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GYROMAGNETIC RESONANCE ABSORPTION OF ELECTROMAGNETIC WAVES IN A PLASMA

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The absorption of characteristic waves in a uniform magnetoactive plasma at frequencies close to the gyromagnetic frequency and multiples of this frequency is investigated, taking account of the thermal motion of the electrons. Collisions and specific plasma absorption mechanisms are considered.

In an earlier work of the author[1] (referred to hereinafter as I) the absorption of electromagnetic waves in a plasma (with thermal motion of the electrons taken into account) was investigated by means of a general dispersion equation for characteristic waves of all three possible types: ordinary waves, extraordinary waves, and plasma waves (denoted 1, 2 and 3). However, we have not analyzed in I cases in which the frequency is close to the gyromagnetic frequency ω_H or its multiples $2\omega_H$, $3\omega_H$, etc. In the present work we investigate absorption at these gyromagnetic frequencies, so that this paper represents a direct extension of I.

The theory of gyromagnetic resonance absorption of waves in a plasma has already been developed.[2-4] In certain respects the present analysis is more general. Both the damping in time and the absorption in space are determined for all three types of waves. The first resonance and certain problems associated with collision effects are considered in greater detail. In addition, we make estimates of certain quantities, give numerical examples, and refine certain of the results obtained in references 2 and 3.

We determine the absorption by starting with the dispersion equation of I [(Eq. 1.8)], which will not be written out completely here. This equation was obtained by solving a linearized system of the electrodynamic equations and the kinetic equation for the electrons. The problem was formulated as follows: We assume that at an arbitrary time t the value of the non-equilibrium part of the distribution function is given in the plane $z = 0$. Then, as the perturbations propagate along the z axis ($z > 0$), the asymptotic behavior of the fields is determined by a function of the form $e^{i\tilde{k}z} = e^{ikz-qz}$, where k is the wave number and q is the absorption factor. Another formulation of the problem, such as used in references 2—4, is possible. In an infinite space, at $t = 0$, we may prescribe a periodic perturbation with wave number k. Then the field varies in time in accordance with the asymptotic relation $e^{pt} = e^{-i\omega t-\gamma t}$ where γ is the damping factor.

Below, in Section I, we calculate absorption in the frequency region close to ω_H. In Section II we analyze absorption at $\omega \approx 2\omega_H$ and $\omega \approx 3\omega_H$. In the last section we discuss the results and give numerical examples.

I. ABSORPTION IN THE REGION OF THE FIRST GYROMAGNETIC RESONANCE

In this section we investigate the absorption, assuming that the frequency ω is close to the gyromagnetic frequency ω_H, so that

$$|(\omega - \omega_H)/\omega_H| \ll 1. \tag{1.1}$$

Furthermore, in accordance with I, we assume that the following condition is satisfied:

$$\delta = (\varkappa T/m)(k^2 \sin^2\alpha/\omega_H^2) \ll 1, \tag{1.2}$$

where α is the angle between the direction of propagation and the direction of the fixed magnetic field $\mathbf{H}_0$, κ is the Boltzmann constant, T is the electron temperature, and $\omega_H = eH_0/mc$ (e and m are the charge and mass of the electron). Taking account of the conditions given in (1.1) and (1.2), we can omit a number of unimportant terms from the general dispersion equation. We thus obtain the equation used as the starting point for this section:

$$i\omega\omega_0^6\{2\pi I_2 I_0^- I_0^+ - (\tilde{k}^2\sin^2\alpha/2\omega_H^2)\, I_0^- I_1^{+2}\} + \omega_0^4\{I_0^+ I_0^-(\omega^2 - c^2\tilde{k}^2\sin^2\alpha) + \pi I_2(I_0^+ + I_0^-)\langle 2\omega^2 - c^2\tilde{k}^2(1+\cos^2\alpha)\rangle$$

$$+ (\tilde{k}^2\sin^2\alpha I_1^{+2}/4\omega_H^2)\langle c^2\tilde{k}^2(1+\cos^2\alpha) - 2\omega^2\rangle - (c^2\tilde{k}^3/\omega_H)\sin^2\alpha\cos\alpha I_0^- I_1^+\}$$

$$- (i\omega_0^2/2\omega)(c^2\tilde{k}^2 - \omega^2)\{(c^2\tilde{k}^2\sin^2\alpha - 2\omega^2)(I_0^+ + I_0^-) + 4\pi(c^2\tilde{k}^2\cos^2\alpha - \omega^2) I_2 + (2c^2\tilde{k}^3\sin^2\alpha\cos\alpha/\omega_H) I_1^+$$

$$+ \delta\langle 2\pi c^2\tilde{k}^2\cos^2\alpha I_2^+ - c^2\tilde{k}^2\sin^2\alpha(I_0^+ - I_0^{++})\rangle\} - (c^2\tilde{k}^2 - \omega^2)^2 = 0, \qquad (1.3)$$

where $\tilde{k} = k + iq$, and $\omega_0 = \sqrt{4\pi e^2 N/m}$ is the frequency of the plasma oscillations (N is the electron concentration). The integrals $I_0^\pm$, $I_1^\pm$, and $I_2^\pm$ are given by

$$I_0^\pm = \sqrt{\frac{m}{2\pi\varkappa T}}\int_C \frac{\exp(-mv_z^2/2\varkappa T)\,dv_z}{i(-\omega\pm\omega_H) + \nu + i\tilde{k}v_z\cos\alpha},$$

$$I_1^\pm = \sqrt{\frac{m}{2\pi\varkappa T}}\int_C \frac{v_z\exp(-mv_z^2/2\varkappa T)\,dv_z}{i(-\omega\pm\omega_H) + \nu + i\tilde{k}v_z\cos\alpha},$$

$$I_2^\pm = \left(\frac{m}{2\pi\varkappa T}\right)^{3/2}\int_C \frac{v_z^2\exp(-mv_z^2/2\varkappa T)\,dv_z}{i(-\omega\pm\omega_H) + \nu + i\tilde{k}v_z\cos\alpha}, \qquad (1.4)$$

where ν is the effective number of collisions between electrons and other particles. The expressions for I_0, I_1 and I_2 are obtained by putting $\omega_H = 0$ in Eq. (1.4). In computing all these integrals (I), the integration is carried out over a contour C which is chosen in the same way as in reference 1.

We shall be concerned here mostly with weakly attenuated waves. The necessary conditions for weak attenuation, as can be verified by calculation, are the inequalities

$$\omega \gg \sqrt{\varkappa T/m}\, k\cos\alpha, \quad \omega \gg \nu. \qquad (1.5)$$

Using the conditions (1.5), we can obtain the usual approximate expressions for all the I integrals except $I_{0,1,2}^+$. The denominators of the integrands in the latter contain the difference $\omega - \omega_H$. The computation of these terms will be considered in detail. In considering the resonance integrals $I_{0,1,2}^+$, we must choose a path C such that the singularity (pole) $v_z = \hat{v}_z$ is encircled from below. This applies for all the I integrals. If the pole is in the lower half plane, the integration must be carried out over the path C shown in Fig. 1a. However, if the point $v_z = \hat{v}_z$ lies in

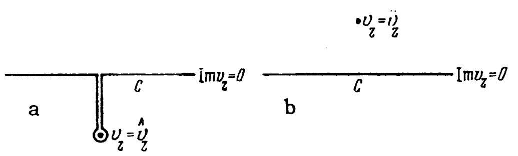

FIG. 1. Integration paths C for the integrals in Eq. (1.4) for $\mathrm{Im}\hat{v}_z < 0$ and $\mathrm{Im}\hat{v}_z > 0$.

the upper half plane, it is sufficient to integrate along the real axis, as in Fig. 1b. From the fact that the denominators in the integrands of $I_{0,1,2}^+$ vanish, we have when $q \ll k$

$$\mathrm{Re}\,\hat{v}_z = \frac{1}{\cos\alpha}\left(\frac{\omega-\omega_H}{k} + \frac{\nu q}{k^2}\right),$$

$$\mathrm{Im}\,\hat{v}_z = \frac{1}{\cos\alpha}\left(\frac{\nu}{k} - \frac{q(\omega-\omega_H)}{k^2}\right). \qquad (1.6)$$

If $q(\omega - \omega_H)/k > \nu$, as is possible only when $\omega > \omega_H$, the integration is carried out over the contour shown in Fig. 1a. Furthermore, it can be shown that if $q \ll k$, then $\mathrm{Re}\,\hat{v}_z = (\omega - \omega_H)/k\cos\alpha$ so that $|\mathrm{Re}\,\hat{v}_z| \gg |\mathrm{Im}\,\hat{v}_z|$. When these considerations are taken into account, it is easy to compute the contribution in $I_{0,1,2}^+$ due to integration around the singularity. Denoting the appropriate parts of the integrals by the symbol I, we have

$$I_{01}^+ = \frac{2}{k\cos\alpha}\sqrt{\frac{m\pi}{2\varkappa T}}\exp\left\langle -\frac{m(\omega-\omega_H)^2}{2\varkappa T k^2\cos^2\alpha}\right\rangle,$$

$$I_{11}^+ = \frac{\omega-\omega_H}{k\cos\alpha} I_{01}^+, \quad I_{21}^+ = \frac{m(\omega-\omega_H)^2}{2\pi\varkappa T k^2\cos^2\alpha} I_{01}^+. \qquad (1.7)$$

If, however, $q(\omega - \omega_H)/k < \nu$ it must be assumed that $I_{0,1,2,I}^+ = 0$.

We now determine the contribution due to integration along the real axis. For example, consider the integral

$$I_{0II}^+ = \sqrt{\frac{m}{2\pi\varkappa T}}\int_{-\infty}^{+\infty} \frac{\exp(-mv_z^2/2\varkappa T)\,dv_z}{-i(\omega-\omega_H) + \nu + i\tilde{k}v_z\cos\alpha}. \qquad (1.8)$$

Following the method of reference 5 for the analysis of such integrals, and substituting $v_z = \sqrt{2\kappa T/m}\,y$, we obtain

$$I_{0II}^+ = \frac{i}{\tilde{k}\cos\alpha}\sqrt{\frac{m}{2\pi\varkappa T}}\int_{-\infty}^{+\infty} dy\, e^{-y^2}/(z-y), \qquad (1.9)$$

where

$$\mathrm{Re}\,z = \frac{(\omega-\omega_H)k + \nu q}{(k^2+q^2)\sqrt{2\varkappa T/m}\cos\alpha},$$

$$\mathrm{Im}\,z = \frac{\nu k - q(\omega-\omega_H)}{(k^2+q^2)\sqrt{2\pi T/m}\cos\alpha}. \qquad (1.10)$$

For weak absorption and $|z| \gg 1$, we have

$$\int_{-\infty}^{+\infty} dy\, e^{-y^2}/(z-y) \approx \frac{\sqrt{\pi}}{z}\left(1 + \frac{1}{2z^2} + \frac{3}{4z^4} + \cdots\right)$$

$$- \pi i e^{-z^2}\,\mathrm{sgn}\,\mathrm{Im}\,z. \qquad (1.11)$$

This formula is applicable only when $|\mathrm{Re}\,z| \gg |\mathrm{Im}\,z|$. If this condition is not satisfied, the exponential term must be omitted (the remaining terms are not changed). When $|z| \ll 1$ we use the expansion

$$\int_{-\infty}^{+\infty} dy e^{-y^2} / (z-y) \approx \sqrt{\pi} z - \pi i \operatorname{sgn} \operatorname{Im} z. \tag{1.12}$$

Using Eqs. (1.11) and (1.12) we can compute $I_0^+ \mathrm{II}$ in various cases. The integrals $I_1^+ \mathrm{II}$ and $I_2^+ \mathrm{II}$ can be expressed in terms of $I_0^+ \mathrm{II}$:

$$I_{1\mathrm{II}}^+ = \frac{1}{i\tilde{k}\cos\alpha} \langle 1 + [i(\omega-\omega_H) - \nu] I_{0\mathrm{II}}^+ \rangle,$$

$$I_{2\mathrm{II}}^+ = \frac{[i(\omega_H - \omega) + \nu]\, m}{2\pi\varkappa T \tilde{k}^2 \cos^2\alpha} \langle 1 + [i(\omega-\omega_H) - \nu] I_{0\mathrm{II}}^+ \rangle. \tag{1.13}$$

We can now proceed with the absorption calculation. We assume at the outset that collisions are relatively unimportant so that

$$\nu \ll \sqrt{\varkappa T/m}\, k \cos\alpha. \tag{1.14}$$

We consider first the frequency region close to ω_H, which lies outside the direct resonance region; we call this the outer region. In this case

$$|\omega - \omega_H| \gg \sqrt{\varkappa T/m}\, k\cos\alpha. \tag{1.15}$$

Assuming that $q \ll k$, we find from Eq. (1.10) that $|z| \gg 1$. Using Eqs. (1.7), (1.11), and (1.13) we arrive at the customary expansion, which is applicable both for $\operatorname{Im} z > 0$ and $\operatorname{Im} z < 0$:

$$I_0^+ = \frac{1}{\nu + i(\omega_H - \omega)} - \frac{\tilde{k}^2 \varkappa T \cos^2\alpha}{m[\nu + i(\omega_H - \omega)]^3} + \frac{1}{k\cos\alpha}\sqrt{\frac{m\pi}{2\varkappa T}} \exp\left(-\frac{m(\omega-\omega_H)^2}{2\varkappa T k^2 \cos^2\alpha}\right)$$

$$I_1^+ = -\frac{\varkappa T \tilde{k}\cos\alpha}{m[\nu + i(\omega_H - \omega)]^2} + \frac{(\omega-\omega_H)}{k^2\cos^2\alpha}\sqrt{\frac{m\pi}{2\varkappa T}} \exp\left(-\frac{m(\omega-\omega_H)^2}{2\varkappa T k^2\cos^2\alpha}\right),$$

$$I_2^+ = \frac{1}{2\pi[\nu + i(\omega_H - \omega)]} - \frac{3\varkappa T \tilde{k}^2 \cos^2\alpha}{2\pi m[\nu + i(\omega_H - \omega)]^3} + \frac{(\omega-\omega_H)^2}{\sqrt{\pi}\, k^3 \cos^3\alpha}\left(\frac{m}{2\varkappa T}\right)^{3/2} \exp\left(-\frac{m(\omega-\omega_H)^2}{2\varkappa T k^2\cos^2\alpha}\right). \tag{1.16}$$

Similar expansions obtain for $I_{0,1,2}$ and $I_{0,1,2}^-$. To convert to the formulas for $I_{0,1,2}$, we must put in Eq. (1.16) $\omega_H = 0$; for $I_{0,1,2}^-$ we must replace in Eq. (1.16) $\omega - \omega_H$ by $\omega + \omega_H$. Since we are interested only in gyromagnetic absorption, exponential terms can be omitted from the last six integrals. Expansions of the type given in (1.16) for $I_{0,1,2}$ and $I_{0,1,2}^-$ are used here throughout.

Substituting expressions such as Eq. (1.16) for the I integrals in Eq. (1.3), we neglect absorption completely in the first approximation (we assume that $\nu = 0$ and omit the exponential terms). We thus arrive at the equation

$$\beta^2 v R n^6 - (1 - u - v + uv\cos^2\alpha) n^4 + [2(1-v)^2 + uv(1+\cos^2\alpha) - 2u] n^2 + (1-v)[u - (1-v)^2] = 0, \tag{1.17}$$

which determines the square of the index of refraction, $n^2 = c^2k^2/\omega^2$, for the extraordinary ($n^2 = n_1^2$), ordinary ($n^2 = n_2^2$), and plasma ($n^2 = n_3^2$) waves. In Eq. (1.17) $v = \omega_0^2/\omega^2$, $u = \omega_H^2/\omega^2$, and $\beta = \sqrt{\kappa T/mc^2}$ is the ratio of the mean thermal velocity to the velocity of light c. The general expression for R is given in references 1 and 6–8 (cf. also below). When $\beta^2 \ll 1$, the root of Eq. (1.17) corresponding to the plasma wave is

$$n_3^2 = \frac{1 - u - v + uv\cos^2\alpha}{\beta^2 v R} = \frac{1 - u - v + uv\cos^2\alpha}{\beta^2 v \langle 3\sin^4\alpha/(1-4u) + \sin^2\alpha\cos^2\alpha[1 + (5-u)/(1-u)^2] + 3(1-u)\cos^4\alpha\rangle}. \tag{1.18}$$

According to Eq. (1.1), $|1-u| \approx 2|1-\sqrt{u}| \approx 2|\omega - \omega_H|/\omega_H \ll 1$. For values of α which are not too close to $\alpha = 0$ or $\alpha = \pi/2$, we have from Eq. (1.18) $n_3^2 \approx -(1-u)^2/4\beta^2\cos^2\alpha$. Since $n_3^2 < 0$, plasma waves cannot propagate when $u \approx 1$. At small α ($\alpha \ll |1-u|/v$), this relation does not apply. We then have $n_3^2 \approx (1-v)/3\beta^2 v$, corresponding to the isotropic case. Here it is clear that gyromagnetic resonance absorption is not important. When $\alpha \approx \pi/2$, we have $n_3^2 = 1/\beta^2$. Although $n_3^2 > 0$, wave 3 (plasma wave) does not propagate because one of the initial conditions (1.2) is violated and there should be appreciable absorption ($q \sim k$). For this reason there is no need to compute the absorption of plasma waves at $\omega \approx \omega_H$ in detail. In this section we discuss therefore gyromagnetic resonance absorption of wave 1 and wave 2 only.

Because plasma waves are neglected [we also neglect the transiton region between waves 3 and 1 (or 3 and 2)[6–8]] we can neglect the $\beta^2 v R n^6$ term. Then, in the second approximation, when $q \neq 0$, we obtain from (1.3), using (1.16) and similar expansions,

$$\frac{q}{k} = \frac{s}{2vn^2}\,\frac{2(v-1)n^4 + 2(v^2 - 4v + 2)n^2 - 3v^2 + 6v - 2}{-2\sin^2\alpha n^2 + 2 + \sin^2\alpha - 2v + 4(1-\sqrt{u})(1-v)n^2/v} + \sqrt{\frac{\pi}{2}}\,\frac{(1-\sqrt{u})^2}{v\beta n^3\cos\alpha} \times \frac{(v\cos 2\alpha - 1)n^4 + (-1/4\, v^2\sin^2\alpha - 2v\cos^2\alpha + 2)n^2 + (v-1)(1 - v^2/4)}{-2\sin^2\alpha n^2 + 2 + \sin^2\alpha - 2v + 4(1-\sqrt{u})(1-v)n^2/v} \times \exp\langle -(1-\sqrt{u})^2/2\beta^2 n^2\cos^2\alpha\rangle, \tag{1.19}$$

where $s = \nu/\omega$. In obtaining Eq. (1.19) we have discarded small terms [terms proportional to $(1-\sqrt{u})^2$ and certain terms proportional to $(1-\sqrt{u})$]. We have used the first-approximation equation (1.17), where we put $\beta^2 v R n^6 = 0$. We may note that it would not be correct to take $u = 1$ at the outset in this equation. It is obvious that the first term in Eq. (1.19) is due to collisions while the second derives from a specific absorption mechanism. It can easily be established that actually $q \ll k$ under the restrictions given above.

When $\alpha = 0$, for wave 1 when $n_1^2 \approx 1 - v/(1-\sqrt{u})$, we have for the slow waves ($n_1^2 \gg 1$), from Eq. (1.19),

$$\frac{q_1}{k} = \frac{s}{2(\sqrt{u}-1)} + \frac{\sqrt{u}-1}{\beta n}\sqrt{\frac{\pi}{8}}\exp\left\langle -\frac{(1-\sqrt{u})^2}{2\beta^2 n^2}\right\rangle. \quad (1.20)$$

This formula has been obtained by Shafranov[9] for the case $s = 0$ and has been considered as a special case in I. From Eq. (1.15) we find that $|1-\sqrt{u}| \gg \beta n \cos\alpha$. If, however, we use Eq. (1.20) with $s = 0$ in the region where $|1-\sqrt{u}| \sim \beta n$, we find that $q \sim k$. Thus when $|\omega - \omega_H| \sim \sqrt{\kappa T/m}\,k$ the absorption of wave 1 must be appreciable. The same holds at small values of α, except that there is appreciable absorption of wave 1 only when $v < 1$; when $v > 1$ it is wave 2 that is strongly absorbed, rather than wave 1 (this effect is considered in greater detail in I; the statement in I that at small values of α only wave 2 is absorbed for any value of v is in error). If however $n_{1,2}^2 \sim 1$, as is the case for values of α which are not too small, then we obtain from Eq. (1.19), for $|1-\sqrt{u}| \sim \beta n \cos\alpha$, a specific absorption $q/k \sim \beta$, i.e., $q \ll k$. This estimate is in agreement with a calculation for the inner resonance region, which we now consider.

In the inner region we assume that the following condition [the inverse of (1.15)] is satisfied:

$$|\omega - \omega_H| \ll \sqrt{\varkappa T/m}\,k\cos\alpha, \quad (1.21)$$

all other restrictions remaining the same as before. Now we must use the expansion (1.13). As before, we again assume that the absorption is weak. This assumption will be violated only at small values of α (if $\alpha = 0$ exactly, this is the case for wave 1), and also in the case of quasi-transverse propagation, i.e., when $\alpha \approx \pi/2$. The first case can be considered by means of our general formulas. We shall not do this because when $\sqrt{u} > 1$, and $n_1^2 > 0$ if thermal motion is neglected, our results agree with those obtained in reference 9 where it is assumed that $\alpha = 0$ exactly. The second case ($\alpha \approx \pi/2$) cannot be analyzed correctly in a nonrelativistic approximation. We may note, however, that according to Eq. (1.21) the inner regions, in which the absorption is large, become very narrow when $\cos\alpha \ll 1$.

Because of the condition (1.14), we can neglect the effect of collisions since under the restrictions of Eq. (1.21) the calculation yields $q_{spec} \sim \beta k$, while the collision contribution is $q_{coll} \sim sk$, so that $q_{spec} \gg q_{coll}$. Putting $\nu = 0$ and assuming that $q \ll k$, we obtain the following expressions for the integrals $I_{0,1,2}^{+}$:

$$I_0^+ = \frac{1}{\tilde{k}\cos\alpha}\sqrt{\frac{\pi m}{2\varkappa T}},$$

$$I_1^+ = \frac{1}{i\tilde{k}\cos\alpha}, \qquad I_2^+ = -\frac{im(\omega-\omega_H)}{2\pi\varkappa T\tilde{k}^2\cos^2\alpha} \quad (1.22)$$

After substitution of the above values in the dispersion equation (1.3), we again obtain as a first approximation Eq. (1.17), where we may take $\beta = 0$ and $u = 1$. In the next approximation, we obtain the following expression for the absorption factor q:

$$q/k = \sqrt{2/\pi}\,(\beta\cos\alpha/vn)\,(2v - 2 - \sin^2\alpha + 2\sin^2\alpha n^2)^{-1}$$
$$\times\Big\{\Big[1 - \Big(1 - \tfrac{7}{4}\sin^2\alpha\Big)v\Big]n^4 - \Big[2 + v\Big(\tfrac{7}{4}\sin^2\alpha - \tfrac{5}{2}\Big)$$
$$+ \tfrac{1}{4}v^2(2\cos 2\alpha - \tan^2\alpha)\Big] + \Big[1 - \tfrac{3}{2}v + \tfrac{1}{2}v^2(1 - \tan^2\alpha)$$
$$+ \tfrac{1}{4}v^3\tan^2\alpha\Big]\Big\}. \quad (1.23)$$

It follows from Eq. (1.23) that $q/k \sim \beta$ when $n_{1,2}^2 \sim 1$. Equation (1.23) is similar to that obtained earlier by Sitenko and Stepanov.[2] The difference lies in the expression inside the curly brackets and is apparently due to the fact that the I_1^{+2} terms have not been considered in reference 2 [(cf. Eq. 1.3)]. We may note that our formula, as has already been indicated, does not apply when $\alpha \approx \pi/2$ (roughly, when $\cos\alpha \lesssim \beta$). The analogous formula in reference 2 does not contain a similar singularity, which appears at $\alpha \to \pi/2$ or $\omega \approx 2\omega_H$, $3\omega_H$, etc. in the inner resonance regions.

Up to this point we have considered the propagation of waves of a given frequency ω and have determined the spatial attenuation of the field. However, as was mentioned in the introduction, another formulation of the problem is possible: in this approach the wave number k is assumed known and the time-damping term $e^{-\gamma t}$ is determined.[10] Solving the problem in this second formulation, as has been done earlier by a number of authors[2-4,6,10] and assuming that the damping γ is weak, we have in the inner resonance region

$$\frac{\gamma}{\omega} = \frac{s}{2}\,\frac{2(v-1)n^4 + 2(v^2 - 4v + 2)n^2 - 3v^2 + 6v - 2}{(v-1)n^4 + [2v^2 - (6+\sin^2\alpha)v + 2]n^2 + v(v-2)^2 + (2v-1)(1-v)} + \sqrt{\frac{\pi}{2}}\,\frac{(1-\sqrt{u})^2}{\beta n\cos\alpha}$$
$$\times\frac{(v\cos 2\alpha - 1)n^4 + (-{}^1\!/_4 v^2\sin^2\alpha - 2v\cos^2\alpha + 2)n^2 + (v-1)(1 - v^2/4)}{(v-1)n^4 + [2v^2 - (6+\sin^2\alpha)v + 2]n^2 + v(v-2)^2 + (2v-1)(1-v)}\exp(-(1-\sqrt{u})^2/2\beta^2 n^2\cos^2\alpha). \quad (1.24)$$

It is easy to show that q in Eq. (1.19) and γ in Eq. (1.24) are related by the expression

$$\gamma = q d\omega/dk, \tag{1.25}$$

where $d\omega/dk$ is the projection of the group velocity in the direction of the vector **k**. The latter can be computed from Eq. (1.17) if we assume for waves 1 and 2 that $\beta^2 \to 0$. An expression similar to Eq. (1.24) has been obtained earlier by Stepanov.[3] Our expression appears in somewhat different form. However, if we compare Eq. (1.24) with $s = 0$ with the analogous relation in reference 3, and take the transformations used here into account, there is a discrepancy in the denominators of the expressions which multiply the exponentials. We may note that there is also a discrepancy of this kind for the other gyromagnetic resonances (cf. below).

When $\alpha = 0$, we have from Eq. (1.24), for the slow wave 1

$$\gamma = \nu + \sqrt{\pi/2}\,[(1-\sqrt{u})^2/\beta n]\,\exp\langle -(1-\sqrt{u})^2/2\beta^2 n^2\rangle.$$

This formulas has been given in I.

A direct calculation of γ for the inner resonance region leads to an equation which can be solved only by numerical methods. However, for not too small values of α, we may take for waves 1 and 2 tentatively $\gamma/\omega \sim \beta n$, which corresponds to the estimate made on the basis of Eq. (1.24) for $|1-\sqrt{u}\,| \sim \beta n$. It should be noted that Eq. (1.25) does not apply in this case. This formula is valid only for weak attenuation; however, the condition $\omega \gg \gamma$, which is satisfied for the inner region when α is not small, is insufficient. The condition $\gamma \ll \sqrt{\kappa T/m}\,k$ must also be satisfied. The latter condition is not satisfied in the inner region when $\omega \approx \omega_H$, since $\gamma \sim \beta n \omega = \sqrt{\kappa T/m}\,k$.

Up to this point we have used the assumption given in Eq. (1.14). We now consider the inverse case, when

$$\nu \gg \sqrt{\varkappa T/m}\,k\cos\alpha. \tag{1.26}$$

Turning now to Eq. (1.10), we obtain for the parameter z, which appears in the integrals $I_{0,1,2}$, (assuming that $q/k \sim \gamma/\omega \ll 1$),

$$\mathrm{Re}\,z = [(\omega-\omega_H)k + \nu q]/k^2\sqrt{2\varkappa T/m}\cos\alpha,$$

$$\mathrm{Im}\,z = \nu/k\sqrt{2\varkappa T/m}\cos\alpha \gg 1.$$

In this case we may use an expansion such as (1.11), since $|z| \gg 1$. If we assume that $|\omega-\omega_H| \ll \nu$, then $|\mathrm{Im}\,z| \gg |\mathrm{Re}\,z|$ and, in general, the exponential terms, which take account of the specific absorption, do not appear. If $|\omega-\omega_H| \gg \nu$, however, although the $\sim e^{-z^2}$ term does not vanish, its contribution is negligibly small. Hence, the specific absorption can be neglected both for $|\omega-\omega_H| \gg \nu$ and $|\omega-\omega_H| \ll \nu$.

We thus conclude that if (1.26) is satisfied, the absorption of waves 1 and 2 in the region $\omega \approx \omega_H$ is determined exclusively by collisions. For the very simple approximation of the collision integral we have used (cf. I), there is no point at all in carrying out a kinetic analysis; we can limit ourselves to the formulas of the elementary theory, which does not take account of the thermal spread in the electron velocities.[11]

2. ABSORPTION IN FREQUENCY REGIONS CLOSE TO $2\omega_H$ AND $3\omega_H$

In this section we analyze absorption in frequency regions near $\omega \approx 2\omega_H$ and $\omega \approx 3\omega_H$ when the following conditions are satisfied, respectively:

$$|(\omega-2\omega_H)/\omega| \ll 1, \qquad |(\omega-3\omega_H)/\omega| \ll 1. \tag{2.1}$$

To consider cases simultaneously we must supplement Eq. (1.8) of I with I_0^{+++} terms. As a result we start from the following equation:

$$\begin{aligned}
&2\pi i\omega\omega_0^6 I_2 I_0^- I_0^+ + \omega_0^4\{I_0^- I_0^+(\omega^2 - c^2\tilde{k}^2\sin^2\alpha) \\
&\quad + \pi I_2(I_0^+ + I_0^-)[2\omega^2 - c^2\tilde{k}^2(1+\cos^2\alpha)]\} \\
&\quad - (i\omega_0^2/2\omega)(c^2\tilde{k}^2-\omega^2)\{(c^2\tilde{k}^2\sin^2\alpha - 2\omega^2)(I_0^+ + I_0^-) \\
&\quad + 4\pi(c^2\tilde{k}^2\cos^2\alpha - \omega^2)I_2 + \delta c^2\tilde{k}^2(2\pi\cos^2\alpha\, I_2^+ \\
&\quad - \sin^2\alpha\, I_0^+)\} - (c^2\tilde{k}^2-\omega^2)^2 \\
&\quad + \delta(I_0^{++} + \tfrac{3}{8}\delta I_0^{+++})\{2\pi i\omega\omega_0^6 I_0^- I_2 \\
&\quad - \omega_0^4\langle(c^2\tilde{k}^2\sin^2\alpha - \omega^2)I_0^- - \pi[c^2\tilde{k}^2(1+\cos^2\alpha) - 2\omega^2]I_2\rangle \\
&\quad + (i\omega_0^2/2\omega)(c^2\tilde{k}^2-\omega^2)(2\omega^2 - c^2\tilde{k}^2\sin^2\alpha)\} = 0.
\end{aligned} \tag{2.2}$$

The integrals I_0^{++} and I_0^{+++} in Eq. (2.2) can be computed by starting from the expression for I_0^+ in Eq. (1.4) and replacing ω_H by $2\omega_H$ or ω_H by $3\omega_H$.

Assuming that (1.5) is satisfied, we first consider the case (1.14) when the number of collisions is relatively small. In Eq. (2.2), for $I_{0,1,2}^+$ we use the approximations in (1.16); for $I_{0,1,2}$ and $I_{0,1,2}^-$ we use expansions similar to (1.16), omitting the exponential terms. The analysis of the resonance integrals I_0^{++} (when $\omega \approx 2\omega_H$) or I_0^{+++} (when $\omega \approx 3\omega_H$) is carried out the same way as in Sec. 1 (for I_0^+).

Considering the absorption of waves 1 and 2, and assuming that $q \ll k$ we use, as before, a successive-approximation method. Neglecting absorption, we arrive at Eq. (1.17), in which we must assume that $\beta^2 \nu R n^6 = 0$. In the next approximation we obtain relations for the absorption factor q. In this case, in contrast with the first gyromagnetic resonance, the absorption in the inner and outer regions for $\omega \approx 2\omega_H$ (or $\omega \approx 3\omega_H$) can be described by a single formula.

Finally, for waves 1 and 2 we have

$$\frac{q}{k} = \frac{s}{2} A\left(u = \frac{1}{4}\right) + \frac{1}{2}\sqrt{\frac{\pi}{8}}\, v\, \frac{\beta n \sin^2\alpha}{\cos\alpha} B\left(u = \frac{1}{4}\right) \exp\left\langle -\frac{(1-2\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right\rangle, \tag{2.3}$$

$$\frac{q}{k} = \frac{s}{2} A\left(u = \frac{1}{9}\right) + \frac{27}{16}\sqrt{\frac{\pi}{8}}\, v\, \frac{\beta^3 n^3 \sin^4\alpha}{\cos\alpha} B\left(u = \frac{1}{9}\right) \exp\left\langle -\frac{(1-3\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right\rangle. \tag{2.4}$$

For the functions $A(u)$ and $B(u)$ we have

$$A(u) = \frac{(2v+u-3)n^4 + (2v^2 - 8v - 2u + 6)n^2 - 3v^2 + 6v + u - 3}{n^2\left[2(1-u-v+uv\cos^2\alpha)n^2 - 2(1-v)^2 - (1+\cos^2\alpha)uv + 2u\right]},$$

$$B(u) = \frac{(u-1)}{un^2}\,\frac{{}^1\!/_2 n^4 \sin^2\alpha + \{v[{}^1\!/_2 + {}^1\!/_2\cos^2\alpha + \sin^2\alpha/(1+\sqrt{u})]}{2(1-u-v+uv\cos^2\alpha)n^2} \rightarrow$$

$$\rightarrow \frac{-(1+{}^1\!/_2\sin^2\alpha)\}n^2 + (1-v)[1-v/(1+\sqrt{u})]}{-2(1-v)^2 - (1+\cos^2\alpha)uv + 2u}. \tag{2.5}$$

When $\omega \approx 2\omega_H$, we take $u = 1/4$ in Eq. (2.5) and when $\omega \approx 3\omega_H$ we use $u = 1/9$.

It follows from Eqs. (2.3) and (2.4) that the change in the specific absorption with frequency, q_{spec}, is determined in the region $\omega = 2\omega_H$ or $\omega = 3\omega_H$ by an exponential resonance factor. Under the conditions considered here, the absorption due to collisions is of non-resonant nature and the value of q_{coll} depends to a considerable extent on the quantity $s = \nu/\omega$. As a very rough maximum, we can take $q_{spec} \sim \beta$ at $\omega \approx 2\omega_H$ and $q_{spec} \sim \beta^3$ at $\omega \approx 3\omega_H$ (for simplicity we assume that $\alpha \sim 1$). Because $\beta^2 = \kappa T/mc^2$ is small, the value of q_{spec} falls off rapidly as the number of the resonance increases. It can be shown that when $\omega \approx 4\omega_H$, $q_{spec} \lesssim \beta^5$ and when $\omega \approx 5\omega_H$, $q_{spec} \lesssim \beta^7$. Thus the largest specific absorption for waves 1 and 2 obtains for $\omega \approx \omega_H$ and $2\omega_H$, in which case $q_{spec} \lesssim \beta$. If one speaks of specific absorption, Eqs. (2.3) and (2.4) are not meaningful in the inner resonance regions [cf. the condition in Eq. (1.21), where we must replace ω_H by $2\omega_H$ or $3\omega_H$] when $\alpha \approx \pi/2$. This is approximately the case for the second gyromagnetic resonance when $\cos\alpha \lesssim \beta$ and for the third when $\cos\alpha \lesssim \beta^3$. When $\cos\alpha \ll 1$, the inner resonance regions are very narrow.

We now consider the absorption of the plasma wave (wave 3). For this wave $n_3^2 \gg 1$ in all cases. We shall not dwell on the properties of these waves, but shall concentrate our attention on the question of absorption. In considering absorption, one must keep in mind the fact that these waves are damped if $\beta n \sim 1$,[2,6-8] regardless of the gyromagnetic resonance mechanism. The attenuation is small when

$$|1-u-v+uv\cos^2\alpha| \ll 1. \tag{2.6}$$

At the same time, the quantity $|1-u-v+uv\cos^2\alpha|$ must not be too small if we are to avoid the transition region between waves 3 and waves 1 (or 2).[6-8] We shall not analyze absorption in the transition region.

For wave 3, analysis of the resonance is more simple at $\omega \approx 3\omega_H$ than that at $\omega \approx 2\omega_H$. This situation results from the fact that the term with I_0^{+++} in Eq. (1.2) does not change the first-approximation equations (1.17), unlike the terms with I_0^{++}. For $\omega \approx 3\omega_H$, the absorption for waves 3, as well as waves 1 and 2, can be given by a single formula. From Eq. (2.2), and Eq. (1.17) we have, when $n_3^2 \gg 1$ and $q_3 \ll k$

$$\frac{q_3}{k} = \frac{s}{2} C\left(u = \frac{1}{9}\right) + \frac{27}{16}\sqrt{\frac{\pi}{8}}\, \frac{\beta^3 n^3 v \sin^4\alpha}{\cos\alpha} D\left(u = \frac{1}{9}\right) \exp\left(-\frac{(1-3\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right), \tag{2.7}$$

with

$$C(u) = \frac{3-2v-u}{1-u-v+uv\cos^2\alpha},$$

$$D(u) = \frac{(1-u)\sin^4\alpha}{2u(1-u-v+uv\cos^2\alpha)}. \tag{2.8}$$

If α is small, we must introduce certain corrections in the second relation of (2.8). When $\alpha \ll 1$, however, because of the presence of the $\sin^8\alpha$ factor, the absorption is relatively small.

When $\omega \approx 2\omega_H$, we again obtain a formula similar to Eq. (2.7); this formula applies only when $|\omega - 2\omega_H| \gg \sqrt{\kappa T/m}\, k \cos\alpha$ (in the outer region):

$$\frac{q_3}{k} = \frac{s}{2} C\left(u = \frac{1}{4}\right) + \sqrt{\frac{\pi}{8}}\, \frac{v\beta n \sin^2\alpha}{2\cos\alpha} D\left(u = \frac{1}{4}\right) \exp\left(-\frac{(1-2\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right). \tag{2.9}$$

As the frequency $\omega = 2\omega_H$ is approached (in the outer region), the absorption becomes appreciable ($q_3 \sim k$). We shall not give formulas for this case, but simply indicate the following point of interest. From Eq. (1.18), we have when $\omega \approx 2\omega_H$

$$n_3^2 \approx \left\{\left[\frac{3}{4}(1-v) - \frac{1}{4} v \sin^2\alpha\right] \Big/ 3\beta^2 v \sin^4\alpha\right\}(1-4u).$$

Whence it is apparent that n_3^2 changes sign on going through $u = 1/4$. Thus, the region of strong specific absorption is not isolated, but is contiguous with the region in which $n_3^2 < 0$ and propagation is forbidden. Under these conditions it becomes more difficult to isolate the specific absorption effect than in the cases when there are isolated, broad absorption lines (this is the case for waves 1 and 2 for large values of α, or waves 3 at $\omega \approx 3\omega_H$).

We now consider the formulas for the damping factor γ, limiting ourselves to waves 1 and 2. Solving the problem in the second formulation (cf. Introduction and I) we have

$$\frac{\gamma}{\omega} = \frac{s}{2} F\left(u = \frac{1}{4}\right) + \sqrt{\frac{\pi}{8}}\, \frac{v\beta n \sin^2\alpha}{2\cos\alpha}\, G\left(u = \frac{1}{4}\right) \exp\left(-\frac{(1 - 2\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right), \tag{2.10}$$

$$\frac{\gamma}{\omega} = \frac{s}{2} F\left(u = \frac{1}{9}\right) + \sqrt{\frac{\pi}{8}}\, \frac{27 v\beta^3 n^3 \sin^4\alpha}{16\cos\alpha}\, G\left(u = \frac{1}{9}\right) \exp\left(-\frac{(1 - 3\sqrt{u})^2}{2\beta^2 n^2 \cos^2\alpha}\right), \tag{2.11}$$

where

$$F(u) = \frac{(2v + u - 3)\, n^4 + (2v^2 - 8v - 2u + 6)\, n^2}{(2 - v - u)\, n^4 - \langle 2(1 - v)^2 + uv(1 + \cos^2\alpha)} \rightarrow$$

$$\rightarrow \frac{-3v^2 + 6v + u - 3}{-4(u + v - 1)\rangle n^2 - (v - 2)[(1 - v)^2 - u] - (1 - v)(u + 2v - 2)}. \tag{2.12}$$

$$G(u) = \frac{(1 - u)\{{}^1\!/_2\, n^4 \sin^2\alpha + \langle v[{}^1\!/_2 + {}^1\!/_2 \cos^2\alpha + \sin^2\alpha/(1 + \sqrt{u})]}{u\{(2 - v - u)\, n^4 - \langle 2(1 - v)^2 + uv(1 + \cos^2\alpha)} \rightarrow$$

$$\rightarrow \frac{-(1 + {}^1\!/_2 \sin^2\alpha)\rangle n^2 + (1 - v)[1 - v/(1 + \sqrt{u})]\}}{-4(u + v - 1)\rangle n^2 - (v - 2)[(1 - v)^2 - u] - (1 - v)(u + 2v - 2)\}}. \tag{2.13}$$

Equation (2.10), which reflects in part the contribution of a specific damping mechanism, applies only in the outer region; on the other hand, Eq. (2.11) can be applied to both the outer and inner resonance regions when $\omega \approx 3\omega_H$. In this connection, we note that Eq. (1.25) does not apply for the inner resonance region when $\omega \approx 2\omega_H$. Thus, the last relation is violated for the inner resonance region when $\omega \approx \omega_H$ and $\omega \approx 2\omega_H$. It does apply to all other cases considered here (for $\nu \ll \omega$).

A comparison of Eqs. (2.10) and (2.11) (with $s = 0$) with the analogous relations from reference 3 indicates a discrepancy, due to the differences between the denominators in Eq. (2.13) and the corresponding formula of reference 3.

In concluding this section, we note that if the inequality (1.26) is satisfied, the absorption of waves 1 and 2 is determined by collisions exclusively, as in Sec. 1. If the collision frequency ν is assumed given, the absorption calculation can be carried out in the framework of the elementary theory.[11] A similar conclusion applies to the gyromagnetic resonance absorption of waves 3. However, because the latter waves are slow ($n_3^2 \gg 1$), inequality (1.26) is satisfied only at relatively high values of the collision frequency ν. In passing we also note that in the case of wave 3 and in the transition region collisions may affect absorption and propagation even when $\omega \gg \nu$.[12]

3. EXAMPLES AND ESTIMATES

In this section we discuss the results obtained, using certain estimates and numerical examples. We use the relations derived in Secs. 1 and 2, as well as the results obtained in I, in which the non-resonance absorption of waves in a plasma is considered.

In comparing absorption due to collisions and due to specific damping mechanisms, in accordance with Eqs. (1.14) and (1.26), we must compare ν and the quantity $\sqrt{\kappa T/m}\, k \cos\alpha = \omega\beta n \cos\alpha$. Typical values of ν and β for different conditions are given in the table. The table is in no way complete nor highly accurate, but is given merely for purposes of illustration. A high-temperature, completely ionized plasma[13] which is the best approximation to the example in the table is the plasma in the solar corona.

	ν, sec^{-1}	β
Solar corona	10—10^{-3}	10^{-2}
Solar chromosphere	10^3	2—$4\cdot 10^{-3}$
Ionosphere E layer	10^5	$2{,}5\cdot 10^{-4}$
Ionosphere F layer	10^3—10^4	4—$6\cdot 10^{-4}$
Plasma in a gas-discharge tube	10^9—10^{10}	1—$2\cdot 10^{-3}$

For non-resonance absorption associated with the Cerenkov losses in the plasma,[9] damping due to the specific absorption mechanism is important only for relatively slow waves. When $\beta n \cos\alpha \ll 1$ the smallness of this absorption is determined by the factor $\exp\{-1/2\beta^2 n^2 \cos^2\alpha\}$; however, if $\beta n \gtrsim 1$, the absorption can be noticeable. When $\beta n \sim 1$ the phase velocity $v_{ph} = c/n$ is of the same order of magnitude as the thermal velocity of the electrons, $v_t = \sqrt{\kappa T/m}$. If the inequality $\beta n \ll 1$ is well satisfied, the specific non-resonance absorption is insignificant. For plasma waves which do not satisfy (2.6), specific absorption becomes significant so that $q_3 \gtrsim k$ ($\gamma_3 \gtrsim \omega$). The strong absorption found when $\beta n_3 \sim 1$ can be seen from Eqs. (2.12) and (2.14) of I, in spite of the fact that here we are somewhat outside the range of applicability of these equations. If the condition $\beta n_3 \ll 1$ is well satisfied, the non-resonance specific absorption is vanishingly small and is unimportant compared with absorption due to collisions. As is clear from the above, the inverse is true only when $\beta n_3 \gtrsim 1$ and $\omega \gg \nu$.

In contrast with plasma waves, the slow low-frequency ordinary waves (cf. I) are not strongly absorbed even when $\beta n_2 \cos\alpha \sim 1$. Thus, from Eq. (2.18) of I it follows that $q_{2,\text{spec}}/k \lesssim \omega/\omega_H \times \cos\alpha$. At the same time, in accordance with Eq. (2.15) of I, $\omega_H \cos\alpha \gg \omega$, so that $q_{2,\text{spec}}/k \ll 1$. It is apparent that the specific absorption obtains also when $\omega_H \cos\alpha \sim \omega$. To carry out an exact calculation of the absorption here, it would be necessary to extend the analysis of the particular case given in I. At the same time, as we have indicated, it is possible to have a case in which $q_2/k \ll 1$ over an appreciable frequency range, but in which the absorption is not a vanishingly small quantity. In the propagation of low-frequency noise in the upper atmosphere (atmospheric whistlers), which are believed to be groups of type-2 waves, as a rule $\omega_H \cos\alpha \gg \omega$. At the same time, in the passage of an atmospheric at great heights above the earth, it is also possible to have $\omega_H \cos\alpha \sim \omega$. In this case the frequencies $f = \omega/2\pi \sim 10^4$ in the spectrum of the atmospheric approach the high-frequency limit (frequencies exceeding this limit are blocked in the ionosphere). A noticeable contribution due to the specific absorption can be expected only when $\beta n_2 \gtrsim 1$. This condition can hardly be realized in the upper atmosphere under non-turbulent conditions. However, the situation may change if there are strong corpuscular perturbations. A calculation of the specific absorption under such conditions can be found in a paper by the author.[14]

We now return to the gyromagnetic resonance absorption and consider the absorption of waves 1 and 2. For these waves, (if we neglect propagation at angles close to $\alpha = 0$ and $\alpha = \pi/2$) the following rough approximations hold: $q_{\text{spec}} \lesssim \beta k$ for $\omega \approx \omega_H$ and $2\omega_H$, and $q_{\text{spec}} \lesssim \beta^3 k$ for $\omega \approx 3\omega_H$. The effect of collisions is roughly determined by $q_{\text{coll}} \sim sk = k\nu/\omega$. To compare the contributions of the different absorption mechanisms it is necessary to estimate the ratios $r_{1,2} = \beta\omega/\nu$ and $r_3 = \beta^3\omega/\nu$. If we are interested in radio waves $(\omega \approx 10^6 - 10^{10})$, $r_3 \ll 1$ for all the cases indicated in the table, except the solar corona.

It is clear even from this estimate that the greatest specific absorption can occur when waves propagate in a hot, highly ionized plasma such as the solar corona. Thus, under the conditions which

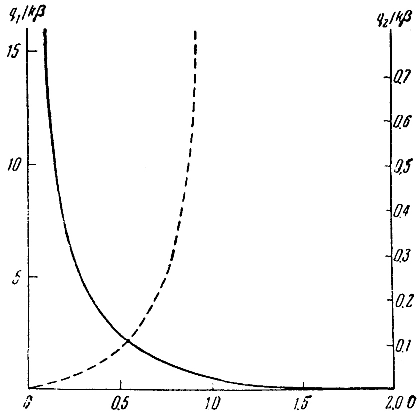

FIG. 2. Curves characterizing the specific absorption of wave 1 (solid curve) and wave 2 (dashed curve) as a function of the parameter $v = \omega_0^2/\omega^2$ for $\omega = \omega_H$ and $\alpha = 45°$.

obtain in the solar corona, in contrast with the other examples, it is quite possible to have a case in which $r_3 \gg 1$ and moreover $r_4 = \beta^5\omega/\nu \gtrsim 1$. The ratio $r_{1,2}$ can be large under the conditions found in the chromosphere. In the ionosphere [excluding very high frequencies, when the ionosphere has no noticeable effect on the propagation $(\omega < 10^8)$] we find that the condition $r_{1,2} \gtrsim 1$ can be realized only at the level of the maximum of the F-layer or higher. Collisions are always predominant in absorption of radio waves in the lower ionospheric layers.

It should be emphasized that the estimates given above are extremely coarse. It is possible that an exact calculation with the formula given in Secs. 1 and 2 would show great differences in the values of the absorption factors. This statement can be illustrated by curves such as those shown in Figs. 2 and 3. In Fig. 2 we show the curves $q_1/k\beta$ and $q_2/k\beta$ (in different scales), which characterize the specific absorption of wave 1 and wave 2, on the basis of Eq. (1.23), for the inner resonance region $\omega \approx \omega_H$ at $\alpha = 45°$. For a given value of α, the absorption given by the curves in Fig. 2 represents the maximum or at least the order of magnitude of the maximum. The shape of the specific absorption line close to $\omega = \omega_H$ has not been obtained, but the width of this line is of order $\Delta\omega \sim \omega\beta n \cos\alpha$.

In Fig. 3 we show similar curves for wave 1 and wave 2, also when $\alpha = 45°$ but with $\omega = 2\omega_H$. The shape of the absorption line close to $\omega = 2\omega_H$ is given by the factor

$$\exp\{-(\omega - 2\omega_H)^2 / 2\omega^2\beta^2 n^2 \cos^2\alpha\},$$

and it is apparent that the curves in Fig. 3 determine the maximum absorption values. It follows from Figs. 2 and 3 that the absorption of wave 1 differs greatly from that of wave 2 under the same

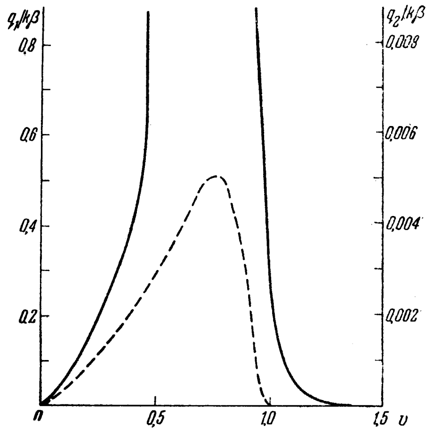

FIG. 3. Curves characterizing the specific absorption of wave 1 (solid curve) and wave 2 (dashed curve) as a function of the parameter $v = \omega_0^2/\omega^2$ for $\omega = 2\omega_H$ and $\alpha = 45°$.

conditions; this is why different scales must be used. The absorption of a given wave in a plasma at its wavelength λ ($q/k = 2\pi q\lambda = 2\pi q\lambda_0/n$) changes considerably as the parameter v is changed.

The differences indicated above in the values of the absorption factors again emphasize the need for exact calculations.

We note in connection with Figs. 2 and 3 that the curves corresponding to different characteristic waves lack those sections where $n^2 < 0$ and propagation is forbidden (in Fig. 2 this is the case, for example, for wave 2 at $v > 1$). The relatively high values of absorption in Fig. 2 are due to the fact that $n_1 \to 0$ and $n_2 \to 0$ as $v \to 0$ and $v \to 1$, respectively (we determine the absorbtion per wavelength in the medium, $\lambda = \lambda_0/n$). If, however, we relate the absorption coefficient to the wave number in free space, $k_0 = 2\pi/\lambda_0$, the increase in the values of $q/k_0\beta$ is not so great. In Fig. 3 the increase in $q/k\beta$ close to $v = 6/7$ arises because at this value $n_1^2 \to \infty$. Actually, it is necessary to take account of the effect of thermal motion on propagation in the region $v \approx 6/7$. In this case plasma waves exist in the region $v < 6/7$. In this region the gyromagnetic absorption must be analyzed by means of Eq. (2.9) (cf. also the remarks following this formula).

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[1] B. N. Gershman, JETP **37**, 695 (1959), Soviet Phys. JETP **10**, 497 (1960).

[2] A. G. Sitenko and K. N. Stepanov, JETP **31**, 642 (1956), Soviet Phys. JETP **4**, 512 (1957).

[3] K. N. Stepanov, JETP **35**, 283 (1958), Soviet Phys. JETP **8**, 195 (1959).

[4] K. N. Stepanov and V. S. Tkalich, J. Tech. Phys. (U.S.S.R.) **28**, 1789 (1958), Soviet Phys.-Tech. Phys. **3**, 1649 (1959).

[5] A. I. Akhiezer and Ya. B. Faĭnberg, JETP **21**, 1262 (1951).

[6] B. N. Gershman, JETP **24**, 659 (1953).

[7] Gershman, Ginzburg, and Denisov, Usp. Fiz. Nauk **61**, 561 (1957).

[8] B. N. Gershman, Сб. памяти А. А. Андронова (Coll. In Memoriam A. A. Andronov) Acad. Sci. Press, p. 599; JETP **31**, 707 (1956), Soviet Phys. JETP **4**, 582 (1957).

[9] V. D. Shafranov, JETP **34**, 1475 (1958), Soviet Phys. JETP **7**, 1019 (1958).

[10] L. D. Landau, JETP **16**, 574 (1946).

[11] Al'pert, Ginzburg, and Faĭnberg, Распространение радиоволн (Propagation of Radio Waves) Gostekhizdat (1953).

[12] B. N. Gershman and M. S. Kovner, Изв. высш. шк., Радиофизика (News of the Higher Schools, Radiophysics) **2**, 28 (1959).

[13] R. F. Post, Revs. Modern Phys. **28**, 338 (1956); Usp. Fiz. Nauk **61**, 491 (1957).

[14] B. N. Gershman, loc. cit. ref. 12, **1**, 49 (1958).

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