INTERACTION OF AN ELECTRON BEAM WITH A PLASMA

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The results of an experimental investigation of the interaction of modulated and unmodulated high-energy electron beams with the plasma in a high-frequency discharge are presented. It is shown that the passage of an unmodulated beam through a plasma results in the production of plasma oscillations in the beam at a frequency close to the plasma frequency. The dependence of oscillation amplitude on frequency and the plasma parameters is determined. The coherent energy losses of electrons in modulated and unmodulated beams moving through a plasma have also been studied.

1. The first experiments in which plasma oscillations were produced by the interaction of an electron beam and a plasma were those reported by Langmuir, Tonks, and Penning.[1-3] Merill and Webb[4] and other authors[5] have investigated the oscillation regions and their relation to the anomalous dissipation of electron energy in a gas discharge. Interpretations of these results have been given by Vlasov[6] and by Bohm and Gross,[11] who showed that the oscillations are excited by a mechanism similar to that which operates in the klystron.

Looney and Brown[8] and Gabor and his coworkers,[9] using an external electron source, found that plasma oscillations are excited only when there are sheaths at the electrodes or at the surfaces which confine the discharge. As has been shown by Akhiezer and Faĭnberg[10] and Bohm and Gross,[7] an initially unmodulated beam characterized by a uniform density and constant velocity is unstable in passage through a uniform plasma (a plasma with no density gradients or sheaths). Under these conditions longitudinal electromagnetic waves of increasing amplitude are excited in the beam and in the plasma. The plasma is excited by the beam when the beam velocity is greater than the thermal velocity of the plasma, $v_0 \gg s$ and also when $v_0 \ll s$. However, when $v_0 \gg s$ the beam-plasma interaction is much more effective. The ratio of the excitation coefficients for these two cases is $v_0/s\,(\Omega/\omega_0)^{-1/3} \gg 1$, where ω_0 is the plasma Langmuir frequency and Ω is the plasma frequency of the beam.

Instabilities can be avoided if there is a velocity spread in the beam. The interaction mechanism described here also applies to the passage of an ion beam through a plasma.

In interacting with the plasma the beam becomes density modulated so that the initially uniform beam is broken up into separate bunches. Under these conditions coherent interactions can become important. The possibility of using the coherence effect for intensifying the interaction between charged particles and a plasma was first indicated by Veksler.[12] If the coherence conditions are satisfied the energy loss of an individual particle in a bunch is given by

$$\left|\frac{dE}{dx}\right| = \frac{e^2 N}{2v_0^2}\,\omega_0^2 \ln\left(1 + \frac{v_0^2}{\omega_0^2 b_\perp^2}\right), \tag{1}$$

where N is the number of particles in the bunch and $b_\perp$ is the transverse dimension of the bunch. The fields produced by a coherent bunch can be intensified if the coherence conditions are satisfied for a number of such bunches simultaneously.[13]

A comparison of this interaction mechanism with that proposed by Tuck[14] indicates that the present excitation coefficient is much larger. The ratio of the excitation coefficients is $(c/v_0)^2 \times (\omega_0 r_0/v_0)(\omega_0/\Omega)^{2/3} \ll 1$, where r_0 is the classical electron radius. This difference is due to the fact that the Tuck analysis does not take account of the possibility of a coherent interaction.

The nature of the instability is of great importance in an analysis of the interaction of a particle beam with a plasma. A method of distinguishing between absolute and convective instability in hy-

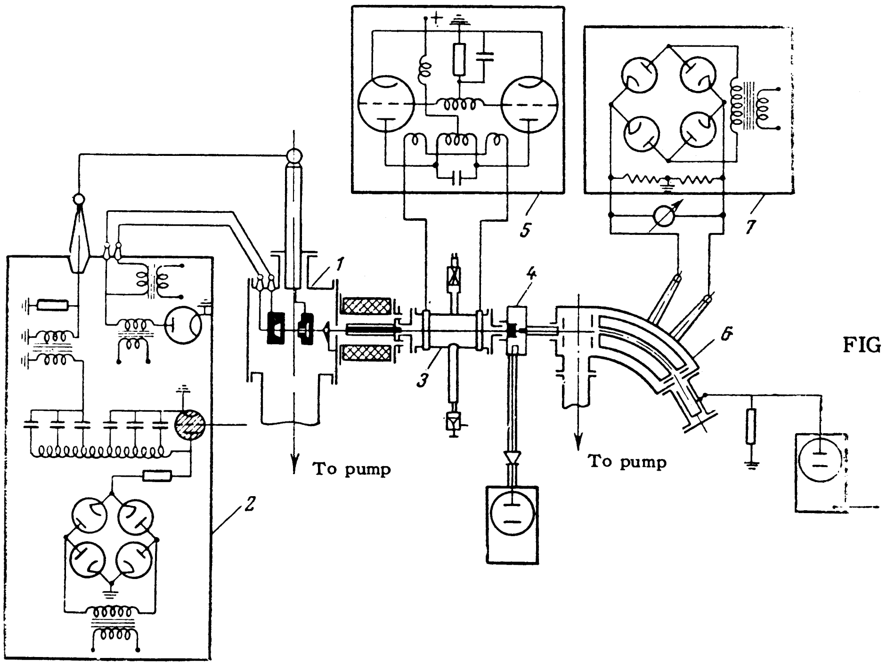

FIG. 1. Diagram of the experimental arrangement.

drodynamics has been pointed out by Landau and Lifshitz.[15] Essentially, this method is based on an investigation of the limiting behavior of the integral $\int \exp\{-i\omega(k)t\}\,dk$. Absolute instability obtains if the integral becomes infinite as $t \to \infty$. The difference between convective and absolute instabilities in beam interactions has been studied by Sturrock.[16] This criterion has been applied by Drummond[17] in an analysis of the excitation of a cold plasma by a beam and indicates that the instability is convective in this case. Investigations carried out by us,[18] using the Landau-Lifshitz technique, lead to the same result.

Experimental attempts to observe the interaction mechanism predicted by the theory[10,11] have been unsuccessful up to the present time.[8] It is probable that the reason for these failures is the fact that the interaction lengths were too small in these experiments and that the initial perturbations, due exclusively to fluctuations in the beam and plasma, were relatively small. This second suggestion has been verified by the work of Boyd, Field and Gould.[19]

2. We now consider the basic relations needed for an analysis of the experimental data.[10]

The gain factor, which characterizes the growth of the longitudinal waves in space (imaginary part of the wave vector **k** for a fixed value of the frequency ω) is given by the following expression when $v_0 \gg s$:

$$\gamma = \frac{3^{1/2}}{2^{4/3}} \frac{\omega_0}{s} \left(\frac{s}{v_0}\right)^{1/3} \left(1 - \frac{s^2}{v^2}\right) \left(\frac{\Omega}{\omega_0}\right)^{2/3}. \tag{2}$$

The excitation coefficient, which characterizes the growth of the oscillations in time (imaginary part of ω for a given **k**) is given by the expression:

$$\varepsilon = \Omega \left\{ \left(\frac{\omega_0}{\omega}\right)^2 \frac{1}{1 - s^2/v_0^2} - 1 + i\eta \right\}^{-1/2}, \tag{3}$$

where η characterizes the damping due to pair interactions or collective interactions. In the latter case, η is given by the Landau formula

$$\eta = \sqrt{\frac{3\pi}{2}} \frac{\omega}{ks} (ak)^{-2} \exp\left\{-\frac{3}{2}(\omega/ks)\right\}^2, \tag{4}$$

where a is the Debye radius.

For a cold plasma ($v_0 \gg s$, $s \to 0$) we have

$$\gamma = \Omega / v_0 \sqrt{(\omega_0/\omega)^2 - 1}, \tag{5}$$

$$\varepsilon = \Omega / \sqrt{(\omega_0/\omega)^2 - 1}. \tag{6}$$

As $\omega \to \omega_0$ the gain factor increases without limit while the excitation coefficient remains finite, being given by

$$\varepsilon = 2^{-4/3} \sqrt{3}\, \omega_0^{1/3} \Omega^{2/3}. \tag{7}$$

The frequency corresponding to maximum excitation is shifted with respect to ω_0 by an amount $\Delta\omega$:

$$\Delta\omega / \omega_0 = -2^{-4/3} (\Omega/\omega_0)^{2/3}. \tag{8}$$

The half width of the curve which describes the excitation coefficient as a function of frequency is given by the following relation (if damping is neglected):

$$\frac{\varepsilon_\omega}{\varepsilon_{\omega\, max}} = \frac{2^{4/3}}{3^{1/2}} \left(\frac{\omega_0^2}{\omega^2} - 1\right)^{-1/2} \left(\frac{\Omega}{\omega_0}\right)^{1/3}. \tag{9}$$

In the amplification case, as $\omega \to \omega_0$, if collisions are neglected, $\gamma \to \infty$. Under these conditions the half-width is determined by the damping due to collisions of plasma particles between themselves or with the walls.

3. We now describe the experimental investigation of the interaction of an electron beam with a plasma.

A schematic diagram and a general picture of the experimental arrangement are shown in Fig. 1. An electron beam with an energy of 80 kev at a current of one ampere (pulsed, pulse length of 2 μsec) is formed by the three-electrode gun 1. The electron acceleration pulse is produced by the double artificial line 2. At the output of the source the beam is focused by a longitudinal magnetic field of about 200 gauss. The plasma is formed in a quartz tube 3, 65 mm in diameter, by a radio-frequency oscillator 5 in which two GU27B tubes are used in push-pull. The oscillator provides up to one kilowatt of CW power into a real load at 16 Mcs. In these experiments the length of the tube is 10 cm and 20 cm.

In normal operation the pressure differential between the electron gun and the tube is produced by means of a copper tube (120 mm long with an aperture of 5 mm) through which the electron beam passes (the beam diameter at the output of the source is less than 5 mm). In this way a pressure variation from 10^{-2} mm Hg in the discharge tube to 10^{-5} mm Hg in the source chamber is obtained. The pressure in the discharge tube is controlled by a mechanical leak.

Beyond the tube there is a tunable coaxial cavity 4. If oscillations at the resonant frequency of the cavity are excited in the beam as it passes through the plasma the cavity is excited. Part of the energy of the cavity oscillations is coupled out by means of a crystal detector. After passing through the discharge tube and the cavity the beam strikes the Faraday cylinder in an electrostatic analyzer 6. The rectifier 7 supplies voltage for the plates of the analyzer. The current from the cylinder is fed to a dc amplifier with an integrating input and then to a loop oscilloscope. A saw-tooth voltage is applied to the plates of the analyzer so that the oscilloscope tape records the $I_{av}(U)$ curve, i.e., the electron energy spectrum averaged over several pulses. The analyzer and the source are isolated from the discharge tube which provides the necessary pressure differential between the tube and the analyzer.

The current pulse and the sweep of the measurement oscilloscope must be synchronized; for this reason all units of the system are triggered by a special synchronizing pulse generator.

Using the system described here we have been able to produce radio-frequency oscillations in an unmodulated beam passing through a plasma. The oscillations are observed in a plasma interaction tube 20 cm in length. If smaller interaction lengths are used (10 cm) it is possible to amplify the radio-frequency component of the beam current in a pre-modulated beam, but oscillations cannot be produced. Figure 2 shows oscillograms of the current pulse (upper oscillogram) and the radio-frequency oscillation pulse at the output of the cavity resonator (lower oscillogram). Oscillations are observed over the entire duration of the current pulse. There is some delay of the radio-frequency pulse with respect to the current pulse because of the time required for oscillations to build up in the cavity (several tenths of a microsecond).

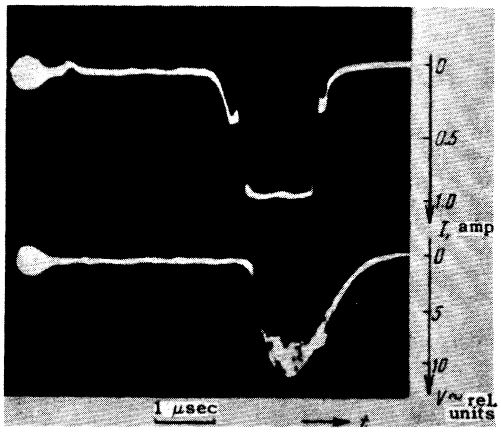

FIG. 2. Oscillogram showing the current pulse and the radio-frequency oscillations in the beam.

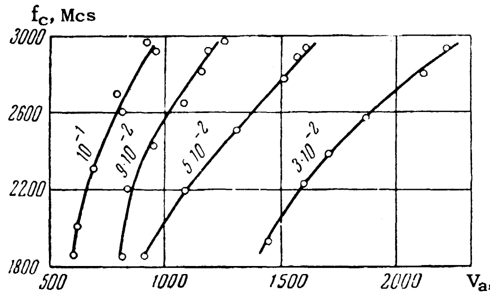

FIG. 3. Curves of the maximum oscillation amplitude at different pressures as a function of oscillator voltage (the figures on the curves are given in mm Hg).

In Fig. 3 the frequency f_c of the measuring cavity corresponding to the maximum oscillation amplitude is given as a function of the oscillator plate voltage V_a at different pressures in the discharge tube. The plate voltage determines the power supplied to the plasma by the oscillator, that is to say, the plasma density. The measurements have been carried out in the frequency range from 1800 to 3000 Mcs, the frequency limits of the oscillator.

At a given frequency the maximum oscillation amplitude is observed for some definite plate voltage or plasma density. In principle this density should correspond to the natural plasma frequency which, in the present case, is the resonant frequency of the measurement cavity. The displacement of the curves with pressure is due to the reduction in plasma density when the pressure in the discharge tube is reduced.

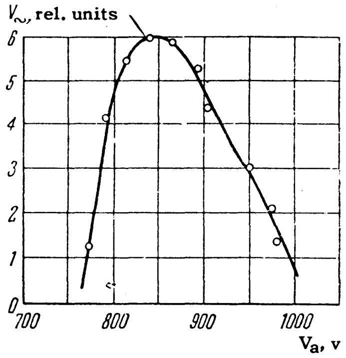

FIG. 4. Curve for oscillation amplitude at 2150 Mcs as a function of oscillator voltage ($p = 9 \times 10^{-2}$ mm Hg).

In Fig. 4, the oscillation amplitude at the output of the cavity $V_\sim$ is plotted in relative units as a function of the oscillator plate voltage with the cavity tuned to $f_c = 2150$ Mcs at a discharge tube pressure $p = 9 \times 10^{-2}$ mm Hg. It is apparent from the curve that at a given frequency and pressure a maximum oscillation amplitude is observed at one voltage, or plasma density. Similar curves are obtained at other frequencies and pressures. All these curves are characterized by a sharp rise in oscillation amplitude as the oscillator voltage increases to the resonance value and a slow reduction as the voltage is increased beyond this point. The nature of the oscillation spectrum can not be determined from the shapes of these curves because the plasma density is not a linear function of oscillator voltage.

A direct examination of the radio-frequency spectrum of the beam oscillations is made by measuring $V_\sim$ at the cavity output as the cavity is tuned to different frequencies with the plasma density held constant. Such a line trace is shown in Fig. 5; this figure refers to a generator voltage $V_a = 1000$ v and a pressure $p = 9 \times 10^{-2}$ mm Hg.

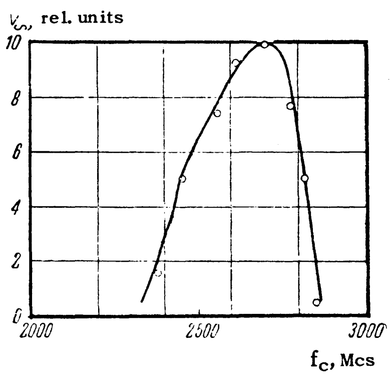

FIG. 5. The oscillation spectrum at a fixed plasma density ($p = 9 \times 10^{-2}$ mm Hg, $V_a = 1000$ v).

These parameters correspond to a plasma density $n = 7 \times 10^{10}$. There is no change in the nature of this spectrum at other plasma densities. Thus, direct measurement of the oscillation spectrum shows broadening at frequencies below the resonance frequency ω_0. The half width of the spectrum is approximately 250 Mcs on the low-frequency side and approximately 100 Mcs on the high-frequency side. This pattern is in qualitative agreement with the frequency dependence of the gain factor γ and the excitation factor ϵ. In the present experiments $v_0 \gg s$; hence, in accordance with Eqs. (5) and (6), γ and ϵ are real for $\omega > \omega_0$ and there is no amplification or excitation. When $\omega < \omega_0$, however, oscillations can be excited.

Inasmuch as γ and ϵ exhibit the same dependence on frequency, when $\omega \neq \omega_0$ this dependence cannot serve as a criterion to determine whether we are dealing with amplification or with self-excited oscillations due to an additional feedback mechanism.

The width of the experimental resonance curve is 350 Mcs. The half-width of the resonance curve for excitation of a cold plasma is given by Eq. (9). With a beam density of 10^8 or 10^9 electrons/cm^3 the half width can vary from 1000 to 1400 Mcs. Since $\gamma \rightarrow \infty$ if damping is neglected for amplification in a cold plasma at $\omega = \omega_0$, the notion of a half-width is not meaningful. The half-width for the gain factor as determined from Eq. (2) applies for a plasma at a finite temperature; in the present case $v_0/s \approx 10^3 - 10^4$ is extremely small. The half width observed in the present experiment cannot be explained by damping due to collective interactions (because in the present case $\eta_{coll} \approx \exp\{-(v_0/s)^2\} \ll 1$). Hence, if one assumes that the system is amplifying, then to explain the observed dependence of signal amplitude on frequency (Fig. 5) it must be assumed that the damping is due to pair collisions or collisions with the walls.

If we assume that oscillations are actually being excited by a feedback mechanism, the discrepancy between the experimental data and theory may be explained by the change in the density of the beam as it moves through the plasma. In order to compare the frequencies of the oscillations excited by the beam with the "natural" plasma frequencies, we have measured the plasma density by means of a radio-frequency double-probe method.[20] The results of these measurements are shown in Fig. 6. Curve 1 shows the density measured by the radio-frequency double-probe method while curve 2 shows the density determined from the frequency of the oscillations excited by passage of the beam through the plasma.

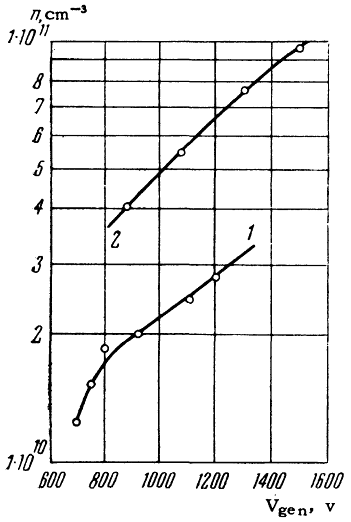

FIG. 6. Curves showing the dependence of plasma density on oscillator voltage ($p = 5 \times 10^{-2}$ mm Hg).

The discrepancy between those results may be due to the fact that the density is a maximum at the axis of the discharge tube while the radio-frequency double-probe method gives a value which represents an average over a diameter. The increase in plasma density due to the beam is insignificant since the beam density is 10^9 electrons/cm^3.

According to theory, when $v_0 \gg s$, excitation should be strongest near ω_0. In the present case the displacement with respect to the Langmuir frequency should be

$$\Delta\omega = 2^{-1/3}(\omega_0\Omega^2)^{1/3} \approx 10^9 \text{ cps}$$

This relation also applies for a cold plasma.

The frequency of maximum amplification in the cold plasma corresponds to $\omega = \omega_0$ (if collisions are neglected).

In Fig. 5, $\Delta\omega = 3 \times 10^9$ and the frequency corresponding to maximum excitation is higher than the Langmuir frequency indicated by the radio-frequency double-probe method rather than lower, as is indicated by the theory.

By comparing the oscillation amplitude in the cavity when it is excited by the beam and the amplitude when it is excited by a radio-frequency pulse generator of known power we have been able to estimate the power level of the oscillations in the cavity. This power level is several watts. It may be noted, however, that the amplitude of the oscillations excited in the beam when it passes through the plasma are comparable with the amplitude of oscillations which are excited when the beam is modulated by voltages up to 10 kv, indicating that the power in the beam is considerably higher; because of the poor beam-cavity coupling, however, only a small part of the true power is coupled out. The existence of coherent losses indicates a rather high field intensity in the plasma.

In a short tube (no oscillation) the energy loss of an unmodulated beam in a plasma is very small. We have not been able to observe this loss experimentally. If the tube length is increased, however, oscillations are excited and these are accompanied by losses. The measurements carried out in the tube 200 mm in length indicate that each electron loses 40 ev/cm at a plasma density of 2×10^{10} electrons/cm^3. A loss of this magnitude can be due only to a coherent interaction. The beam excites oscillations and the resulting field modulates the beam; eventually electron bunches of dimensions smaller than half a plasma wavelength are formed with a separation which represents a multiple of the plasma wavelength. This mechanism, as has been indicated above, leads to a strong interaction between the beam and plasma. According to theory[21] [Eq. (1)] coherent losses of this kind for a bunch consisting of 10^9 particles at plasma densities from 10^{10} to 10^{11} may amount to 10 or 100 ev/cm respectively. In measurements of the coherent energy losses in a modulated beam with different interaction lengths (10 and 20 cm) it has been found that the specific energy loss remains approximately equal to 80 ev/cm. The higher energy loss for bunches which are preformed (before entrance into the plasma) is due to a higher degree of coherence.

Experiments have also been carried out in which plasma oscillations are excited by bunching a beam with an external radio-frequency field at a frequency close to the natural plasma frequency (cf. reference 22). Bunching is accomplished by means of a cavity which is located at the point at which the beam leaves the electron gun (not shown in the diagram). The construction of this cavity is similar to that of the measurement cavity; the bunching cavity is driven by a pulsed magnetron.

The curves showing the amplification of the radio-frequency component in the beam as a function of plasma density are similar to the curve in Fig. 4 and the corresponding curve in reference 19. As has already been indicated, it is impossible to produce oscillations in an unmodulated beam with an interaction length of 10 cm. It would appear that this interaction length is too small and that the initial fluctuations in the beam and plasma are also small and cannot be observed experimentally, even when amplified. A prebunched beam provides a significant increase in the initial signal so that the amplification effect can be observed.

The data which have been obtained still do not furnish an answer to the question of whether we are dealing here with absolute or convective instability. In order to answer this question it will be necessary to measure the amplitude of the signal at different points along the plasma. It should be noted that in the present case we may be dealing with self-excitation due to a feedback mechanism. The plasma is bounded in the beam direction and its length is of the order of a perturbation wavelength.[23] Under these conditions, because of "feedback" due to reflection, an amplification process would lead to oscillations of this kind.

The excitation of plasma oscillations by an unmodulated beam is of great interest; interactions of precisely this type are undoubtedly instrumental in the excitation of plasma oscillations in various kinds of gas discharges in which "runaway" electrons appear.[24,25] This interaction is apparently responsible for the effective exchange of energy between the ordered motion of the charged particles and the rest of the plasma.

The experimental production of oscillations in an unmodulated beam passing through a plasma can be used directly for studying plasma parameters (electron density and ion temperature), for producing radio waves in the millimeter region, and for acceleration of particles in a plasma.[26]

In conclusion, we wish to thank K. D. Sinel'nikov and A. I. Akhiezer for discussion of the results of this work.

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