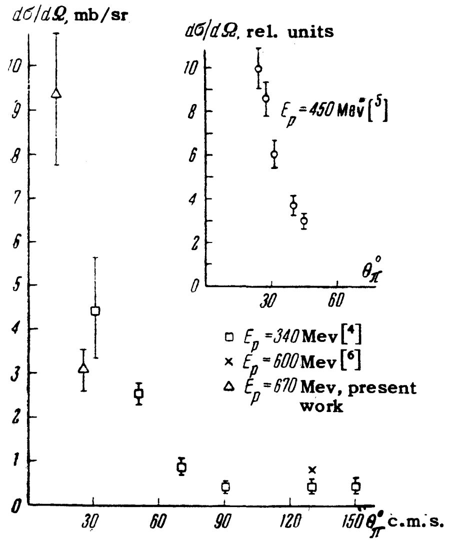

Since, for example, if the ratio of the pion formation cross sections in reaction (1) for incident proton energy 340 Mev for 0° and 180°, calculated on the basis of this theory with a core radius $0.5\hbar/m_\pi c$ is equal to ~ 10, then this ratio for 670-Mev protons increases to ~ 120 if the same wave function parameters are used and if the dependence of the angular distribution of reaction (3), which is indispensable for the calculation, is obtained by extrapolating the data for the inverse reaction for the meson energy region 174 to 370 Mev.[9] The differential cross sections calculated by this model for the incident proton energy 670 Mev, is

$$d\sigma(12^\circ)/d\Omega = 3.1\cdot 10^{-30}, \quad d\sigma(25^\circ)/d\Omega = 2.4\cdot 10^{-30}\,\mathrm{cm}^2/\mathrm{sr}$$

The quantitative disagreement between the calculated values and the experimental data is evidently due to the fact that in all these calculations one looks at the formation of positive pions from the collision of the incident proton with the proton of the deuteron only as reaction (3), and one does not take into account the contribution from pion formation in the reaction $p + p \rightarrow n + p + \pi^+$, whose total cross section exceeds by a factor of a few tens the total cross section for (3) in the incident proton energy region near 900 Mev, that used in the impulse approximation theory calculations.

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ON THE MAGNETIC SUSCEPTIBILITY OF A RELATIVISTIC ELECTRON GAS

A. A. RUKHADZE and V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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LANDAU[1] was the first to show that a gas consisting of free electrons is diamagnetic, if one neglects the electron spins, and that its diamagnetism is equal to one third of the spin paramagnetism. Landau evaluated the magnetic susceptibility of an electron gas, starting from the expression of the energy spectrum of a non-relativistic electron in a magnetic field.

To evaluate the magnetic susceptibility of a relativistic electron gas one could use the solution of the Dirac equation for an electron in a magnetic field.[2] It is, however, simpler to use the method of the quantum transport equation with a self-consistent interaction.[3] Starting from the Dirac equations for an electron in an external transverse electromagnetic field, we obtain the following transport equation with a self-consistent interaction for the quantum distribution function which depends on the momentum **p**, the coordinate **r** and the spin indices α, β, γ

$$\frac{\partial}{\partial t} f_{\alpha\beta}(\mathbf{p},\mathbf{r}) = \frac{1}{(2\pi)^6}\frac{i}{\hbar}\int d\boldsymbol{\tau}\, d\boldsymbol{\eta}\, d\mathbf{k}\, d\mathbf{r}_1 e^{i\boldsymbol{\tau}(\boldsymbol{\eta}-\mathbf{p})+i\mathbf{k}(\mathbf{r}_1-\mathbf{r})}$$
$$\times\Big\{\Big[\Big(\boldsymbol{\alpha}, c\Big(\boldsymbol{\eta}+\frac{\hbar\mathbf{k}}{2}\Big)-e\mathbf{A}\Big(\mathbf{r}_1-\frac{\hbar\boldsymbol{\tau}}{2}\Big)\Big)+\beta\mu\Big]_{\beta\gamma} f_{\alpha\gamma}(\boldsymbol{\eta},\mathbf{r}_1)$$
$$-\Big[\Big(\boldsymbol{\alpha}, -c\Big(\boldsymbol{\eta}-\frac{\hbar\mathbf{k}}{2}\Big)+e\mathbf{A}\Big(\mathbf{r}_1+\frac{\hbar\boldsymbol{\tau}}{2}\Big)\Big)-\beta\mu\Big]_{\gamma\alpha} f_{\gamma\beta}(\boldsymbol{\eta},\mathbf{r}_1)\Big\}, \quad (1)$$

where $\boldsymbol{\alpha}$ and β are the Dirac matrices, $\mu = mc^2$, **A** is the vector potential of the transverse electromagnetic field. To determine the magnetic susceptibility it is necessary to evaluate the current density which in the case of the relativistic electron gas is of the form

$$\mathbf{j} = ec \int d\mathbf{p}\, (\boldsymbol{\alpha}_{\alpha\beta} f_{\alpha\beta}). \quad (2)$$

The magnetic susceptibility of the gas χ^* is determined using a solution of the stationary linearized equation (1) as follows: $\mathbf{j}_k = ck^2\chi\mathbf{A}_k$, where $\mathbf{j}_k$ and $\mathbf{A}_k$ are the Fourier components of the current and the vector potential. After simple calculations we find the magnetic susceptibility of a relativistic electron gas

$$\chi = \pi e^2\hbar^2 \left\{1 - \frac{1}{3}\right\} \int_0^\infty \frac{f_0(p)}{E_p}\, dp, \qquad E_p = \sqrt{c^2p^2 + \mu^2}, \quad (3)$$

where $f_0(p)$ is the equilibrium momentum distribution function of the electrons normalized to the total number of electrons per unit volume. The one in the curly brackets in (3) is caused by the electron spins and corresponds to the spin paramagnetism of the electron gas, while the second term $1/3$ corresponds to the diamagnetism of the free electrons. The diamagnetism of a relativistic, as of a non-relativistic, electron gas is thus equal to one third of its spin paramagnetism. In the non-relativistic limit $E_p = \mu = mc^2$ and Eq. (3) goes over into Landau's well-known expression.

For a relativistic degenerate electron gas a simple evaluation of the integral in (3) gives

$$\chi_\Phi = \left(\frac{e\hbar}{2mc}\right)^2 \frac{m^2c}{\pi^2\hbar^3} \left\{1 - \frac{1}{3}\right\} \ln \frac{p_0 + \sqrt{p_0^2 + m^2c^2}}{mc}, \quad (4)$$

where $p_0 = \hbar(3\pi^2N)^{1/3}$. In the ultrarelativistic limit, $p_0 \gg mc$, we get from Eq. (4)

$$\chi_\Phi = \left(\frac{e\hbar}{2mc}\right)^2 \frac{m^2c}{\pi^2\hbar^3} \left\{1 - \frac{1}{3}\right\} \left[\ln \frac{2\hbar(3\pi^2N)^{1/3}}{mc} + \frac{m^2c^2}{4\hbar^2(3\pi^2N)^{2/3}}\right]. \quad (5)$$

It follows from Eq. (5) that the magnetic susceptibility of an ultrarelativistic degenerate electron gas increases logarithmically with increasing density

$$\chi_\Phi \approx 0.5 \cdot 10^{-3} \ln (2\hbar(3\pi^2N)^{1/3}/mc).$$

In real cases $\chi_\Phi \ll 1$.

For an ultrarelativistic Boltzmann electron gas ($\kappa T \gg mc^2$) Eq. (3) goes over into the following expression for the magnetic susceptibility

$$\chi_B = \left(\frac{e\hbar}{2mc}\right)^2 \frac{N}{2\kappa T} \left(\frac{mc^2}{\kappa T}\right)^2 \left\{1 - \frac{1}{3}\right\} \left[\ln \frac{\kappa T}{mc^2} + 0.116\right]. \quad (6)$$

In the equilibrium state of the system the number of electron-positron pairs formed through collisions is for $\kappa T \gg mc^2$ equal to[4] $N^+ = N^- = 0.183\,(\kappa T/\hbar c)^3$. Taking this into account in Eq. (6) we conclude that the magnetic susceptibility of the system increases logarithmically with increasing temperature, $\chi_B \approx 10^{-4} \ln(\kappa T/mc^2)$. In real systems $\chi_B \ll 1$.

We express our gratitude to V. L. Ginzburg for his interest and for discussions of this paper.

*We emphasize that we are dealing with the susceptibility of an electron gas in a thermodynamic equilibrium state. The magnetic moment of the system may in a non-equilibrium state be appreciably higher.

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ELECTRICAL RESISTANCE MAXIMUM FOR FERROMAGNETS AT THEIR CURIE POINTS AT LOW TEMPERATURES

E. I. KONDORSKIĬ, O. S. GALKINA, and L. A. CHERNIKOVA

Moscow State University

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WE showed earlier[1] that for nickel the ratio $\Delta\rho/\Delta I$ (where $\Delta\rho$ is the change of electrical resistivity for a change in magnetization of ΔI produced by a magnetic field, in the region of magnetic saturation) is approximately equal to the ratio $(\rho_T - \rho_0)/(I_0 - I_T)$, where ρ_T and I_T are the specific resistivity and saturation magnetization at temperatures $T < 20°K$, ρ_0 is the residual resistivity and I_0 is the saturation magnetization derived by extrapolation to absolute zero. It was also established that at hydrogen temperatures and below, the law $\rho_T - \rho_0 = aT^{3/2}$ holds for iron and nickel, where a is the constant of proportionality, and that above hydrogen temperatures the difference $\rho_T - \rho_0 - aT^{3/2}$ is roughly proportional to T^5. From this it was deduced that at hydrogen and he-