# PERIPHERAL NUCLEON INTERACTION IN THE TWO-MESON APPROXIMATION

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Triplet nucleon-nucleon scattering phase shifts are computed at nonrelativistic energies in the two-meson approximation. A comparison of the two-meson and one-meson phase shifts shows that the one-meson approximation is excellent for all mixing parameters (starting with $^3S$-$^3D$). This permits the use of theoretical values of mixing parameters in order to obtain a unique solution in a phase-shift analysis. The peripheral part of the two-meson potential, which corresponds in first Born approximation to the correct scattering amplitude, is also derived.

GALANIN et al.[1] have proposed a method of calculating nucleon-nucleon scattering phase shifts for large orbital angular momenta $l$ in the two-meson approximation. The method makes it possible to obtain, for two-meson phase shifts, the lowest order term in a series expansion in the parameter

$$1/L = \sqrt{1 + p^2/\mu^2}/(l + 1),$$

where $\mu$ is the pion mass and p is the nucleon momentum in the barycentric system. Specific calculations were carried out in reference 2 for the singlet state, in the nonrelativistic approximation including corrections of order $\sim p^2/m^2$, where an expansion in a second parameter $\zeta = \epsilon\sqrt{L}/2$ was simultaneously introduced ($\epsilon = \mu/m$, m = nucleon mass). Thus the singlet scattering phase shifts are obtained in the form of a double series expansion in the parameters $1/L$ (lowest order term) and $\zeta$ (first three terms) which insures good accuracy in the intermediate region $1 \ll L \ll 4/\epsilon^2$.

In the approximation indicated, calculations can be performed in an analogous fashion for the triplet state which makes it possible to find the nucleon-nucleon scattering operator due to the exchange of two mesons. For comparison with theoretical investigations and phenomenological models we also calculate the nucleon-nucleon potential corresponding to the two-meson scattering operator obtained in this work. An expansion in $1/L$ of the phase shifts corresponds to an expansion in $1/x$ of the potential, where x is the distance between the nucleons in units of $1/\mu$; i.e., the present method makes it possible to obtain the peripheral part of the two-meson potential for distances $x > 1$ with an accuracy of order $\sim 1/x$.

## 1. SCATTERING AMPLITUDES AND PHASE SHIFTS

The calculation of the scattering operator $M^{(2)}(q^2)$ in the vicinity of the nearest singular point* $q^2 = -4\mu^2$ differs from the corresponding singlet amplitude calculation[2] only in the replacement of the singlet matrix elements $w$, $w_1$, $w_2$, and $w_3$ [see Eqs. (1.7), (1.16), and (1.23) in reference 2] by the corresponding spin operators. Also, in the calculation of the contribution from one of the perturbation theory diagrams ($B_4$ in the notation of reference 2) it is necessary to include terms of order $\sim s^2$, where $s^2 = 1 + q^2/4\mu^2$. The nonexchange part of the scattering operator near the point $q^2 = -4\mu^2$, with corrections of order $\sim p^2/m^2$ included, may be written as follows:

$$M^{(2)}(q^2) = M_S(q^2) + \frac{i}{m^2}(\mathbf{S}[\mathbf{p}'\times\mathbf{p}])\, M_{LS}(q^2) + \frac{(\boldsymbol{\sigma}^{(1)}\mathbf{q})(\boldsymbol{\sigma}^{(2)}\mathbf{q})}{m^2} M_T(q^2), \qquad (1.1)$$

where

$$M_S(q^2) = -\frac{3g^4}{8mE}\left\{c_{0S}s + c_{1S}\varepsilon \ln(\varepsilon + 2s) + \frac{3+2\lambda_\tau}{6}\frac{\varepsilon s}{\varepsilon + 2s} + \frac{3-2\lambda_\tau}{12E}\frac{\varepsilon}{v}\left[\ln\frac{2Es + \varepsilon v}{E(1+v)\mu/p} - \frac{i\pi}{2\sqrt{1-s^2}}\right]\right\},$$

$$c_{0S} = (\alpha - 1)^2, \qquad c_{1S} = \alpha - 1;$$

$$M_{LS}(q^2) = -\frac{3g^4}{8mE}\left\{c_{0(LS)}s + c_{1(LS)}\varepsilon \ln(\varepsilon + 2s) + \frac{3+2\lambda_\tau}{12}\frac{\varepsilon s}{\varepsilon + 2s} + \frac{3-2\lambda_\tau}{12E}\frac{\varepsilon m^2}{p^2}\left(\frac{1}{v} - v\frac{\mu^2}{\mu^2+p^2}\right)\left[\ln\frac{2Es+\varepsilon v}{E(1+v)\mu/p} - \frac{i\pi}{2\sqrt{1-s^2}}\right]\right\}.$$

$$c_{0(LS)} = -\frac{1}{2}(\alpha - 1)^2 + \frac{3-2\lambda_\tau}{12}\frac{\mu^2}{p^2+\mu^2} + \frac{2}{3}\lambda_\tau\beta_2,$$

*We use the notation introduced by Galanin et al.[1,2] except for a change in sign of the momentum transfer $q^2$. Our choice has the convenient feature that $q^2 = \mathbf{q}^2$.

$$c_{1(LS)} = -\frac{1}{2}\alpha - (1-2s^2)\frac{3-2\lambda_\tau}{12E}\frac{m^2}{p^2+\mu^2} - \frac{\lambda}{3}\left(\beta_2 + 2\beta\frac{s^2}{\varepsilon^2}\right);$$

$$M_T(q^2) = -\frac{3g^4}{8mE}\Big\{c_{0T}s + c_{1T}\varepsilon\ln(\varepsilon+2s) - \frac{3-2\lambda_\tau}{48E}\frac{\varepsilon v m^2}{\mu^2+p^2}\left[\ln\frac{2Es+\varepsilon v}{E(1+v)\mu/p} - \frac{i\pi}{2\sqrt{1-s^2}}\right]\Big\},$$

$$c_{1T} = \frac{3+2\lambda_\tau}{48} + (1-2s^2)\frac{3-2\lambda_\tau}{48}\frac{m^2+\mu^2/2}{\mu^2+p^2} - \frac{\lambda_\tau}{24}\beta_1\left(1-4\frac{s^2}{\varepsilon^2}\right);$$

$$c_{0T} = -\frac{1}{8}\left(1+\frac{1}{2}\frac{\mu^2}{\mu^2+p^2}\right) - \frac{\lambda}{12}\left(1-\beta_1-\frac{1}{2}\frac{\mu^2}{\mu^2+p^2}\right),$$

$$v = \sqrt{4s^2p^2+\mu^2}/\mu; \qquad E^2 = 1+p^2/m^2; \qquad \alpha = 1{,}2;$$

$$\beta_1 = 0.025, \qquad \beta_2 = -0.029, \qquad \beta = -\beta_1-\beta_2;$$

and $\lambda_T = 1, -3$ corresponds to isotopic spin $T = 1, 0$.

The coefficient $a_l(S, S'_Z, S_Z, T)$ in an expansion of the amplitude in associated Legendre polynomials may be obtained by calculating the matrix elements of the operator (1.1) in the angular momentum representation and integrating over $|s|$ entirely analogously to what was done in references 1 and 2. The triplet amplitudes corresponding to a change in the spin projection $|\Delta S_Z| = 1$ and 2 are expanded in terms of the polynomials $P_l^{(1)}$ and $P_l^{(2)}$ and the expansion coefficients are expressed in terms of integrals over the cut of the associated Legendre functions of the second kind $Q_l^{(1)}$ and $Q_l^{(2)}$. Replacing the associated functions by $Q_l$ we obtain, accurate to order $\sim 1/L$,

$$a_l(1,1,0,T) = -p\frac{4(\mu/p)^3}{i\pi l}\sqrt{1+\frac{\mu^2}{p^2}}\,Q_l(1+2\mu^2/p^2)\times\int_0^\infty e^{-L|s|^2}\Delta[M(1,1,0,T)/\sin\theta]\,|s|\,d|s|,$$

$$a_l(1,1,-1,T) = -p\frac{2(\mu/p)^2}{i\pi l^2}Q_l(1+2\mu^2/p^2)\times\int_0^\infty e^{-L|s|^2}\Delta M(1,1,-1,T)\,|s|\,d|s|.$$

where we denote by the symbol $\Delta f$ the discontinuity in the function $f$ across the cut $q^2 \le -4\mu^2$ ($s^2 \le 0$). Finally, the various scattering phase shifts* are expressed in terms of the coefficients $a_l(S, S'_Z, S_Z, T)$ with the following result

$$\eta_l^J = \frac{3g^4\varepsilon\mu}{8E\sqrt{\pi}pL^{3/2}}Q_l(1+2\mu^2/p^2)\{d_0^J + 2\sqrt{\pi}\zeta d_1^J + \zeta^2 d_2^J\}. \tag{1.2}$$

Here it is understood that the superscript $J$ is to be omitted for the singlet state. We find for the coefficients of the powers of $\zeta$ appearing in Eq. (1.2)

$$d_0 = (\alpha-1)^2, \qquad d_1 = \alpha - 1 + \frac{3-2\lambda_\tau}{6}\left(1-\frac{p^2}{2m^2}\right)\Psi(z),$$

$$d_2 = -4(\alpha-1) + p^2/m^2 + \frac{1}{3}\lambda_\tau(4-2p^2/m^2),$$

$$z^2 = L\mu^2/4p^2, \qquad \Psi(z) = ze^{-z^2}\int_0^z e^{x^2}dx; \qquad d_i^l = d_i,$$

$$d_i^{l\pm1} = d_i \pm \varepsilon\sqrt{\varepsilon^2+p^2/m^2}D_i \qquad (i = 0, 1, 2);$$

$$D_0 = (\alpha-1)^2 - \frac{4}{3}\lambda_\tau\left(\beta_2 - \frac{1}{2}\beta\right),$$

$$D_1 = \alpha - (3-2\lambda_\tau)\left(1-\frac{p^2}{2m^2}\right)\frac{m^2}{3p^2}\left(\Psi(z)-\frac{1}{2}\right) - \frac{\lambda_\tau}{3}\frac{\beta}{\zeta^2},$$

$$D_2 = -2(2\alpha+1). \tag{1.2'}$$

The triplet phase shift with $J = l$ and the singlet phase shift $\eta_l$ coincide, and the triplet phase shifts $\eta_l^{l\pm1}$ (with $J = l\pm1$) differ from them by terms due to the spin-orbit interaction.

The mixing parameter is given by

$$\xi_J = \frac{3g^4\varepsilon^3\mu}{8E\sqrt{\pi}pL_J^{3/2}}Q_{J+1}\left(1+2\frac{\mu^2}{p^2}\right)\left(1+2\frac{\mu^2}{p^2}+2\frac{\mu}{p}\sqrt{1+\mu^2/p^2}\right)\{h_0 + 2\sqrt{\pi}\zeta_J h_1\},$$

$$h_0 = 1 - \frac{2}{3}\lambda_\tau\beta_1,$$

$$h_1 = -\frac{9+2\lambda_\tau}{24} - \frac{(3-2\lambda_\tau)(1-p^2/2m^2)m^2\Psi(z_J)}{12(\mu^2+p^2)z_J^2} + \frac{\lambda_\tau\beta_1}{6\zeta^2},$$

$$L_J = (J+2)/\sqrt{1+p^2/\mu^2}, \quad \zeta_J = \varepsilon\sqrt{L_J}/2, \quad z_J^2 = L_J\mu^2/4p^2. \tag{1.3}$$

Equations (1.2) and (1.3) represent the first terms of the expansion of two-meson phase shifts in $1/L$ and $\zeta$. Therefore we omit corrections of order $\epsilon^2$ in the calculations since such terms are of the same order of smallness as $\sim 1/L$ corrections to higher order terms in the expansion in $\zeta$ (note that $\epsilon^2 = 4\zeta^2/L$). An interesting feature of our results is the strong cancellation in the phase shifts $\eta_l$, $\eta_l^J$ of the contributions from perturbation theory and from the "smooth" part of the meson-nucleon amplitude containing the coefficient $\alpha$ (the main terms contain $\alpha - 1 \ll \alpha, 1$). The contributions from the meson-nucleon amplitude containing $\beta_1$ and $\beta_2$ are small and may be neglected in rough estimates.

Expression (1.3) for the mixing parameter does not contain the coefficient $\alpha$. This is explained by the fact that the corresponding part of the meson-nucleon amplitude is spin-independent and does not contribute to the tensor forces. Since the coefficient $\beta_1$ entering into (1.3) makes a small contribution, the mixing parameters are approximated well by perturbation theory. This makes it possible to obtain more accurate expressions for them than Eq.

*For that purpose we make use of Eq. (6) of Grashin[3] accurate to terms of order $1/l$. The notation for the phase shifts and expansion coefficients of the amplitude coincides with that introduced by Grashin;[3] only their real parts concern us here.

TABLE I

| $E_{lab}$, Mev | 10 | 40 | 100 | 200 | 300 | 400 | 670 |
|---|---|---|---|---|---|---|---|
| $^1D_2$ | 0.04 | 0.1 | 0.25 | 0.5 | — | — | — |
| $^1G_4$ | 0 | 0.01 | 0.04 | 0.1 | 0.2 | 0.3 | — |
| $^3P_0$ | 0.06 | 0.03 | 0.02 | — | — | — | — |
| $^3P_1$ | −0.07 | −0.1 | −0.15 | — | — | — | — |
| $^3P_2$ | 1 | 2 | 2 | — | — | — | — |
| $\xi_2$ | −0.001 | −0.003 | −0.01 | −0.02 | −0.04 | −0.05 | — |
| $^3F_2$ | 0.02 | 0.04 | 0.06 | 0.07 | 0.08 | 0.07 | — |
| $^3F_3$ | −0.01 | −0.02 | −0.04 | −0.08 | −0.1 | −0.15 | — |
| $^3F_4$ | 0.05 | 0.2 | 0.4 | 0.7 | 1 | 1 | — |
| $\xi_4$ | 0 | 0 | −0.002 | −0.008 | −0.015 | −0.02 | −0.04 |
| $^3H_4$ | 0 | 0.01 | 0.03 | 0.06 | 0.07 | 0.08 | 0.1 |
| $^3H_5$ | 0 | 0 | −0.01 | −0.03 | −0.05 | −0.07 | −0.1 |
| $^3H_6$ | 0.01 | 0.02 | 0.1 | 0.3 | 0.4 | 0.5 | 0.9 |

TABLE II

| $E_{lab}$, Mev | 10 | 40 | 100 | 200 | 300 | 400 | 670 |
|---|---|---|---|---|---|---|---|
| $^1P_1$ | −0.15 | −0.2 | −0.2 | — | — | — | — |
| $^1F_3$ | 0 | −0.02 | −0.04 | −0.07 | −0.1 | −0.1 | — |
| $^1H_5$ | 0 | 0 | −0.01 | −0.02 | −0.03 | −0.04 | −0.1 |
| $\xi_1$ | −0.005 | −0.02 | −0.02 | −0.01 | — | — | — |
| $^3D_1$ | −0.07 | −0.01 | 0.05 | 0.07 | — | — | — |
| $^3D_2$ | 0.02 | 0.03 | 0.03 | 0.04 | — | — | — |
| $^3D_3$ | 0.3 | −0.6 | −0.8 | −0.8 | — | — | — |
| $\xi_3$ | 0 | −0.003 | −0.006 | −0.006 | −0.005 | −0.002 | — |
| $^3G_3$ | −0.01 | −0.02 | 0.01 | 0.03 | 0.05 | 0.06 | — |
| $^3G_4$ | 0 | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 | — |
| $^3G_5$ | 0.02 | −0.08 | −0.2 | −0.3 | −0.3 | −0.4 | — |

(1.3) by taking into account higher order terms in the $1/L$ expansion.*

Tables I and II show the ratios of the two-meson phase shifts, Eqs. (1.2) and (1.3), to the corresponding one-meson phase shifts calculated by Grashin.[3] Ratios smaller than 1% for $\eta_l$, $\eta_l^J$, and smaller than 0.1% for the mixing parameters $\xi_J$, were replaced by zero. The ratios were not calculated for the cases when $1/L \gg 1$. However, the ratios given in the tables also correspond to rather large values of $1/L$ $(0.4 - 0.8)$ and therefore the results should be viewed as rough estimates only and may be used with sufficient reliability only to establish the limits of applicability of the one-meson approximation.

It can be seen from the tables that the ratios are very large for the phase shifts with $J = l + 1$. As was already pointed out[3] this is due to the fact that the one-meson contributions to these phase shifts are anomalously small since at low energies they have the "anomalous" energy dependence $p^{2l+3}$. An interesting feature of our results is the high accuracy of the one-meson mixing parameters (beginning with $\xi_1$), which is very important for a comparison of theoretical and experimental data, since polarization data make it possible to obtain the mixing parameters with sufficient accuracy experimentally. Thus a comparison of the theoretical and experimental $\xi_2$ for pp scattering at 310 Mev[4] makes it possible, from just this one parameter, to reject two of the eight solutions of Stapp et al., namely the second and the fourth. The subsequent more accurate analysis with higher phase shifts taken into account in the one-meson approximation[5] showed that the two solutions become one (No. 2 in the notation of Cziffra et al.[5]).

One is inclined to believe, as suggested by one of the authors[3] and by Cziffra et al.,[5] that fewer solutions will also be found at other energies if use is made of peripheral one-meson phase shifts (for example, those for which Tables I and II give ratios smaller than 10%) in the phase shift analysis.

We next discuss briefly the analytic properties of two-meson phase shifts as a function of $p^2$. All present calculations, as well as those of Galanin et al.,[1,2] were carried out for physical energies; however it is trivial to generalize the method used to arbitrary energies and therefore Eqs. (1.2) and (1.3) are also valid for complex $p^2$ accurate to order $1/|L|$, $|\xi|^2$. In the unphysical region the two-meson phase shifts have a branch point at

*To this end it is necessary to obtain higher order terms in the expansion of the perturbation theory amplitudes in powers of $s^2$. For one of the diagrams, which gives an anomalously large contribution to Eq. (1.3), the exact singular part (all powers of $s^2$) is calculated in reference 2.

$p^2 = -\mu^2$ and a cut from $-\mu^2$ to $-\infty$. The function $Q_l(1 + 2\mu^2/p^2)$ is single-valued in the entire cut $p^2$ plane, and for an analytic continuation of the function $\Psi(z)$ which appears in Eqs. (1.2′) and (1.3) it is convenient to use an expansion in the neighborhood of $p^2 = 0$ $(z = \infty)$: $\Psi(z) = 1/2 + 1/4 z^{-2} + \ldots$ However the integration over $dq^2$ by the saddle-point method used in the calculation of the two-meson shifts, as well as the expansion in $s^2$ for one of the perturbation theory diagrams, are only valid under the condition $|p^2 + \mu^2| \gg \mu^2/|L|$, and consequently Eqs. (1.2), (1.3) cannot be used in the vicinity of the singular point $p^2 = -\mu^2$.

## 2. NUCLEON-NUCLEON POTENTIAL

The calculated two-meson phase shifts are determined by the interaction in the peripheral region $x > 1$, which may be considered as weak. This makes it possible to use the Born approximation to construct a potential* corresponding to the scattering amplitude. By inverting the usual Born formula we find for the potential

$$U = -\frac{1}{2\pi^2 m}\int e^{i\mathbf{q}\mathbf{r}} M(q^2)\,d\mathbf{q}. \tag{2.1}$$

We introduce the scattering operator (1.1) into Eq. (2.1) and write the two-meson potential as follows:

$$U = U_S + (\mathbf{LS})\,U_{LS} + (\sigma^{(1)}\nabla_x)(\sigma^{(2)}\nabla_x)\,U_T, \tag{2.2}$$

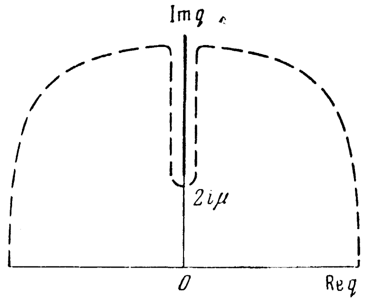
where $(\mathbf{LS})$ is the spin-orbit operator with eigenvalues $\frac{1}{2}[J(J+1) - l(l+1) - S(S+1)]$ and the functions $U_S$, $U_{LS}$, and $U_T$, after integration over the angles of the vector $\mathbf{q}$, are given by

$$U_S = \frac{i}{\pi m r}\int_{-\infty}^{+\infty} e^{iqr} M_S(q^2)\,q\,dq,$$

$$U_{LS} = \frac{i}{\pi m^3 r}\frac{\partial}{\partial r}\frac{1}{r}\int_{-\infty}^{+\infty} e^{iqr} M_{LS}(q^2)\,q\,dq,$$

$$U_T = -\frac{i\mu^2}{\pi m^3 r}\int_{-\infty}^{+\infty} e^{iqr} M_T(q^2)\,q\,dq. \tag{2.3}$$

To integrate over $dq$ we close the contour in the upper half-plane going around the cut on the imaginary axis $\mathrm{Im}\,q \geq 2\mu$ in the manner shown in the figure. The integral over the semicircle at infinity may only give a delta-function-like contribution in those cases when $M(q^2)$ does not fall off sufficiently rapidly as $q^2 \to \infty$. Therefore the




peripheral part of the potential is determined by the integral over the cut only and for $x \gg 1$ only a small portion near the singular point $q = 2i\mu$ plays an important role. Going over to the variable $s$ and using the saddle point method we obtain accurate to order $\sim 1/x$:

$$\int_{-\infty}^{+\infty} e^{iqr} M(q^2)\,q\,dq = 4\mu^2 e^{-2x}\int_0^{-\infty} e^{-x|s|^2}\Delta M(s^2)\,|s|\,d|s|. \tag{2.4}$$

The remaining integration over $d|s|$ in Eq. (2.4) coincides with the integral of the discontinuity in the amplitude, $\Delta M(s^2)$, which was encountered in calculating phase shifts and differs from it by the formal replacement of $L$ by $x$. Making use of the previous results and denoting by $d_1(x)$, $D_1(x)$, and $h_1(x)$ the functions obtained from $d_1$, $D_1$, and $h_1$ [see Eqs. (1.2′) and (1.3)] by the formal substitution $L \to x$, we obtain for $1 \ll x \ll 4/\epsilon^2$

$$U_S = -\frac{3g^4\varepsilon^2\mu}{4\sqrt{\pi}}\frac{e^{-2x}}{x^{5/2}}\left\{d_0 + \varepsilon\sqrt{\pi x}\,d_1(x) + \frac{\varepsilon^2 x}{4}d_2\right\}, \tag{2.5}$$

$$U_{LS} = -\frac{3g^4\varepsilon^4\mu}{4\sqrt{\pi}}\frac{e^{-2x}}{x^{7/2}}\left\{D_0 + \varepsilon\sqrt{\pi x}\,D_1(x) + \frac{\varepsilon^2 x}{4}D_2\right\}, \tag{2.6}$$

$$U_T = -\frac{3g^4\varepsilon^4\mu}{16\sqrt{\pi}}\frac{\varepsilon^{-2x}}{x^{5/2}}\{h_0 + \varepsilon\sqrt{\pi x}\,h_1(x)\}. \tag{2.7}$$

In accordance with the expansion of the phase shifts in $1/L$ and $\zeta$, the coefficients of the exponentials in the potentials are lowest order terms in an expansion in $1/x$ and $\epsilon\sqrt{x}/2$. Therefore only lowest order terms in $1/x$ should be kept when differentiating with respect to $x$ the tensor or spin-orbit parts [see Eq. (2.3)] of the potential (2.2) [e.g., differentiation in Eq. (2.3) is equivalent to multiplication by $-2$].

The two-meson potential (2.2), (2.5), (2.6), and (2.7) represents the next correction to the well-known one-meson potential (see, e.g., reference 6), and consists of a central, spin-orbit, and tensor part. Interactions of the type $\sigma^{(1)}\sigma^{(2)}$ and $(\sigma^{(1)}\mathbf{L}) \times (\sigma^{(2)}\mathbf{L})$ give corrections to the phase shifts $\eta_l$, $\eta_l^J$ of order $\epsilon^2$, $1/L$, and are therefore not included in our approximation. The tensor interac-

*We note that, strictly speaking, a potential in the conventional sense (local) describing the nucleon interaction does not exist. This is reflected in the fact that our equivalent potential depends on $p^2$ and is therefore nonlocal. Furthermore, in higher orders of smallness in our expansion the operation of constructing a potential is not even unique.

tion contributes to $\eta_l$, $\eta_l^J$ corrections of the same order, but, in contrast, fully determines the mixing parameters. For this reason we kept the tensor terms in the scattering operator (1.1) and the potential (2.2), but ignored interaction terms of the form $\sigma^{(1)}\sigma^{(2)}$ and $(\sigma^{(1)}\mathbf{L})(\sigma^{(2)}\mathbf{L})$. A characteristic feature of the two-meson potential is its substantial energy dependence contained in the function $\Psi(\mu\sqrt{x}/2p)$, as well as its dependence on isotopic spin.

We emphasize in conclusion that this potential is equivalent to the scattering amplitude only in the first Born approximation. The question of a more correct construction of the potential will be discussed separately.

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[1] Galanin, Grashin, Ioffe, and Pomeranchuk, JETP **37**, 1663 (1959), Soviet Phys. JETP **10**, 1179 (1960); Nucl. Phys. (in press)

[2] Galanin, Grashin, Ioffe, and Pomerancuhk, JETP **38**, 475 (1960), Soviet Phys. JETP **11**, 347 (1960); Nucl. Phys. (in press).

[3] A. F. Grashin, JETP **36**, 1717 (1959), Soviet Phys. JETP **9**, 1223 (1959).

[4] Stapp, Ypsilantis, and Metropolis, Phys. Rev. **105**, 302 (1957).

[5] Cziffra, MacGregor, Moravcsik, and Stapp, Phys. Rev. **114**, 880 (1959).

[6] Bethe and de Hoffmann, Mesons and Fields, v. 2 (Russ. transl.) IIL, 1957, p. 363.

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