method of relative measurements ensured a higher accuracy of the results.

[1] Tsien, Ho, Chastel, and Vigneron, J. phys. radium **8**, 165, 200 (1947).
[2] L. L. Green and D. L. Livesey, Nature, No. 4036, 332 (1947).
[3] L. L. Green and D. L. Livesey, Phil. Trans. **241**, 323 (1948).
[4] Farewell, Segré, and Wiegand, Phys. Rev. **71**, 327 (1947).
[5] L. Marshall, Phys. Rev. **75**, 1339 (1949).
[6] K. W. Allen and J. T. Dewan, Phys. Rev. **80**, 181 (1950).
[7] E. W. Titterton, Nature **168**, 590 (1951).
[8] D. Hill, Phys. Rev. **87**, 1049 (1952).
[9] C. B. Fulmer and B. L. Cohen, Phys. Rev. **108**, 370 (1957).
[10] Dmitriev, Drapchinskiĭ, Petrzhak, and Romanov, Dokl. Akad. Nauk SSSR **127**, 531 (1959), Soviet Phys.-Doklady **4**, 823 (1960).

Translated by J. G. Adashko
192

LONGITUDINAL RELAXATION OF NUCLEAR SPINS IN A PARAMAGNETIC CRYSTAL AT VERY LOW TEMPERATURES

B. I. KOCHELAEV

Kazan' State University

Submitted to JETP editor December 11, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 999-1000 (March, 1960)

IT is well known that at sufficiently low temperatures, the specific heat associated with the lattice of a paramagnetic crystal becomes negligible compared with the specific heat of spin systems. In this case, the spin system of paramagnetic atoms will be the thermostat determining the equilibrium population distribution among nuclear spin levels, and the longitudinal nuclear relaxation time T_1 will be the time required to establish thermodynamic equilibrium between the electronic and nuclear spin systems. It is the purpose of this note to describe some calculations of this time.

At low temperatures, the energy levels of most paramagnetic atoms in an external magnetic field can be described by an effective spin $S = \frac{1}{2}$ with an anisotropic g factor, i.e., with the magnetic moment of an atom being given by $\mu = \beta \mathbf{gS}$, where $\mathbf{g}$ is a tensor of the second rank and β is the Bohr magneton. We assume that the electric field inside the crystal has axial symmetry and that the Zeeman energy is much greater than either kT or the mean interaction energy of magnetic dipoles. In this case Bogolyubov's approximate method for second quantization can be used to find the energy spectrum of the interacting electron spins. This spectrum turns out to be equivalent to the spectrum of a system of non-interacting Bose particles with Hamiltonian

$$\mathcal{H} = E_0 + \sum_{\mathbf{k}} E_{\mathbf{k}} N_{\mathbf{k}}, \quad N_{\mathbf{k}} = b_{\mathbf{k}}^{+} b_{\mathbf{k}},$$

$$E_0 = g_{\parallel} \beta H N, \quad E_{\mathbf{k}} \approx \frac{10\pi}{a} g_{\perp}^2 \beta^2 (k^2 - 3k_z^2), \qquad (1)$$

where $b_{\mathbf{k}}^{+}$, $b_{\mathbf{k}}$ are creation and destruction operators for Bose particles with wave vector $\mathbf{k}$, N is the number of paramagnetic atoms, a is the shortest distance between them and $g_{\parallel}$, $g_{\perp}$ are the principal values of the tensor $\mathbf{g}$. In our calculations we used the fact that to find the spectrum of a spin system in a strong magnetic field it is only necessary to consider the diagonal and semi-diagonal parts of the operator for the dipole-dipole interaction.[2] It was also assumed that the paramagnetic atoms were arranged in a simple cubic lattice, that the magnetic field $\mathbf{H}$ is directed along the symmetry axis of the crystal field, and the lattice sums can be approximated by integrals.

Suppose the nuclear spins are coupled to the paramagnetic atoms only through the dipole-dipole interaction. We shall find the probability of a transition in which one boson is created, another destroyed, the energy difference between them being equal to the energy required to re-orient a nuclear spin in the external field, $E_{\mathbf{k}_1} - E_{\mathbf{k}_2} = 2g_N \beta_N H$ (where g_N is the nuclear magnetic moment in nuclear magnetons β_N). Assuming that $2g_N \beta_N H \ll \beta^2 g_{\perp}^2 / a^3$ and using the standard formulas of perturbation theory, we find that the probability of a transition is

$$P(m+1, m) = \frac{9}{20\pi^2\hbar} \left(\frac{g_{\parallel}}{g_{\perp}}\right)^2 g_N^2 \beta_N^2 \frac{a^3}{r^6} \sin^2 2\theta$$

$$\times \exp\left(\frac{2g_{\parallel}\beta H}{kT}\right)(I + m + 1)(I - m), \qquad (2)$$

where $\mathbf{r}$ is the radius vector from a paramagnetic atom to a nuclear spin $\mathbf{I}$; θ is the angle between the direction of the magnetic field and $\mathbf{r}$, and m is the magnetic quantum number of the nucleus.

Experimentally, the method of a saturated nuclear magnetic resonance has been used to meas-

ure a nuclear spin relaxation time in a paramagnetic crystal. The measurement was made on the protons in the water of crystallization of $Ce_2Mg_3(NO_3)_{12} \cdot 24H_2O$ at 0.086°K.[3] It was impossible to achieve saturation, and only an upper bound on the relaxation time was obtained: $T_1 < 32$ sec. A rough estimate using formula (2) yields $T_1 \approx 10^{-1}$ sec. The estimate is rough because $g_{||} = 0.25$ for the Ce^{3+} ion[4] and the condition $2g_{||}\beta H \gg kT$ is poorly fulfilled. This result does not contradict the experimental one.

The author would like to thank S. A. Al'tshuler for discussion of his results.

[1] N. N. Bogolyubov, Лекции по квантовой статистике (Lectures on Quantum Statistics) Kiev, 1949.

[2] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).

[3] Estle, Hart, and Wheatley, Phys. Rev. **112**, 1576 (1958).

[4] K. D. Bowers and J. Owen, Reports on Progress in Physics **18**, 304 (1955).

Translated by R. Krotkov
193

SEMICONDUCTOR AMPLIFIERS AND GENERATORS WITH CARRIERS HAVING NEGATIVE EFFECTIVE MASS

N. G. BASOV, O. N. KROKHIN, and Yu. M. POPOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor December 17, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1001-1002 (March, 1960)

KRÖMER[1] proposed the use of the negative effective mass of carriers in a semiconductor for the amplification and generation of electromagnetic waves, for when such carriers move in a field they give up their energy to the field interacting with them, i.e., they possess negative losses. To obtain such states, the use of a constant electric field is proposed.

In the present communication we shall show that it is impossible to obtain a state with negative losses by using a constant electric field in a semiconductor.

In fact, negative losses mean that, at least for some ϵ_1 and ϵ_2 ($\epsilon_2 > \epsilon_1$), the following condition is fulfilled:

$$wf(\varepsilon_2)[1 - f(\varepsilon_1)]n(\hbar\omega) - wf(\varepsilon_1)[1 - f(\varepsilon_2)]n(\hbar\omega) = wn(\hbar\omega)\{f(\varepsilon_2) - f(\varepsilon_1)\} > 0, \quad (1)$$

where $f(\epsilon)$ is the distribution function for electrons, $n(\hbar\omega)$ is the number of photons of energy $\hbar\omega = \epsilon_2 - \epsilon_1$, and w is the probability of spontaneous emission. We do not consider spontaneous transitions, since they are not relevant to the process of amplification. From (1) it follows that at least at some points in the interval $\epsilon_2 - \epsilon_1$, $\partial f(\epsilon)/\partial\epsilon > 0$ should hold. At thermodynamic equilibrium, $\partial f/\partial\epsilon < 0$ always holds for any ϵ, independently of the type of distribution of the particles, and consequently amplification is impossible.

The creation of a semiconductor amplifier or generator using the effect of negative losses should be considered from the viewpoint of the possible destruction of thermodynamic equilibrium and the attainment of states with $\partial f/\partial\epsilon > 0$ in some interval of energy. However, in the steady state in a constant electric field E, as assumed by Krömer,[1] it is impossible to obtain such states for a semiconductor, as direct calculation shows.[2-5] The distribution functions found up to the highest values of fields, have $\partial f(\epsilon, E)/\partial\epsilon < 0$ for all values of ϵ [$\partial f(\epsilon, E)/\partial\epsilon \to 0$ as $E \to \infty$]. In very strong fields when the processes of electron scattering at lattice phonons are unimportant, it is also impossible to obtain an amplifying state, even if impact ionization and the Zener effect are ignored. In this case the electron will oscillate periodically between the upper and lower edges of the permitted band, so that electron states with positive and negative masses are equally probable.

Everything that has been said above can be generalized directly to the case of an anisotropic band in which, for some values of the quasi-momentum **p**, some components of the tensor-effective mass are negative; here, too, it is impossible to have $\partial f/\partial\epsilon > 0$ in a constant electric field, since in semiconductors the constants of interaction with acoustical and optical phonons are of the same order of magnitude.

Thus, to create semiconducting systems with negative losses, it is necessary to obtain a state with negative temperature, where $\partial f/\partial\epsilon > 0$. Such states can in principle be attained by excitation of the electrons with sufficiently powerful monochromatic radiation, causing transitions between the levels of one band or different bands using a pulsed system or a steady state system, as in molecular generators and amplifiers, or using pulsed excitation by an electric field.[6]

We note that to obtain a negative temperature using a pulsed electric field, it is expedient to use