and the applied electric field which characterizes the extent of the effect (cf. reference 2) was estimated to be 1.2×10^{-5} at a temperature of 0° C.

The sample used had an irregular form and in estimating the coefficient α no correction for the demagnetizing factor was introduced. The large nonuniformity of the applied electric field was also not taken account of.

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[1] L. D. Landau and E. M. Lifshitz, электродинамика сплошных сред (Electrodynamics of Continuous Media) Gostekhizdat, 1957.

[2] I. E. Dzyaloshinskiĭ, JETP **37**, 881 (1959), Soviet Phys. JETP **10**, 628 (1960).

[3] B. N. Brockhouse, J. Chem. Phys. **21**, 961 (1953).

[4] McGuire, Scott, and Grannis, Phys. Rev. **98**, 1562 (1955).

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185

ON THE MOMENTUM SPECTRUM OF π^+ MESONS FROM THE REACTION $\pi^+ + p \to 2\pi^+ + n$

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In the observation of the reaction $\pi^- + \pi^+ + \pi^- + n$ for an incident-meson energy $E_\pi = 1.37$ Bev[1] in the laboratory system, the histogram representing the momentum spectrum of the π^+ and π^- meson was found to have two maxima: broad and low at small values of momentum and narrower and higher at large values of momentum. This was explained by Sternheimer and Lindenbaum[2] by means of the real isobaric nucleon model ($T = J = 3/2$). It should be mentioned that, according to this model, a similar momentum spectrum should also be observed in the reactions $\pi^- + p \to \pi^- + \pi^0 + p$ and $\pi^+ + p \to 2\pi^+ + n$. The first of these reactions was studied in reference 1. The shape of the total momentum spectrum in this case is in better agreement with the statistical theory of Fermi, which gives one maximum at medium energies. Disparities with the conclusions of the isobaric theory are also mentioned in reference 3. In this connection, we should draw attention to the formal possibility, existing in theory, of not employing the notion of a real isobaric nucleon.

For simplicity, we consider the reaction $\pi^+ + p \to 2\pi^+ + n$, which occurs in the isotopic state with total angular momentum $T = 3/2$ and total meson angular momentum $\Lambda = 2$. We denote the momentum of the incident meson in the center-of-mass system by $\mathbf{k}_0$ and the momenta of the emitted mesons by $\mathbf{k}_1$ and $\mathbf{k}_2$. As is known, this reaction is described by the matrix $\langle \mathbf{k}_1, \mathbf{k}_2 | T^{3/2;2} | \mathbf{k}_0 \rangle$, whose elements in the total angular momentum representation are $T^{3/2;2}_{J,lL(l_1,l_2)}(k_1, k_2, k_0)$, where J is the total angular momentum, l is the orbital angular momentum of the incident meson, L is the total and l_1, l_2 the partial orbital angular momenta of the radiated mesons (see references 4 and 5); $L = l \pm 1$, $|l_1 - l_2| \le L \le l_1 + l_2$. The probability of observing a π^+ meson in a final state with a momentum of absolute magnitude k is given by the expression

$$w(k) = \frac{v}{2} \int_0^\pi \sin\theta d\theta \left\{ \left(\frac{d\sigma(k_1, k_2, \theta)}{dk_1} \right)_{k_1=k} + \left(\frac{d\sigma(k_1, k_2, \theta)}{dk_2} \right)_{k_2=k} \right\}, \tag{1}$$

where v is the velocity of the incident π meson, and θ the angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$; k_1 is determined from k_2 (and conversely) by the relativistic energy-momentum conservation law.

We shall be interested in the qualitative comparison of the spectra resulting from the individual partial states, i.e., $w_{L(l_1 l_2)}(k)$. We consider part of the matrix of the process under study $T' = \{A + B(\mathbf{k}_1\mathbf{k}_0)^2\}(\sigma\mathbf{k}_0)$. Representing $(\mathbf{k}_1\mathbf{k}_0)^2$ in an expansion in Legendre polynomials, we obtain

$$T' = \left\{ \left(A + \tfrac{1}{3} k_1^2 k_0^2 B \right) P_0(\cos\vartheta_{10}) + \tfrac{2}{3} k_1^2 k_0^2 B P_2(\cos\vartheta_{10}) \right\} (\sigma\mathbf{k}_0).$$

If the coefficient of P_2 is very much less than the coefficient of P_0, then $T' \approx A(\sigma\mathbf{k}_0)$. Applying a similar argument to the matrix as a whole, we find that if

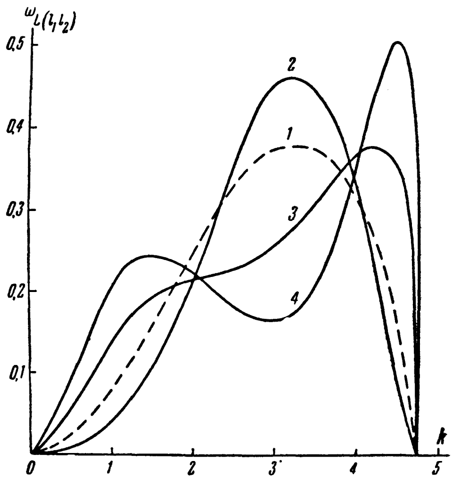
$$|T^{3/2;2}_{J',l'L'(l_1'l_2')}| \ll |T^{3/2;2}_{J,lL(l_1 l_2)}|$$

for $l_1' + l_2' \geq l_1 + l_2 + 2$, and if the matrix does not have singularities, then

$$T^{3/2;2}_{J,lL(l_1l_2)}(k_1, k_2; k_0) \approx N k_1^{l_1} k_2^{l_2} \tau^{3/2;2}_{J,lL(l_1l_2)}(k_0), \quad (2)$$

where N is a normalizing factor and $\tau(k_0)$ is a function very weakly dependent on k_1, k_2. For energies ~ 1.3 Bev, π mesons with $l \leq 5$ take part in π-N scattering (see the survey article of Gell-Mann and Watson,[6] Fig. 1). This gives 4 and 6 for the maximum $L = l \pm 1$. Taking into account the expenditure of energy on the production of the additional π meson, we thus assume (underestimating somewhat) that the states with $l_1' + l_2' \geq 4$ do not essentially contribute to the matrix element. Then, for energies ~ 1.3 Bev, approximation (2) can be used for the cases in which $l_1 + l_2 \geq 2$.

In the figure are shown the momentum spectra for



Curve 1 – Statistical theory; Curve 2 – $L(l_1l_2) = 2(11)$; Curve 3 – $L(l_1l_2) = 2(20), 2(02)$; Curve 4 – $L(l_1l_2) = 3(30), 3(03)$.

some partial states and for $E_\pi = 1.37$ Bev, calculated from approximation (2). The normalization makes the area of each curve equal to unity. The spectra corresponding to states with $l_1 = l_2$ have single maxima, and the greater the angular momentum, the sharper the peak. For $l_1 \neq l_2$ the shape of the spectrum changes with increasing $\Delta l = |l_1 - l_2|$, and acquires the character of a double-humped curve.* The total momentum spectrum is formed by the superposition of the partial spectra (with the interference of the partial states taken into account) with weights determined by the specific character of the interaction; the spectrum can take on all intermediate shapes, from a curve with one sharp maximum to a curve with two characteristic maxima. If it is assumed that the weight of the orbital angular momentum is due to one of the mesons, then the total momentum spectrum of the π^+ mesons, for sufficiently large energies, will be represented by a double-humped curve. A similar result can be expected for the total momentum spectra of the π mesons in other reactions mentioned above.

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*This result may be explained by an extreme simplification of the problem. If the mesons are assumed to be ultra-relativistic and the nucleon is assumed to be at rest, then the law of conservation of energy gives $k_1 + k_2 = \epsilon$, where ϵ is the total energy minus the mass of the nucleon. In this case

$$w_{L(ll)}(k) \sim k^{2l}(\varepsilon - k)^{2l}\rho(k), \quad w_{L(l0)+L(0l)}(k) \sim [k^{2l} + (\varepsilon - k)^{2l}]\rho(k),$$

where $\rho(k)$ is the state density function; $\rho(0) = \rho(\epsilon) = 0$, and $\rho(\epsilon/2) = \rho_{max}$. For sufficiently large l, the second probability has two maxima.

[1] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).

[2] R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. **109**, 1723 (1958).

[3] R. C. Whitten and M. M. Block, Phys. Rev. **111**, 1676 (1958).

[4] B. d'Espagnat, Kgl. Danske Videnskab Selskab., Mat-fys Medd. **28**, 11 (1954).

[5] V. A. Astaf'ev, JETP **36**, 98 (1959), Soviet Phys. JETP **9**, 70 (1959).

[6] M. Gell-Mann and K. M. Watson, Annual Review of Nuclear Science **4**, 219 (1954) [Russ. Transl., Usp. Fiz. Nauk **59**, 399 (1956)].

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186

THEORETICAL INTERPRETATION OF ELASTIC π^--p SCATTERING EXPERIMENTS ON THE PROTON SYNCHROTON OF THE JOINT INSTITUTE FOR NUCLEAR RESEARCH

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EXPERIMENTS on π^--p scattering at high energies are of great interest for the study of nucleon structure. In particular, it is possible from the analysis of such experiments to obtain data on the distribution of nuclear matter in the proton and also the value of the mean square of its radius.[1]