{\infty} NW_N Q_N(2) \Big/ \sum_{N=1}^{\infty} NW_N$$

or

$$\bar{e}_{I_0}(\theta) \approx \cos\theta\, A \zeta_1(I_0), \quad \bar{F}_{I_0}(\theta) \approx 1 + P_2(\theta) B\zeta_2(I_0).$$

The value of W_N has been estimated in a study by Nosov and Strutinskiĭ. In particular, for $L = 1$, according to Groshev et al.[1], $W_N = \text{const} \times \exp\{-2(N - \bar{N})^2/\bar{N}\}$. If we make use of the experimental value $\bar{N} \approx 3.5$, we obtain the following values for W_N:

$N =$	1	2	3	4	5	6
W_N / const =	0.05	0.28	0.87	0.87	0.28	0.027

Although Strutinskiĭ and Nosov obtained W_N with a particular model for the level density $\rho(\epsilon)$, it is evident that the distribution of W_N cannot be seriously altered by the use of other models for $\rho(\epsilon)$ which approximate the observed spectrum of nuclear levels. From the values of W_N given above, it can be seen that essentially it is only necessary to take into account chains having $N = 2$, 3, 4, and 5 quanta.

In order to get some idea of the order of magnitude of $e_{I_0}(\theta)$ and $F_{I_0}(\theta)$, and of the sensitivity of $e_{I_0}(\theta)$ and $F_{I_0}(\theta)$ to changes in the parameter α in the spectrum $\rho(I, \alpha)$, a calculation was made of the mean values $\bar{e}_{I_0}$ and $\bar{F}_{I_0}$ for a dipolar cascade of quanta ejected from a nucleus with initial spin $I_0 = 4$ and $I_0 = 3$. The calculations were made for the two models of the distribution $\rho(I, \alpha)$ discussed above — the spectra of types a and b. The results are shown in Tables I and II, where the values of the coefficients A and B for the quantum polarization and the angular distribution are given. Of particular interest is the estimate of the mean polarization of the

TABLE I. The coefficients A and B for $I_0 = 4$, for spectra of types a and b

α	4	6	8	10	∞	ρ_I = const*
A_a	0.49	0.23	0.13	0.09	0	0.215
A_b	0.38	0.23	0.15	0.12	0.05	0.215
B_a	0.046	—	0.014	—	0	0.033

*In this column, for comparison, are given the values of the A and B coefficients for the case where the nuclear level spectrum does not depend on $I: \rho_I(\varepsilon) = \rho(\varepsilon)$.

quanta in a cascade following the capture of a polarized neutron by an unpolarized target nucleus with spin J (we denote the degree of polarization

TABLE II. The A coefficients for $I_0 = 3$, for spectra of types a and b

α	4	6	8	10	∞
A_a	0.40	0.17	0.115	0.075	0
A_b	0.37	0.195	0.15	0.12	0.07

of the neutron by η_n). In this case $\zeta_0(I_0) \equiv 1$; $\zeta_2(I_0) \equiv 0$, and for $\zeta_1(I_0)$ we have

$$\zeta_1(I_0) = \eta_n \frac{(2I_0+1)}{6(2J+1)} \sqrt{(2J+3)/(2J+1)} \text{ for } I_0 = J + {}^1/_2;$$

$$\zeta_1(I_0) = -\eta_n \frac{(2I_0+1)}{6(2J+1)} \sqrt{(2J-1)/(2J+1)} \text{ for } I_0 = J - {}^1/_2.$$

For the limiting quantum polarization with the given values of the parameter α we find ($\theta = 0°$)

$$\xi_{I_0}(\alpha) \equiv \overline{e}_{I_0}(\theta = 0^0)/\eta_N$$

$$= 3C_{1110}^{11}(\zeta_1(I_0)/\eta_n) \sum_{N=1}^{\infty} NW_N Q_N(1) / \sum_{N=1}^{\infty} NW_N.$$

The results of the calculations for $\xi_{I_0}(\alpha)$ are shown in Fig. 2 for $J = {}^7/_2$.

The estimates which have been made of angular distribution and circular polarization show that these values lie within experimentally attainable limits. An investigation of $e_{I_0}(\theta, \hbar\omega)$ and $F_{I_0}(\theta, \hbar\omega)$ could give information about the relationship $\rho_I(\epsilon)$. It is important to note that the sign of the mean polarization $\overline{e}_{I_0}$ makes it possible to establish the spin of the initial state of the compound nucleus I_0.

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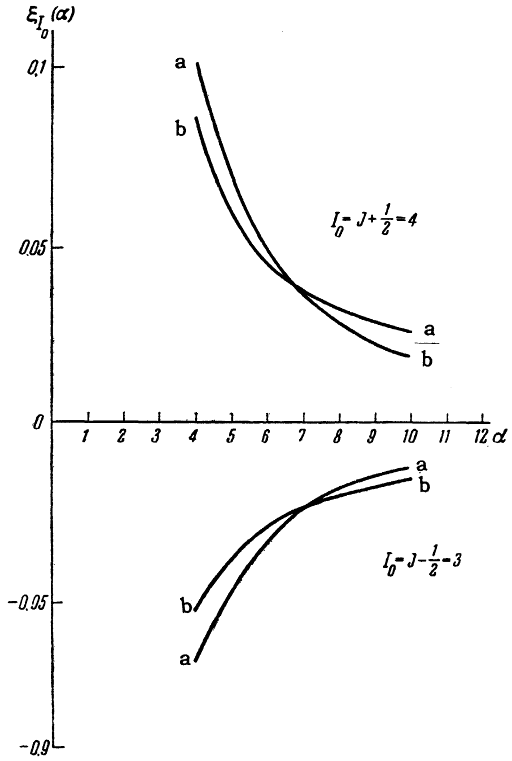

FIG. 2. Dependence of the limiting spectral-mean polarization of the cascade quanta on the parameter α, for nuclear spectra $\rho(I, \alpha)$ of the types a and b, for the case of capture of polarized neutrons by an unpolarized nucleus with spin J = 7/2. The upper curves correspond to an initial spin of $I_0 = J + ½ = 4$ for the compound nucleus; the lower curves to $I_0 = J - ½ = 3$.

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