ON THE PROCESSES INVOLVING TRANSFER OF MOMENTUM TO THE MEDIUM

M. I. RYAZANOV

Moscow Engineering Physics Institute

Submitted to JETP editor July 18, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 854-862 (March, 1960)

The change of the transition probability caused by the Coulomb scattering of particles by the atoms of the medium is found for a certain class of processes in which there are one charged particle and an arbitrary number of neutral particles in the initial and final states.

1. INTRODUCTION

EXPERIMENTAL studies of processes of interaction of high-energy particles are often made in rather dense substances. As a rule the data obtained in such experiments are compared with the theoretical results of calculations of the probability of the given process in vacuum. This means an implicit use of the hypothesis that possible participations of the atoms of the surrounding substance in the process lead only to vanishingly small corrections to the transition probability. In particular, this applies to the possible transfer of some part of the momentum to atoms of the medium through Coulomb scattering of the particles involved in the process.

The incorrectness of this assumption was first pointed out by Landau and Pomeranchuk,[1] who showed that multiple scattering of a particle by atoms of the medium causes a considerable change in the bremsstrahlung. In the case of bremsstrahlung a transfer of momentum to the medium would occur even if the Coulomb scattering were not taken into account. Therefore it is convenient to take account of the additional transfer of momentum to the medium in the treatment of the basic process. The position is different, however, when one treats a process that occurs spontaneously in vacuum. Transfer of momentum to atoms of the medium leads to the appearance of a new type of process, qualitatively different from the basic process. For example, spontaneous π-μ decay is possible in vacuum. In matter one has besides the spontaneous type of decay the possibility of an "induced" or "deceleration" type of decay, in which atoms of the medium take part in the process, receiving part of the momentum of the initial particle. Since in the usual experimental arrangement the momentum transferred to the medium is not measured, the experimentally observed transition probability must include both the spontaneous and the "induced" processes.

We present below a general method for calculating probabilities of processes with transfer of momentum to atoms of the medium taken into account, and we consider the change of the transition probability owing to this effect for the simplest cases. We note that this effect occurs already in the approximation linear in the density of medium atoms, in which the action of each atom is considered independently. Therefore in most cases it is not necessary to deal with multiple scattering, and we can confine ourselves to first-order perturbation theory in the potential of the atoms of the medium.

We consider only cases of very fast charged particles, with momenta large in comparison with the reciprocal of the Thomas-Fermi radius of the atom, $\lambda = me^2Z^{1/3}$ ($\hbar = c = 1$). This means that we can consider only Coulomb scattering by the nuclei of the atoms, including the action of the electron shells in terms of a screening factor only.

We also note the following fact: The occurrence of finite energy losses in the motion of a charged particle through matter has the consequence that, unlike the situation for vacuum, the imaginary term in the mass of the particle, which Feynman introduced to prescribe the correct way of going around the poles for the Green's function, is now a finite, though indeed small, quantity. The finite value of this quantity is also due to the presence of the electron shells of the atoms. We shall neglect here effects associated with the finiteness of the imaginary term in the mass.

In treating the interaction of a charged particle with an individual atom we shall neglect retardation and the recoil of the atom, restricting the treatment to not very large momentum transfers. Under the assumptions that have been made the

total potential of all the atoms of the medium, $U(x) = \sum_a U_0(x - x_a)$ [$U_0(x-x_a)$ is the potential of an individual atom], can be regarded as an external field. The total potential $U(x)$ depends on the specific (but unknown) distribution of the atoms of an amorphous medium. Therefore in the final results there must be an averaging over the coordinates x_a of the atoms; this is denoted by the symbol $\langle \ldots \rangle$. Because the medium as a whole is neutral, $\langle U(x) \rangle$ is a constant independent of x. To simplify the writing it is convenient to introduce instead of U the potential $V(x) = U(x) - \langle U(x) \rangle$, including the constant $\langle U(x) \rangle$ in the definition of the fourth component of the momentum of the charged particle. Thus the problem reduces to the calculation of the transition probability in an external field $V(x)$.

2. THE MATRIX ELEMENT OF THE PROCESS IN AN EXTERNAL FIELD

Let us consider a process whose initial and final states involve one charged particle, the initial and final four-momenta being p_1 and p_2. Besides this particle, there can be an arbitrary number of neutral particles in both the initial and the final states. Let us denote the difference of the four-momenta of the neutral particles in the initial and final states by k. Assuming that the Coulomb scattering of virtual particles can be neglected, we shall deal with the effects of the scattering of the initial and final charged particles by the potential V.

The matrix element of the process in vacuum can always be put in the form

$$M_0 = \int dx \bar{\psi}_{02}(x)\, O(k)\, \psi_{01}(x) \exp(ikx), \qquad (2.1)$$

where $\psi_{01}(x)$ and $\psi_{02}(x)$ are the wave functions of the initial and final charged particles in vacuum. In what follows we shall assume them normalized so that there is one particle in a large volume Ω. For example, for spin $\frac{1}{2}$ we have $\psi_{01}(x) = \Omega^{-1/2} u_1 \times \exp(-ip_1 x)$. In this connection we shall always understand the three-dimensional integral over momenta to mean the sum,

$$\int d^3p \to (2\pi)^3 \Omega^{-1} \sum_{\mathbf{p}}, \qquad \delta(\mathbf{p} - \mathbf{p}') = \Omega (2\pi)^{-3} \delta_{\mathbf{p},\mathbf{p}'}.$$

The matrix element of the process in the external field can be obtained from Eq. (2.1) by replacing the vacuum wave functions ψ_{01} and ψ_{02} by the wave functions of the charged particles in the external field. Assuming that the effect of the external field is small, we can determine the desired wave functions by perturbation theory. Assuming the momentum l transferred to the external field small in comparison with the momentum of the particle, in the first nonvanishing approximation in l/p and in second-order perturbation theory in V we can get the expression

$$\psi_{1(2)}(x) = C_{1(2)} \{1 + f_{1(2)}(x)\} \psi_{01(2)}(x); \qquad (2.2)$$

where f(x) denotes the quantity

$$f(x) = 2E \int \frac{dl V(l) \exp(ilx)}{l^2 + 2pl + i\delta} \left\{ 1 + 2E \int \frac{dl' V(l') \exp(il'x)}{(l+l')^2 + 2p(l+l') + i\delta} \right\}, \qquad (2.3)$$

and the normalization constant C is determined from the requirement that the wave function of the particle in the medium be normalized just like that of the particle in vacuum (one particle in the volume Ω). It follows that the normalization constant C is given by

$$C^{-1} = \Omega^{-1/2} \left[\int dx |1 + f(x)|^2 \right]^{1/2}. \qquad (2.4)$$

It may seem that the normalization constant C should be set equal to unity, since the difference $1 - C^2$ contains the large volume Ω in its denominator. Actually this must not be done, since in the numerator of the difference $1 - C^2$ there is a quantity proportional to the total number N of the atoms of the medium. The ratio of these quantities is proportional to the number of atoms per unit volume, and remains constant as the normalization volume goes to infinity. Therefore we must not neglect the difference between the normalization constant C and unity.

From what has been said it follows that the matrix element of the process in the external field can be written in the form

$$M = \int dx \exp(ikx) (\bar{\psi}_{02}(x)\, O(k)\, \psi_{01}(x)) (1 + f_2^*(x)) (1 + f_1(x)). \qquad (2.5)$$

Assuming for definiteness that the charged particles have spin $\frac{1}{2}$, and denoting the momentum $p_1 - p_2 - k$ transferred to the external field by s, we can transform Eq. (2.5) to the form

$$M = \int dx \exp(-isx) (\bar{u}_2 O(p_1 - p_2 - s) u_1) \times (1 + f_2^*(x)) (1 + f_1(x)). \qquad (2.6)$$

3. THE TRANSITION PROBABILITY IN THE EXTERNAL FIELD

From Eq. (2.6) it is easy to get the square of the absolute value of the matrix element and the transition probability for the process in the external field. As is well known, the differential transition probability is connected with the matrix element by the formula

$$dW = \frac{d}{dt} |M|^2 d^3p_1 d^3p_2 d^3k_1 \ldots d^3k_\nu (2\pi)^{-3(\nu+2)}, \quad (3.1)$$

where ν is the number of neutral particles in the final state, and $k_1, \ldots, k_\nu$ are the momenta of these particles. We can replace the integration over one of the momenta of the neutral particles in the final state by integration over k, and then go over from integration over k to integration over the momentum s transferred to the medium. Since for a process in vacuum the integration over the momentum of one particle is always taken between infinite limits by means of a δ function, we can perform the integration over s between infinite limits without destroying the correspondence with the vacuum case.

Using this idea, we easily get from Eq. (3.1)

$$dW = \frac{d}{dt} C_1^2 C_2^2 \iint dx\, dx' \int d^3s \exp[is(x-x')]$$
$$\times |(\bar{u}_2 O(p_1 - p_2 - s)\, u_1|^2 \, (1 + f_1(x))$$
$$\times (1 + f_1^*(x'))(1 + f_2(x'))$$
$$\times (1 + f_2^*(x))(2\pi)^{-3(\nu+2)} d^3p_1\, d^3p_2\, d^3k_1 \ldots d^3k_{\nu-1}. \quad (3.2)$$

Carrying out the integration over x and x', one easily convinces oneself that only small values of s play any part in the integral over s. Therefore, provided the difference $p_1 - p_2$ is not small, the quantity $|\bar{u}_2 O(p_1 - p_2 - s) u_1|^2$ is a slowly varying function of s as compared with the exponential. Taking this slowly varying function from under the integral sign, we can relate the transition probability in the external field to the transition probability dW_0 in vacuum:

$$dW = \frac{dW_0 \Omega \int d^3x (1 + b_1(x))(1 + b_2(x))}{\int d^3x (1 + b_1(x)) \int d^3x' (1 + b_2(x'))}, \quad (3.3)$$

where we have used the notation

$$b(x) = |1 + f(x)|^2 - 1.$$

It is easy to see from Eq. (3.3) that inclusion of the normalization leads to cancellation from the final result of all terms associated with the effect of the external field on only the initial or only the final particle. In fact, using the smallness of b(x), we can get from Eq. (3.3) the formula

$$dW = dW_0 (1 + \Omega^{-1} \int d^3x b_1(x) b_2(x) - \Omega^{-2}$$
$$\times \int d^3x b_1(x) \int d^3x' b_2(x')). \quad (3.4)$$

When we substitute in Eq. (3.4) the explicit expression for $b_1(x)$ and $b_2(x)$ we must remember that we are using perturbation theory for the external field, so that in Eq. (3.4) we must include only the part of b(x) linear in the field, dropping terms quadratic in the field. Besides this we must note that in the cases we are concerned with, in which the external field is that of an individual atom, or else we are using the external potential $V(x) = U(x) - \langle U(x) \rangle$, the last term in Eq. (3.4) is zero. Therefore the final expression for the transition probability in the medium is connected with the transition probability in vacuum by the formula

$$dW = dW_0 \{1 + (2\pi)^3 \Omega^{-1} 4E_1E_2 \int d^3l\, |V(l)|^2 [(l^2 + 2\mathbf{p}_1 l - i\delta)^{-1}$$
$$+ (l^2 - 2\mathbf{p}_1 l + i\delta)^{-1}][(l^2 + 2\mathbf{p}_2 l + i\delta)^{-1}$$
$$+ (l^2 - 2\mathbf{p}_2 l - i\delta)^{-1}]\}. \quad (3.5)$$

Let us apply this formula to the case in which we are dealing with the external field of an individual atom. Then in virtue of the fact that in our approximation the effects of the atoms on the process are treated independently, to get the total effect of N atoms we must multiply the correction for one atom by the total number of atoms. Thus we get from Eq. (3.5) the result

$$dW = dW_0 \{1 + n(2\pi)^3 4E_1E_2\, 2\,\mathrm{Re} \int d^3l\, |U_0(l)|^2$$
$$\times (l^2 + 2_2 \mathbf{p} l - i\delta)^{-1}[(l^2 + 2\mathbf{p}_1 l - i\delta)^{-1}$$
$$+ (l^2 - 2\mathbf{p}_1 l - i\delta)^{-1}]\}, \quad (3.6)$$

where n is the number of atoms per unit volume.

Exactly the same formula is obtained if we consider the process in the external potential $V(x) = U(x) - \langle U(x) \rangle$. If we use the fact that in the momentum representation the total potential U is connected with the potential U_0 of an individual atom by the relation $U(p) = U_0(\mathbf{p})\, \delta(p_4) \times \sum_a \exp(i\mathbf{p} \cdot \mathbf{r}_a)$, we easily show that $\langle |V(\boldsymbol{l})|^2 \rangle = N |U_0(\boldsymbol{l})|^2$. By using this result one gets Eq. (3.6) from Eq. (3.5).

4. APPLICATION OF THE DIAGRAM TECHNIQUE

The result obtained above can also be found by a different method, by using an analogy with the diagram technique of quantum field theory. In the derivation of Eq. (3.6) the averaging over the positions of the medium atoms was carried out only in the concluding stage of the calculations. We can arrive at the analogy with quantum field theory if we do the averaging over the positions of the atoms somewhat earlier. Let us consider the average

value of the product of two potentials $V(l)$ and $V(l')$ in the momentum representation. Using the fact that

$$\left\langle \sum_a \exp(i\mathbf{p}\mathbf{r}_a) \right\rangle = N\delta_{\mathbf{p},0} = n(2\pi)^3 \delta(\mathbf{p}),$$

we easily get

$$\langle V(l) V(l') \rangle = F(l)\, \delta^{(4)}(l + l'),$$

$$F(l) = n(2\pi)^3 |U_0(\mathbf{l})|^2 \delta(l_4). \qquad (4.1)$$

The presence of the four-dimensional δ function on the right enables us to regard Eq. (4.1) as an expression analogous to the propagation function of a quasi-particle. If both potentials $V(l)$ and $V(l')$ are in the same matrix element, the averaging of their product leads to a transfer of momentum from one particle to another (from one point of a diagram to another). Therefore the average value (4.1) can be graphically represented by connecting the points of action of the potentials on a Feynman diagram by a dotted line. In other words, such average values are taken into account in the same way as the exchange of a virtual photon, but with the propagation function (4.1).

There is also another possible case, in which one of the potentials to be averaged is in the matrix element M and the other in the Hermitian adjoint matrix element M^+. Using the fact that there is always an integration over the momenta l and l' transferred to the external field, we can put the result of integrating the expression (4.1) in the form

$$n(2\pi)^3 \int d^3l\, |U_0(\mathbf{l})|^2. \qquad (4.2)$$

It is easily seen that the average values (4.2) are to be taken into account in just the same way as the emission of real photons, but with the factor $e(2\omega)^{-1/2}$ replaced by $n^{1/2}(2\pi)^{3/2} U_0(\mathbf{l})^2$ and with an integration over the momenta of the emitted "quasi-particles." Average values of this type correspond to a mean transfer of momentum to atoms of the medium.

These arguments make our problem of the calculation of the probability of a process with account taken of transfer of momentum to the medium entirely analogous to the problem of the calculation of radiative corrections to the process in question; we are here concerned not with corrections caused by the electromagnetic interaction, but with those caused by interactions with "quasi-particles." As is well known, in the calculation of radiative corrections to the experimentally observable cross section one must include not only radiative corrections owing to the exchange of virtual photons, but also those processes occurring in the same order of the perturbation theory and involving the emission of small quanta that are not registered by the experimental apparatus. The corresponding diagrams are shown in Fig. 1.

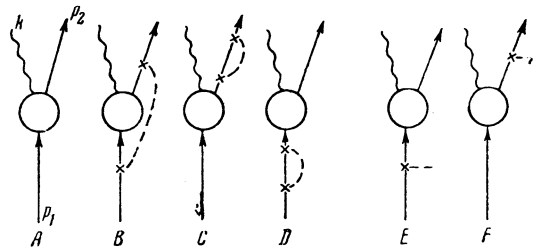

FIG. 1

Diagrams C and D contain proper-energy parts in external lines. A renormalization of the wave function must be carried out in the corresponding parts of the matrix element; it can, for example, be performed by the Feynman method.[2] This renormalization leads to the same results as the normalization of the wave function in the external field which was done in the preceding section.

The cancellation in the final result of all expressions involving the effect of the external field on only one initial or one final particle is analogous to the cancellation of the infrared divergence in the case of the electromagnetic radiative corrections.

It is convenient to represent all the terms contributing to the final result by so-called generalized Feynman diagrams, in which together with the graph corresponding to the matrix element there is also drawn its mirror image, corresponding to the Hermitian adjoint matrix element. Figure 2 shows

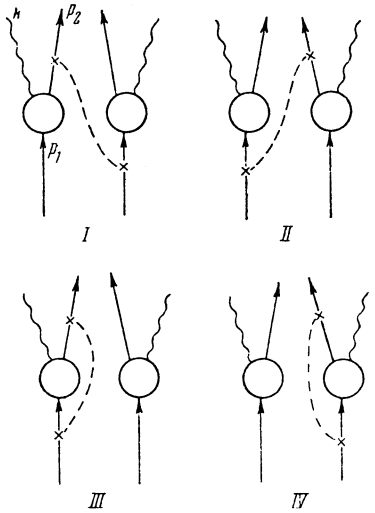

FIG. 2

all the diagrams of this kind; diagrams I and II correspond to transfer of momentum to the medium (emission of "quasi-particles"), and diagrams III and IV correspond to the process without mean momentum transfer to the medium (exchange of virtual "quasi-particles").

It is easily verified that by means of the diagram technique one again gets Eq. (3.6).

5. THE EXPERIMENTALLY OBSERVED TRANSITION PROBABILITY

To obtain the probability of a process in the medium we must substitute in Eq. (3.6) the explicit expression for the potential U_0 of an individual atom:

$$U_0(r) = (Ze^2/r)\exp(-\lambda r); \quad U_0(\mathbf{p}) = (Ze^2/2\pi^2)(\mathbf{p}^2+\lambda^2)^{-1}. \tag{5.1}$$

From Eqs. (3.6) and (5.1) we get the formula for the transition probability in the medium

$$dW = dW_0\Big\{1 + (2/\pi)\,4E_1E_2\,2\,\mathrm{Re}\int d^3l\,(\mathbf{l}^2+\lambda^2)^{-2} \times (\mathbf{l}^2 + 2\mathbf{p}_2\mathbf{l} - i\delta)^{-1}[(\mathbf{l}^2+2\mathbf{p}_1\mathbf{l}-i\delta)^{-1} + (\mathbf{l}^2 - 2\mathbf{p}_1\mathbf{l} - i\delta)^{-1}]\Big\}. \tag{5.2}$$

The calculation of the integral is carried out in the Appendix; for not too small angles ϑ between the momenta of the charged particles, $\mathbf{p}_1$ and $\mathbf{p}_2$ ($2|\mathbf{p}_1||\mathbf{p}_2|\sin\vartheta > \lambda|\mathbf{p}_1+\mathbf{p}_2|$) we have the following formula:

$$dW = dW_0\{1 - 4\pi^2Z^2e^4n\lambda^{-3}E_1E_2/\sqrt{p_1^2p_2^2 - (\mathbf{p}_1\mathbf{p}_2)^2}\}. \tag{5.3}$$

In ordinary units this is

$$dW = dW_0\{1 - 4\pi^2Zn(\hbar/mc)^3(\hbar c/e^2) \times E_1E_2/|\mathbf{p}_1||\mathbf{p}_2||\sin\vartheta|\}. \tag{5.4}$$

In the derivation of this formula we have taken into account only the Coulomb scattering of the initial and final charged particles and have neglected the scattering of virtual particles. Let us estimate in what cases this neglect is justified. It is easy to see that in the averaging of the product of a pair of potentials $V(l)$ and $V(l')$ over the coordinates of the atoms there appear in the matrix element integrals of the type

$$\int \frac{d^4l\,\delta(l_4)\,|U_0(\mathbf{l})|^2}{\prod_i[(p_i+l)^2 - m_i^2]}.$$

We have taken into account only those factors in the denominator for which $p^2 = m^2$, neglecting the terms containing l in the factors for which $p^2 \neq m^2$. This is justified in cases in which the effective values of the momenta of the virtual particles satisfy the inequality $p^2 - m^2 \gg \lambda^2$. If this condition is satisfied the formula (5.4) remains valid for a process of arbitrary order, if in the initial and final states there is one charged particle together with an arbitrary number of neutral particles.

We note that Eq. (5.4) is valid both for particles of spin $\frac{1}{2}$ and also for particles of spin 0. In fact, all formulas up to Eq. (2.6) are valid for arbitrary spin; the change of Eq. (2.6) for spin 0 is trivial, so that Eqs. (3.3) — (3.6) are also valid for arbitrary spin.

Equation (5.4) cannot be used for small values of the difference $\mathbf{p}_1 - \mathbf{p}_2$, that is, in cases in which $\mathbf{k}$ is small in magnitude. In this case the quantity $|\bar{u}_2 O(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{s})u_1|^2$ in Eq. (3.2) cannot be regarded as a slowly varying function of $\mathbf{s}$. Besides this, Eq. (5.4) is invalid in the region of small angles ϑ between $\mathbf{p}_1$ and $\mathbf{p}_2$, such that $2|\mathbf{p}_1||\mathbf{p}_2| \times \sin\vartheta < \lambda|\mathbf{p}_1+\mathbf{p}_2|$. These cases require further consideration, since in this region it may be necessary to include the effect of higher orders of perturbation theory in the external potential of the atoms of the medium.

Let us estimate the size of the corrections to the transition probability for some media. Introducing the speeds β_1 and β_2 of the charged particles, we can write Eq. (5.4) in the form

$$dW = dW_0(1 - B/\beta_1\beta_2|\sin\vartheta|), \tag{5.5}$$

where for lead, water, and liquid hydrogen the coefficient B has the values 0.75×10^{-3}, 0.9×10^{-4}, and 0.6×10^{-5}, respectively. It follows that for small angles the change of the transition probability in dense substances near the end of the periodic table can become appreciable. On the other hand it is well known that for high energies of the initial particle the process occurs only in a small range of angles ($\vartheta \leq (1-\beta_1^2)^{1/2}$). In such cases we can also expect that the corrections to total transition probabilities will be large. We shall illustrate these results with the example of π-μ decay.

6. APPLICATION TO π-μ DECAY

As is well known, the probability of π-μ decay in vacuum can be written in the form

$$dW_0 = \frac{g^2}{8\pi}\frac{\mu^2(M^2-\mu^2)}{E_1p_1}dE_2. \tag{6.1}$$

It follows from Eq. (5.4) that the decay probability in a medium can be written in the form

$$dW = dW_0\{1 - 4\pi^2Z^2e^4n\lambda^{-3}/\beta_1\beta_2|\sin\vartheta|\}. \tag{6.2}$$

As has already been pointed out, at very small angles $\vartheta < \lambda|\mathbf{p}_1+\mathbf{p}_2|/2|\mathbf{p}_1||\mathbf{p}_2|$ Eq. (5.4), and consequently also Eq. (6.2), is incorrect. Such

small angles, however, make only a negligibly small contribution to the total decay probability. Therefore we can get the total decay probability by integrating the approximate formula (6.2). Instead of integrating over the angle ϑ it is convenient to use the conservation laws to express ϑ in terms of the μ-meson energy E_2 and intergrate over E_2 from its minimum value $(E_1/2)[1+\mu^2M^{-2} - \beta(1-\mu^2M^{-2})]$ to its maximum value $(E_1/2)[1+\mu^2M^{-2} +\beta(1-\mu^2M^{-2})]$ (M and E_1 are the mass and energy of the π meson). After the intergration we can write the total decay probability for a meson passing through the medium with speed β_1 in the form

$$W = W_1\left\{\sqrt{1-\beta_1^2} - \frac{2\pi^3Z^2e^4n}{\beta\lambda^3}\frac{M^2+\mu^2}{M^2-\mu^2}\right\}.$$
$$W_1 = (g^2/8\pi)\mu^2M(1-\mu^2M^{-2})^2. \qquad (6.3)$$

Thus the Coulomb scattering of the decaying particle and the decay products leads to a decrease of the total decay probability, i.e., to an increase of the lifetime of the unstable particle.

Let us emphasize once again that this result applies only to the decay of a π meson moving swiftly through the medium, and cannot be extended to a π meson at rest. Furthermore, for a π meson at rest in the medium the influence of the Coulomb scattering can act only on the μ meson produced in the decay, i.e., just on the final charged particle. As has been shown above, in this case there is on the average no change of the transition probability.

The change of the decay probability is especially marked at high π-meson energies, since the main term, the decay probability in vacuum, falls off linearly with increase of the energy, and the correction term approaches a constant value as the energy increases. For lead, water, and liquid hydrogen the coefficient $2\pi^3Z^2e^4n\lambda^{-3}(M^2+\mu^2)(M^2-\mu^2)^{-1}$ has the values 3×10^{-3}, 4×10^{-4}, and 2×10^{-5}, respectively. Therefore in lead, for example, at π-meson energies of the order of 10^{10} ev the correction term is 30 percent of the main effect, and in water at the same energy the correction is about 4 percent of the main effect. The use of perturbation theory to treat the total potential of the atoms of the medium makes the result only qualitatively correct for large values of the correction. Therefore it is interesting to treat processes with transfer of momentum to the medium without handling V by the perturbation method; such a treatment will be presented elsewhere. Finally, for the motion of particles of very high energy in a medium one should take into account losses in the medium. This fact makes the results obtained here inapplicable at very high energies.

We note that in its nature and properties the effect treated here is an analog of the Landau-Pomeranchuk effect for bremsstrahlung. In both cases a change of the transition probability is caused by the Coulomb scattering of the charged particles by the atoms of the medium and the effect reduces to the transfer of momentum to the atoms of the medium. The only difference is that in the case of bremsstrahlung it is convenient to take the transfer of momentum to the medium into account as part of the bremsstrahlung process itself, and in the case of decay we have to deal with a further process of decay in an external field which is qualitatively different from the main decay process.

The effect of transfer of momentum to the atoms of the medium must be taken into account for all processes involving at least two fast charged particles. The extension to the case of several charged particles is obvious. If the scattering of the virtual particles can be neglected, the averaged differential transition probability for the case of several charged particles will have a form analogous to Eq. (5.4). To get the total transition probability a separate treatment is required in each concrete case.

In conclusion, I take this occasion to thank E. L. Feĭnberg and V. M. Galitskiĭ for helpful discussions.

APPENDIX

The integral in Eq. (5.2)

$$J = \mathrm{Re}\int d^3l\,(l^2+\lambda^2)^{-2}(l^2+2\mathbf{p}_1\mathbf{l}-i\delta)^{-1}$$
$$\times\{(l^2+2\mathbf{p}_2\mathbf{l}-i\delta)^{-1}+(l^2-2\mathbf{p}_2\mathbf{l}-i\delta)^{-1}\}$$

can be put in the form

$$J = \mathrm{Re}\left(1+\frac{\lambda}{2}\frac{\partial}{\partial\lambda}\right)\int_{-1}^{+1}du\int_0^1 2xdx\int_0^1 3y^2dy\int d^3l$$
$$\times[l^2-4l(\mathbf{p}_2ux+\mathbf{p}_1(1-x))y+\lambda^2(1-y)-i\delta]^{-4}$$
$$= \mathrm{Re}\frac{i\pi^2}{8}\left(1+\frac{\lambda}{2}\frac{\partial}{\partial\lambda}\right)\int_{-1}^{+1}du\int_0^1 2xdx\int_0^1 3y^2dy$$
$$\times[y^2(\mathbf{p}_2ux+\mathbf{p}_1(1-x))^2-\lambda^2(1-y)+i\delta]^{-5/2}.$$

Integrating over y and taking the real part, we have

$$J = 4\pi^2\left(1+\frac{\lambda}{2}\frac{\partial}{\partial\lambda}\right)\frac{1}{\lambda}\int_{-1}^{+1}du\int_0^1 xdx\,[4(\mathbf{p}_2ux+\mathbf{p}_1(1-x))^2+\lambda^2]^{-2}$$
$$= -\frac{\pi^2}{\lambda}\frac{\partial^2}{\partial\lambda^2}\int_{-1}^{+1}du\int_0^1 xdx\,[4(\mathbf{p}_2ux+\mathbf{p}_1(1-x))^2+\lambda^2]^{-1}.$$

Changing to the variables $\zeta = ux + (1-x)$ and $\eta = ux - (1-x)$ and abbreviating $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{q}$ and $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{k}$, we get after integrating over ζ

$$J = -\frac{\pi^2}{2\lambda}\frac{\partial^2}{\partial\lambda^2}\int_{-1}^{+1} d\eta\,[\lambda^2 q^2 + \eta^2(q^2k^2 - (\mathbf{qk})^2)]^{-1/2}$$

$$\times\left\{\tan^{-1}\frac{\eta(q^2+\mathbf{qk})}{\sqrt{\lambda^2q^2+\eta^2(q^2k^2-(\mathbf{qk})^2)}}\right.$$

$$\left.-\tan^{-1}\frac{\eta(\mathbf{qk}-q^2)}{\sqrt{\lambda^2q^2+\eta^2(q^2k^2-(\mathbf{qk})^2)}}\right\}.$$

For $q^2k^2 - (\mathbf{qk})^2 \gg \lambda^2q^2$ the behavior of the integrand is determined by the square root in the denominators, and the main contribution is from small values of η. Assuming that the inequality just stated holds, we can then get the approximate result

$$J = -(\pi^3/4\lambda^3)(p_1^2p_2^2 - (\mathbf{p}_1\mathbf{p}_2)^2)^{-1/2}.$$

[1] L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 735 (1953).

[2] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

Translated by W. H. Furry
164