SUPERFLUIDITY OF LIGHT NUCLEI

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The physical ideas and the mathematical methods developed in superconductivity theory are applied to a study of the properties of light nuclei. On the basis of the shell model of the nucleus, it is shown that account of the residual interactions of protons and neutrons located near the Fermi surface leads to the appearance of superfluid states. Data on the binding energy of the last neutron in the $22 \le A \le 32$ region indicate the presence of paired (pp), (nn), and (np) correlations with the same quantum numbers s and m connected with the superfluidity of light nuclei.

APPLICATION of physical ideas and mathematical methods developed in superconductivity theory[1] to the theory of heavy nuclei has proved to be very fruitful. The residual interactions of protons and neutrons found in the outer shell of light nuclei can also lead to a large role for pair correlations and to the appearance of a superfluid state, i.e., to an energetically more favorable state than that of complete degeneracy of the Fermi gas. In this connection, it is of interest to consider the possibility of the appearance of a superfluid state of a light nucleus and to investigate its basic properties. The present paper is devoted to the study of these questions.

Taking the shell model of the nucleus as a base, we shall investigate certain properties of light nuclei with atomic weight A lying in the range $16 < A < 40$, where one can still use the hypothesis of isotopic invariance. We shall assume that the protons and neutrons forming the closed shell $Z = 8$, $N = 8$ create a radially symmetric field, which is distorted somewhat by nucleons lying outside this shell. Therefore our consideration applies directly to even weakly deformed nuclei (this restriction evidently has a purely methodological character), i.e., to nuclei lying in the $22 \le A \le 32$ interval.[2] We consider the residual interactions of protons and neutrons lying in the outer shell. The most significant difference of these interactions in light nuclei in comparison with heavy nuclei is the presence of neutron-proton interactions in addition to pp and nn interactions. We shall characterize the state of the nucleon by a choice of quantum numbers s, the quantum number m (the absolute value of the projection of the momentum along the axis of symmetry of the nucleus), and the number $\rho = \pm 1$ (the sign of this projection), while in the case of LS coupling ρ also characterizes the direction of the spin.

The Hamiltonian of the residual interactions of the nucleons close to the Fermi surface, i.e., in the region $E_F - \delta \le E(s, m) \le E_F + \Delta$, will be written in the isotopically symmetric form; that is,

$$\begin{aligned} H = \Sigma \{E(s, m) - \lambda\} \{a_{\rho m}(s)^+ a_{\rho m}(s) + b_{\rho m}(s)^+ b_{\rho m}(s)\} \\ + {}^1/_4 \Sigma J(s_1, s_2, s_2', s_1' \,|\, \rho_1 m_1, \rho_2 m_2;\ \rho_2' m_2', \rho_1' m_1') \\ \times \{a_{\rho_1 m_1}(s_1)^+ a_{\rho_2 m_2}(s_2)^+ a_{\rho_2' m_2'}(s_2')\, a_{\rho_1' m_1'}(s_1') \\ + b_{\rho_1 m_1}(s_1)^+ b_{\rho_2 m_2}(s_2)^+ b_{\rho_2' m_2'}(s_2')\, b_{\rho_2' m_2'}(s_2') \\ + 2a_{\rho_1 m_1}(s_1)^+ b_{\rho_2 m_2}(s_2)^+ b_{\rho_2' m_2'}(s_2')\, a_{\rho_1' m_1'}(s_1')\}. \end{aligned} \quad (1)$$

Here summation is carried out over s, s_1, s_2, s_1', s_2' for all positive values of m, m_1, m_2, m_1', m_2', and for $\rho = \pm 1$, while

$$\rho_1 m_1 + \rho_2 m_2 = \rho_1' m_1' + \rho_2' m_2', \quad \rho_1 m_1 \neq \rho_1' m_1',$$

$a_{m\rho}(s)$, $b_{m\rho}(s)$ are the operators of absorption of a proton and a neutron, λ is a parameter playing the role of a chemical potential, which is defined by the condition

$$n = \sum_{s,\, m,\, \rho} \{\overline{a_{\rho m}(s)^+ a_{\rho m}(s)} + \overline{b_{\rho m}(s)^+ b_{\rho m}(s)}\}. \quad (2)$$

We carry out a linear canonical transformation

$$\begin{aligned} a_{\rho m}(s) = u_m(s)\, \alpha_{m,-\rho} + \rho v_m(s)\, \alpha_{m\rho}^+ + i n_m(s)\, \beta_{m,-\rho} \\ + i\rho w_m(s) \beta_{m\rho}^+, \\ b_{\rho m}(s) = u_m(s)\, \beta_{m,-\rho} + \rho v_m(s)\, \beta_{m\rho}^+ + i n_m(s)\, \alpha_{m,-\rho} \\ + i\rho w_m(s)\, \alpha_{m\rho}^+, \end{aligned} \quad (3)$$

where the real functions u_m, v_m, n_m, w_m are related by the following relation

$$u_m(s)^2 + v_m(s)^2 + n_m(s)^2 + w_m(s)^2 = 1. \quad (4)$$

We determine the new vacuum state by the relation

$$\alpha_{\rho m}(s)\Psi = \beta_{\rho m}(s)\Psi = 0. \quad (5)$$

Solving (5), we obtain a wave function of the new vacuum state in the following form

$$\Psi = \prod_{s,\,m} \{(u_m^2 + n_m^2) + (v_m^2 + w_m^2)\, a_{m+}^+ a_{m-}^+ b_{m+}^+ b_{m-}^+ + (u_m v_m - w_m n_m)(a_{m+}^+ a_{m-}^+ - b_{m+}^+ b_{m-}^+) + i(u_m w_m + n_m v_m)(a_{m+}^+ b_{m-}^+ - a_{m-}^+ b_{m+}^+)\}\Psi_0, \quad (6)$$

where $a_{\rho m}(s)\Psi_0 = 0$. The canonical transformation (3) is so chosen that the np interaction is taken into account, i.e., so that

$$(\Psi^* a_{\rho m} b_{\rho m} \Psi) \equiv \langle a_{\rho m} b_{\rho m} \rangle \neq 0.$$

Further, proceeding as in reference 3, we find the mean value of the energy operator $\overline{H}$ over the new vacuum state, and we determine the $u_m(s)$, $v_m(s)$, $n_m(s)$, and $w_m(s)$ from the condition of the minimum $\overline{H}$; as a result, we get

$$C_m(s) + {}^1/_2 \sum_{s'm'} J(s, s' \mid m, -m; -m', m') \frac{C_{m'}(s')^2}{\sqrt{\xi(s', m')^2 + 2C_{m'}(s')^2}} = 0, \quad (7)$$

$$u_m v_m - w_m n_m = u_m w_m + n_m v_m = -C_m(s)/2\varepsilon(s, m); \quad (8)$$

where

$$C_m(s) = \sum_{s'm'} J(s, s' \mid m, -m; -m', m')(u_{m'}(s') v_{m'}(s') - n_{m'}(s) w_{m'}(s')),$$

$$\varepsilon(s, m) = \sqrt{\xi(s, m)^2 + 2C_m(s)^2}, \quad \xi(s, m) = E(s, m) - \lambda,$$

$$u_m^2 + n_m^2 = {}^1/_2(1 + \xi(s, m)/\varepsilon(s, m)),$$

$$\times\, v_m^2 + w_m^2 = {}^1/_2(1 - \xi(s, m)/\varepsilon(s, m)).$$

Let us consider the approximation

$$J = \text{const}, \qquad \rho = \text{const}, \quad (9)$$

i.e., J and the level density ρ are constant in the energy range $E_F - \delta \le E \le E_F + \Delta$. We solve Eqs. (7) and (2) in the same way as in reference 4 and obtain

$$C = \frac{\sqrt{(4\Omega - n)\, n}}{e^{2/G} - 1} \frac{e^{1/G}}{2\sqrt{2}\rho}; \quad (10)$$

$$E_F - \lambda = \frac{2\Omega - n}{2\rho} \frac{1}{e^{2/G} - 1}, \quad (11)$$

where $G = -\rho J$, n = number of neutrons, while Ω is the number of levels in the outer shell of the nucleus. We note that $C = C_0/\sqrt{2}$, $\epsilon = \epsilon_0$, where C_0 and ϵ_0 correspond to the case of the absence of np interaction.[4]

Thus, if the model considered for a light nucleus is valid, then the residual (after separation of the self-consistent field) np, pp, and nn interactions lead to the formation of a superfluid state of the nucleus. It should be noted that the energy of the ground state of an even nucleus does not depend on how the nucleons are coupled together: protons with protons or neutrons with neutrons or also coupling of protons with neutrons takes place.

The first excited state in light even-even nuclei (and also in odd-odd nuclei, in which N = Z, when the last neutron and proton are described by the same quantum numbers) is separated from the ground state by an energy gap of order 2ϵ, and the behavior of single particle excitation levels in these nuclei ought to be similar to the same degree to which isotopic invariance is valid. However, for the explanation of energy levels and the energy of coupling of nuclei in the region $22 \le A \le 32$, one must take into consideration the quadruple correlation of α-particle-like nucleons in addition to pair correlations; these new correlations evidently play a very important role and somewhat mask the effect of pair correlations.

We now consider the data of Brink and Kerman[2] pertaining to the coupling energy of the last neutron in light nuclei. The binding energy of the last nucleon in ${}_{12}Mg^{26}_{14}$, ${}_{13}Al^{27}_{14}$, and ${}_{15}P^{31}_{16}$, where all the neutrons are coupled, is of the order of 11 Mev, while in the nuclei ${}_{12}Mg^{25}_{13}$, ${}_{12}Mg^{27}_{15}$, and ${}_{14}Si^{29}_{15}$, where the last neutron is not coupled, it is of the order of 7 Mev. Further neutron-proton couplings in ${}_{11}Na^{22}_{11}$, ${}_{13}Al^{26}_{13}$, and ${}_{15}P^{30}_{15}$ reduce to a last-neutron binding energy of the order of 11 Mev, and in odd-odd nuclei, where the last proton and neutron are described by different quantum numbers and therefore the pair correlation between them does not exist, the binding energy of the last neutron is also of the order of 7 Mev.

Thus the data on the binding energies of the last neutron in light nuclei confirm our assumption of the presence of np, pp, and nn pair correlations with identical quantum numbers s and m, leading to a superfluid state of light nuclei.

In conclusion we express our gratitude to N. N. Bogolyubov for his very interesting discussions.

[1] Bogolyubov, Tolmachev, and Shirkov, Новый метод в теории сверхпроводимости (A New Method in the Theory of Superconductivity) Acad. Sci. Press, 1958; English Translation, Consultants Bureau, N. Y., 1959.

[2] D. M. Brink and A. K. Kerman, Nucl. Phys. **12**, 314 (1959).

[3] V. G. Solov'ev, Dokl. Akad. Nauk SSSR **123**, 652 (1958), Soviet Phys.-Doklady **3**, 1197 (1959); Nucl. Phys. **9**, 655 (1959).

[4] V. G. Solov'ev, JETP **36**, 1869 (1959), Soviet Phys. JETP **9**, 1331 (1959).

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ERRATA TO VOLUME 10

page	reads	should read
Article by A. S. Khaĭkin		
1044, title	. . . resonance in lead	. . . resonance in tin
6th line of article	~ 1000 oe	~ 1 oe
Article by V. L. Lyuboshitz		
1223, Eq. (13), second line	$\ldots-\mathrm{Sp}_{1,2}\mathscr{E}(\mathbf{e}_1)$	$\ldots-\mathrm{Sp}_{1,2}\mathscr{E}(\mathbf{e}_2)\ldots$
1226, Eq. (26), 12th line	$\cdots\{(\mathbf{p}+\mathbf{q},\ \mathbf{p}$	$\cdots\{(\mathbf{p}+\mathbf{q},\mathbf{p})-(\mathbf{p}+\mathbf{q},\ \mathbf{n})\cdot\cdot(\mathbf{pn})\}$
1227, Eqs. (38), (41), (41a) numerators and denominators	$(\mathbf{p}^2-\mathbf{q})$	$(\mathbf{p}^2-\mathbf{q}^2)^2$
1228, top line	$\mathbf{m}_2=\frac{\mathbf{q}_1-\mathbf{p}_1}{\mathbf{q}_1-\mathbf{p}_1}$	$\mathbf{m}_2=[\mathbf{m}_3\ \mathbf{m}_1]$

ERRATA TO VOLUME 12

page	reads	should read
Article by Dzhelepov et al.		
205, figure caption	54	5.4
Article by M. Gavrila		
225, Eq. (2), last line	$-2\gamma\Theta^{-4}\,{}^1/_8$	$-2\gamma\Theta^{-4}-{}^1/_8$
Article by Dolgov-Savel'ev et al.		
291, caption of Fig. 5, 4th line	$p_0 = 50 \times 10^{-4}$ mm Hg	$p_0 = 5 \times 10^{-4}$ mm Hg.
Article by Belov et al.		
396, Eq. (24) second line	$\ldots-(4-2\eta)\,\sigma_1+\ldots$	$\ldots+(4-2\eta)\,\sigma_1+\ldots$
396, 17th line (r) from top	. . less than 0.7	. . less than 0.07
Article by Kovrizhnykh and Rukhadze		
615, 1st line after Eq. (1)	$\omega_{0e}^2=2\pi e^2n_e/m_e,$	$\omega_{0e}^2=4\pi e^2n_e/m_e,$
Article by Belyaev et al.		
686, Eq. (1), 4th line	$\ldots b_{\rho_2m_2}(s_2')+\cdots$	$\ldots b_{\rho_1m_1}(s_1')+\cdots$
Article by Zinov and Korenchenko		
798, Table X, heading of last column	$\sigma_{\pi^-\to\pi^+}=$	$\sigma_{\pi^-\to\pi^-}=$
Article by V. M. Shekhter		
967, 3d line after Eq. (3)	$\varepsilon\equiv 2m_pE+m_p^2$	$\varepsilon\equiv(2m_pE+m_p^2)^{1/2}$
967, Eq. (5), line 2	$+(B_V^2+B_A^2)\ldots$	$+(B_V^2+B_A^2)\,Q\ldots$
968, Eq. (7)	$\ldots(C_V^2+C_A^2).$	$\ldots C_V^2+C_A^2-Q^2(B_V^2+B_A^2).$
968, line after Eq. (7)	for $C_V^2+C_A^2\equiv\ldots$	for $C_V^2+C_A^2-Q^2(B_V^2+B_A^2)\equiv\ldots$
Article by Dovzhenko et al.		
983, 11th line (r)	$\gamma=1.8\pm$	$\gamma=1{,}8\pm0{,}2$
Article by Zinov et al.		
1021, Table XI, col. 4	—1,22	1,22
Article by V. I. Ritus		
1079, line 27 (1)	$-\Lambda_{\pm}(t),$	$\Lambda_{\pm}(t),$
1079, first line after Eq. (33)	$\frac{1}{2}(1+\beta).$	$\frac{1}{2}(1\pm\beta).$
1079, 3d line (1) from bottom	$\ldots\mathfrak{N}(q'p;\ pq')\ldots$	$\ldots\mathfrak{N}(p'q;\ pq')\ldots$
Article by R. V. Polovin		
1119, Eq. (8.2), fourth line	$U_{0x}u_xg(\gamma)-[\gamma\cdots$	$-U_{0x}u_xg(\gamma)\,[\gamma\cdots$
1119, Eq. (8.3)	$\cdots\mathrm{sign}\ u.$	$\cdots\mathrm{sign}\ u_g.$
Article by V. P. Silin		
1138, Eq. (18)	$\cdots+\frac{4}{5}c^2k^2$	$\cdots+\frac{6}{5}c^2k^2.$