THE EXISTENCE OF NEW ISOTOPES OF LIGHT NUCLEI AND THE EQUATION OF STATE OF NEUTRONS

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The limits of stability (relative to nucleon emission) of light nuclei are considered. The existence (in the sense of stability against decay with emission of a nucleon) of the following nuclei is predicted: He^8, Be^{12}, O^{13}, $B^{15,17,19}$, C^{16-20}, N^{18-21}, Mg^{20}. The problem of the possibility of existence of heavy nuclei composed of neutrons only is considered. The problem is reduced to that of a Fermi gas with a resonance interaction between the particles. The energy of such a gas is proportional to $\omega^{2/3}$, where ω is its density. The accuracy of the calculations is not sufficient to determine the sign of the energy and answer the question as to the existence of neutron nuclei.

THE problem of the possible isotopes has been treated by Nemirovskiĭ[1,2] for $8 \leq Z \leq 84$, and by Baz'[3] for the region $17 \leq A \leq 40$. The former uses the one-particle approximation, with an attempt to find the dependence of the parameters of the well on the numbers of neutrons and protons. For nuclei with an excess of protons Baz' bases his discussion on the experimental data on the mirror nuclei (with excess of neutrons) and on the well-known expression for the Coulomb energy. For nuclei with an excess of neutrons he extrapolates the binding energy in series of nuclei with constant isotopic spin.

These papers predict the existence of many as yet unknown β-active isotopes. In the table given below the isotopes so predicted are enclosed in dashed-line squares. One of them has very recently been observed experimentally.[4]

In the present paper (Sec. 1) we make additional predictions in the region of the lightest nuclei; the isotopes so predicted are enclosed in solid-line squares in the table. We point out particularly the conclusion that there is a large probability that He^8 exists. For nuclei with an excess of neutrons the writer has tried to take the effect of shells and the pair interaction of neutrons into account as accurately as possible.

In Sec. 2 the question is raised of the existence of nuclei composed solely of neutrons. In the limiting case of a large number of neutrons, by using the data on resonance in the ^1S scattering, one can find the general form of the dependence of the energy on the density of the nuclear matter, but the accuracy of the first approximation obtained in this paper is insufficient to give a definite answer to the question of the existence of such nuclei.

1. LIGHT NUCLEI

Following the method of Baz',[3] one easily convinces oneself that there must exist a nucleus O^{13} with a proton binding energy not smaller than 1.2 Mev and with β^+-decay energy 16 to 17 Mev. Using the data[4] on the mass of O^{20}, we conclude that the mirror nucleus Mg^{20} must exist with proton binding energy not less than 2.7 Mev and β^+-decay energy about 7 Mev. The existence of O^{12}, Ne^{16}, and Mg^{19} is not excluded (empty spaces in the table);* the corresponding mirror isotopes Be^{12}, C^{16}, and N^{19} are predicted in this paper (see later argument), but their energies cannot be predicted with enough accuracy to give a definite conclusion about O^{12}, Ne^{16}, and Mg^{19}. The isotopes Ne^{17}, Na^{19}, Mg^{21}, and Mg^{22} are predicted by Baz'.

Regarding all the other nuclei in the upper right-hand part of the table we can assert with assurance that they are unstable against emission of a proton, i.e., they do not exist, which is shown in the table by the minus signs in all the upper cells.

Let us turn to the nuclei with an excess of neutrons. A nucleus with an excess of neutrons does not exist in the case in which all the discrete levels are already filled up with neutrons. An important point here is that the nuclear forces fall off rapidly with distance, and therefore the number of levels in the field of the nuclear forces is limited (in

*These nuclei may be unstable with respect to the emission of two protons at once. On the other hand, at the limit of stability the expression for the Coulomb energy of the last proton, $1.2(Z-1)A^{-1/3}$, gives too large a result; for example, in the pair $Li^8 - B^8$ we have for Li^8 the binding energy $Q_n = 2$ Mev and for B^8 the value $Q_p = 0.2$ Mev, so that the difference is 1.8 Mev, whereas by the formula we would get $1.2 \times 4 \times 7^{-1/3} = 2.5$ Mev.

Z ↓

	p^1	—	—	—	—	—	—	—	—	—	—	—
N^1	D^2 2.23	He^3	—	—	—	—	—	—	—	—	—	—
—	T^3 6.26	He^4 20,58	—	—	—	—	—	—	—	—	—	—
—	—	—	Li^6	Be^7	B^8	C^9	—	—	—	—	—	—
	?H^5	He^6 0.94	Li^7 7,25	Be^8 18,90	B^9 18.58	C^{10} 21.9	—		—	—	—	—
		—	Li^8 2,04	Be^9 1,67	B^{10} 8.44	C^{11} 13,3	N^{12}	O^{13}	—	—	—	—
		He^8	Li^9 3.58	Be^{10} 6.81	B^{11} 11,50	C^{12} 18,73	N^{13} 20.9	O^{14}	—		—	—
		—	—	Be^{11} 0.55	B^{12} 3.36	C^{13} 4.95	N^{14} 10,55	O^{15} 13,23	—	Ne^{17}	—	
		—		Be^{12}	B^{13} 4.89	C^{14} 8,17	N^{15} 10.84	O^{16} 15.66	F^{17} 17.01	Ne^{18}	Na^{19}	Mg^{20}
				—		C^{15} 1,23	N^{16} 2.50	O^{17} 4,15	F^{18} 9.17	Ne^{19} 11,39	Na^{20}	Mg^{21}
					B^{15}	C^{16}	N^{17} 5.84	O^{18} 8.07	F^{19} 10,42	Ne^{20} 16.92	Na^{21} 18,57	Mg^{22}
				—		C^{17}	N^{18}	O^{19} 3.96	F^{20} 6.61	Ne^{21} 6,76	Na^{22} 11,05	Mg^{23}
					B^{17}	C^{18}	N^{19}	O^{20} 7,65	F^{21} 8.18	Ne^{22} 10,36	Na^{23} 12,42	Mg^{24} 16,4
				—		C^{19}	N^{20}	O^{21}	F^{22}	Ne^{23} 5,20	Na^{24} 6.97	Mg^{25} 7,34
					B^{19}	C^{20}	N^{21}	O^{22}	F^{23}	Ne^{24} 8,90	Na^{25} 9,15	Mg^{26} 11,12
								O^{23}	F^{24}	Ne^{25}	Na^{26}	Mg^{27} 6,44
								O^{24}	F^{25}	Ne^{26}	Na^{27}	Mg^{28} 8,56
								O^{25}	F^{26}	Ne^{27}	Na^{28}	Mg^{29}
								O^{26}	F^{27}	Ne^{28}	Na^{29}	Mg^{30}
								O^{27}	F^{28}	Ne^{29}	Na^{30}	Mg^{31}
								O^{28}	F^{29}	Ne^{30}	Na^{31}	Mg^{32}

→ N

contradistinction, for example, to the case of the Coulomb field). With the spin taken into account the number of levels is always even; therefore if a nucleus exists containing an odd number of neutrons (2n + 1), then there is also a place for a subsequent (2n + 2)-nd neutron. On account of the mutual attraction of a pair of neutrons the binding energy of the (2n + 2)-nd neutron is always larger than that of the preceding (2n + 1)-st neutron.

In each cell of the table that corresponds to an experimentally known isotope there is written the binding energy of the last neutron. It is easily verified that in all cases $E_{2n+2} > E_{2n+1}$. Therefore the existence of the nuclei Be^{12} and C^{16} definitely follows from the existence of Be^{11} and C^{15}. As a rough estimate, the binding energy of a neutron in Be^{12} is about 2 — 3 Mev, and the β-decay energy is 12 — 13 Mev; for C^{16} these values are 3 — 4 Mev and 8 — 9 Mev, respectively.

It is much harder to settle the existence of other isotopes. Extrapolation for fixed isotopic spin[3] T is not reliable, since it involves comparison of neutrons that are in different shells.

For the lightest nuclei the idea of a smooth dependence of the parameters of the well on N and Z[1,2] does not take sufficient account of the individual peculiarities of the shells. We shall try to make maximum use of the experimental data. It is known from the scattering of neutrons by He^4 that for the partial wave $P_{3/2}$ there is a resonance at the energy +1.0 Mev (i.e., in the continuous spectrum) with width 0.55 Mev (which corresponds to an He^5 lifetime of 10^{-21} sec). The nucleus He^5 does not exist, and consequently there is no discrete bound state of a neutron in the field of He^4.

In the same sense, the dineutron does not exist, since from experiments on the scattering of neutrons by protons it is known that in the ^1S state, which is allowed for two neutrons by the Pauli principle, the attraction is not sufficient for the formation of a bound state. Therefore the He^6 nucleus is a remarkable system of three particles ($n + n + He^4$), which cannot be bound together in pairs, but all three together form a bound system. Quite crudely we can imagine that He^6 consists of two neutrons in the state $(P_{3/2})^2$ in the field of He^4. The energy of interaction between the two neutrons (about -3 Mev) is more than enough to compensate for the positive energy of each neutron in the state $P_{3/2}$ (+1 Mev) in the field of He^4.

The $P_{3/2}$ shell has four places in all. Therefore we can raise the question of the possibility of He^7 and He^8. According to Kurath,[5] in the limit of small range of the forces and weakly bound nucleons, and for large radius of the orbits of the shell ($r_0 \ll r_1$, $L = 3K$, in his notation), one gets a simple result: if the energy of interaction of two neutrons is B, then the energy of the interaction of three neutrons is also B, and the energy of the interaction of four neutrons is 2B, i.e., the neutrons combine in pairs, as it were. From this there follows the conclusion that He^7 does not exist, but He^8 exists; the expected binding energy of a neutron in 0.8 — 0.5 Mev, and the β-decay energy is about 12 Mev. It would be extremely desirable to verify the existence of He^8 experimentally and determine its binding energy.

How accurately the rule of the combining of neutrons in pairs in a single shell around a doubly magic (closed) core holds experimentally can be seen from two examples.*

1) The filling up of the $d_{5/2}$ shell on the closed O^{16} (see table). We quote the binding energies (in megavolts). The subscript on E is the number of neutrons in the $d_{5/2}$ shell (the upper index is the atomic weight):

$$E_1^{17} = 4.15,\quad E_2^{18} = 8.07,\quad E_3^{19} = 3.96,\quad E_4^{20} = 7.65.$$

There are no data on E_5 and E_6, which finish the filling of the shell; the nuclei O^{21} and O^{22} have not yet been observed.

2) The filling up of the $f_{7/2}$ shell on Ca^{40}, which has closed shells (this example has been treated partially by Nemirovskiĭ[2]). The binding energies are:

E_1^{41}	E_2^{42}	E_3^{43}	E_4^{44}	E_5^{45}	E_6^{46}	E_7^{47}	E_8^{48}
8.3	11.4	8.0	11.4	7.4	11.0 (?)†	6.8 ?†	10.8

At the end of the filling-up of the $f_{7/2}$ shell the binding energy E falls sharply: $E_9^{49} = 5.1$. Since He^4 is a closed doubly magic nucleus (and an even stabler one than O^{16} and Ca^{40}), these examples speak convincingly for the existence of He^8.

If the proton shell is not filled, then E drops off extremely sharply within the range of the given neutron shell; we may imagine that the first neutrons unite in pairs with the "free" protons (those outside the closed shells), and later neutrons can no longer do this. As an example let us consider the $d_{5/2}$ shell of Ne^{18} — a nucleus with two protons beyond O^{16}. We have:

E_1^{19}	E_2^{20}	E_3^{21}	E_4^{22}	E_5^{23}	E_6^{24}
11.4	16.9	6.8	10.4	5.2	8.9

If the proton shell falls short of being closed by one, two, or three protons, the binding energy of the neutrons is decreased as compared with the binding to a closed shell (cf. C^{15}, N^{16}, and O^{17} in the table). But within the limits of a given neutron shell (on a core with holes in the proton shell) E varies little, in contradistinction to the case in which excess protons are present.

We give examples of the filling of the $f_{7/2}$ shell with neutrons in nuclei with unfilled proton shells:

Nucleus K_{19}^{39}: $E_1^{40} = 7.9,\quad E_2^{41} = 10.0,\quad E_3^{42} = 7.4,\quad E_4^{43} = 10.8.$

Nucleus Ar_{18}^{38}: $E_1^{39} = 6.7,\quad E_2^{40} = 9.7,\quad E_3^{41} = 6.1.$

Thus we can formulate the rule that on nuclei with closed proton shells and with holes in the proton shell (but not on nuclei with excess protons), the binding energies of the odd neutrons are practically constant within the limits of a given neutron shell. The binding energies of the even neutrons are also constant within the limits of a given shell, but are larger by the amount of the pairing energy. Carrying this rule over to the $d_{5/2}$ shell, we come to the conclusion that the experimental fact of the existence of bound $d_{5/2}$ states in the nuclei C^{15} and N^{16}, N^{17} guarantees the possibility of filling up the entire $d_{5/2}$ shell, to C^{20} and N^{21}, respectively.

An examination of the binding energies of neutrons in the table reveals a regular increase of E in each row, with increase of the number of protons (the single exception is the pair $Li^8 - Be^9$, which is due to the special structure of Be^8). Extrapolation of E to the left along the rows makes probable the existence of B^{15}, and from this — by the principle of the constancy of the binding energy in a shell — of B^{17} and B^{19}. The existence of the nuclei with odd numbers of neutrons, B^{14}, B^{16}, B^{18}, remains questionable. With considerable assurance we can assert that the odd (in n) nuclei Be^{13}, Be^{15}, Be^{17}, Li^{10}, He^9 do not exist.

*The mass data are taken from review articles.[6-8]

†The nucleus Ca^{46} has not been studied, so that one knows experimentally only the sum $E_6^{46} + E_7^{47} = 17.8$; the separate terms in the table are obtained by interpolation.

On the whole, however, the assertions that can be made reliably about nuclei with excess neutrons not known to exist are extremely scanty. From studies of scattering only the nonexistence of n^2 and He^5 is quite accurately proved. From principles of the pair interaction of neutrons it is obvious that n^3 and He^7 do not exist. There is no longer such certainty regarding H^5 (H^5 is entered in the table with a question mark), and the hypothesis that it is stable has been suggested.[9] We note that if n^4 and H^5 existed, then there would be isotopically similar quasi-stable systems H^4 with $T = 2$ and He^5 with $T = 3/2$, which would manifest themselves in the scattering of n by T and of n by He^4; this situation has been examined in detail in a separate note.[10] At present there are no experimental data in the required range of neutron energies.

Unlike the upper right-hand part of the table, which is almost solidly filled with minus signs ("does not exist"), in most of the cells of the lower left-hand part we can put neither a minus nor the symbol of a nucleus ("exists"). The obscurity of the problem of the limits of existence of isotopes with excess neutrons is a consequence of the fact that the limiting case is not clear; it is not known whether a heavy nucleus composed solely of neutrons could exist.

2. THE NEUTRON LIQUID

The problem of the limiting number of neutrons that can adhere to a heavy nucleus has been considered by Wheeler;[11] he came to the conclusion that for $Z \sim 90 - 100$ the maximum mass number is $A_{max} \sim 500 - 600$. Wheeler used the Weizsäcker formula; Nemirovskiĭ[2] correctly critizes this formula near the limits of existence, and therefore Wheeler's conclusions are not reliable.

Let us consider the extreme case of a very large nucleus consisting of neutrons alone. If it does exist, it surely does so only with a density much smaller than that of ordinary nuclei. Let us first examine the properties of a neutron liquid of small density; these properties are determined by the pair interactions of the neutrons at small energies (up to a few Mev). In this region only the interaction of pairs of neutrons in the ^1S state is of importance, and here this interaction is completely determined by the scattering length* (cf., e.g., reference 12)

$$a = -(d \ln \varphi / dr)^{-1} = -19 \cdot 10^{-13} \text{ cm};$$

the sign corresponds to the absence of a bound state, and the quantity a corresponds to the so-called energy of a virtual level (μ is the reduced mass, equal to $M/2$):

$$E_v = \hbar^2/2\mu a^2 = 0.11 \text{ Mev}.$$

We cite here the well-known calculation[13,14] of the energy of interaction of particles in the continuous spectrum, confining ourselves at once to the S wave. As usual, we consider first a spherical box for $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of the two particles, i.e., we set $\psi(\mathbf{r}) = 0$ at $|\mathbf{r}| = R$. Without interaction the normalized S-wave function in such a box is

$$\psi = \sin(n\pi r/R) / r\sqrt{2\pi R}.$$

With an interaction corresponding to scattering with the phase shift α we have

$$\psi = \sin[\alpha + R^{-1}(n - \alpha/\pi)\pi r] / r\sqrt{2\pi R},$$

which corresponds to a change of the energy of the n-th state given by

$$\Delta E_n = -\hbar^2 n\pi\alpha / \mu R^2.$$

Let us eliminate the auxiliary quantities R and n from the expression for ΔE_n. The state under consideration is characterized by the momentum of the relative motion

$$p_n = \hbar n\pi / R$$

and the density at the origin of coordinates in the unperturbed motion

$$\rho_n(0) = \psi^2(0) = \pi n^2 / 2R^3.$$

Let us express ΔE_n in terms of p and $\rho(0)$; after this we can set $R \to \infty$, $n \to \infty$, and forget about n. We get

$$\Delta E = -2\pi\hbar^3\alpha\rho(0) / \mu p. \tag{1}$$

We express the phase in terms of the scattering length:

$$\alpha = -\tan^{-1}(ap/\hbar).$$

For $E \ll E_v$, $ap \ll \hbar$ we have

$$\alpha = ap/\hbar, \qquad \Delta E = -2\pi\hbar^2 a\rho(0)/\mu; \tag{2}$$

for $E \gg E_v$, $ap \gg \hbar$ we get

$$\alpha = \pi/2, \qquad \Delta E = -\pi^2\hbar^3\rho(0)/p\mu. \tag{3}$$

Let us apply the expressions (2) and (3) to a Fermi gas consisting of neutrons only with mean density ω. We single out one neutron with a definite spin direction. At the point where this neutron is located, the density of other neutrons with the same spin direction is zero by the Pauli principle; if

*For pp scattering $a = -17.2$, and for np scattering $a = -23.7$; we assume that a depends linearly on the product of the magnetic moments.

there were no interaction, the density of the other neutrons with antiparallel spins would not differ from that of those with parallel spins on the average over all space; that is, $\omega(0) = \omega/2$. We recall that ω is the total density of neutrons with both spin directions and that the formula for ΔE contains just the density in the state without interaction.*

We still have to take into account the fact that the change of energy ΔE relates to a system of two particles; in order not to include the interaction of each pair twice, we recall that the decrease of the energy of one particle is $\Delta E/2$. We finally find that if for a pair of particles in the ^1S state $\Delta E = k\rho(0)$, where k is a coefficient that depends on the momentum, then the change of the energy of all the gas in unit volume on account of the interaction is

$$U = \omega^2 \bar{k}/4; \tag{4}$$

here k is averaged over the Fermi distribution.

The Fermi distribution is characterized by the boundary momentum p_f, the boundary energy E_f, and the total kinetic energy $\mathscr{E}$ of all the gas in unit volume; as is well known

$$\mathscr{E} = \omega\bar{E} = \tfrac{3}{5}\omega E_f, \qquad E_f = p_f^2/2M,$$
$$\omega = p_f^3/3\pi^2\hbar^3, \qquad \mathscr{E} = p_f^5/10\pi^2\hbar^3 M. \tag{5}$$

When we average k we get a result which depends on the ratio of E_f to the energy E_V of the virtual level. For $E_f < E_V$ the quantity k is constant and ($\mu = M/2$)

$$U = -\pi\hbar^2\alpha\omega^2/2\mu. \tag{6}$$

In the limiting case $E_f \gg E_V$ we must average over the Fermi distribution p^{-1}, where p is the momentum of the relative motion of two particles. We have

$$\mathbf{p} = \mu(\mathbf{v}_1 - \mathbf{v}_2) = \tfrac{1}{2}M(\mathbf{v}_1 - \mathbf{v}_2) = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2). \tag{7}$$

Using the electrostatic analogy† we easily find

$$\overline{|\mathbf{p}_1 - \mathbf{p}_2|^{-1}} = 6/5p_f, \qquad \overline{p^{-1}} = 12/5p_f \tag{8}$$

and finally

$$U = -3\pi^2\hbar^3\omega^2/5\mu p_f = -2p^5/15\pi^2\hbar^3 M = -\tfrac{4}{3}\mathscr{E}. \tag{9}$$

This is a remarkable result: the interaction energy is a constant multiple of the kinetic energy.

If we take these results literally, we get the following physical conclusions about the dependence on the density of the average energy of a neutron, $E_1(\omega) = (\mathscr{E} + U)/\omega$: at small density, in the limit

$$E_1 = \tfrac{3}{5}E_m > 0, \qquad E_1 \sim \omega^{2/3}, \tag{10}$$

the interaction is proportional to a higher power of ω (higher than the first power); at the density ω_0 that corresponds to $E_f = 5E_V$, the energy E_1 goes to zero, and then changes sign and at larger densities

$$E_1 = -\tfrac{1}{3}E_m < 0, \qquad E_1 \sim \omega^{2/3}. \tag{11}$$

This expression holds for* $\omega > \omega_0 \approx a^{-3}$. From this it follows that a nucleus can exist that consists of neutrons only, with a binding energy given by $-E_1$.

This treatment does not give the equilibrium density, since according to Eq. (11) as the density increases E_1 continues to decrease (E_1 is negative and its absolute value increases). To find the equilibrium density and the binding energy at this density we must bring in the effective range of nuclear forces and the interaction in states with $l \neq 0$. Qualitatively, however, the fact of the existence of neutron nuclei itself follows just from the change of sign of E_1, which is obtained from a calculation at the density $\omega_0 = a^{-3}$. Since a is extremely large, we have $\omega_0 \approx 0.001\,\omega_n$, where ω_n is the density of ordinary nuclei. In a state corresponding to the density ω_0 for which $E_1 = 0$ the boundary kinetic energy E_f is about 0.5 Mev, so that the contribution from $l \neq 0$ and the influence of the effective range are negligible; thus the assumptions about the interaction of the neutrons that were the basis for the calculation are very well satisfied at $\omega = \omega_0$. We note that if the existence of a range of values of ω in which $E_1 < 0$ is confirmed, then the surface tension of the neutron liquid will give a definite critical size of the neutron droplet, i.e., a minimum number of neutrons for which the existence of a neutron nucleus is possible. Therefore if it is proved that bound states n^4, n^6, or n^8 do not exist, this does not by itself exclude the existence of heavier neutron nuclei.

*Another possible approach is based on the fact that the statistical weights of the triplet and singlet are in the ratio 3 : 1; a given neutron interacts with only ¼ of the others. But in the singlet state without scattering the density at the origin of coordinates is twice as large as the average density throughout the volume, since in the singlet state only even angular momenta l are possible, and therefore the S state, the only one that contributes to $\rho(0)$, makes up twice as large a fraction of all singlet states as in the case of different particles. We finally find (1 is the index for the singlet) $\omega(0) = 2\bar{\omega}_1 = 2(\omega/4) = \omega/2$, which agrees with the result obtained in the text.

†For any body

$$\overline{r_{12}^{-1}} = \iint r_{12}^{-1}dv_1dv_2 = \int \varphi_1 dv_1 = \bar{\varphi},$$

where φ is the potential for unit charge density, which satisfies the equation $\Delta\varphi = -4\pi$ inside the body and $\Delta\varphi = 0$ outside the body.

*A consistent calculation on the assumptions made above gives a value of the coefficient very close to unity.

Nevertheless the main result — the change of sign of E_1 — is by no means to be regarded as established, since only the pair interaction of the neutrons has been considered and no account has been taken of the influence of the other neutrons on the wave functions of the interacting pair. The result is doubly unreliable because for $\omega > \omega_0$ the desired quantity E_1 is the small difference of two nearly equal quantities:

$$E_1 = \overline{E} + U_1, \qquad \overline{E} = \tfrac{3}{5} E_f, \qquad U_1 = -\tfrac{4}{5} E_f = -\tfrac{4}{3} \overline{E}. \tag{12}$$

For $\omega > \omega_0$, $E_f \gg E_V$, the scattering does not depend on the length a, and we can set $a = \infty$, $a^{-1} = 0$, i.e., consider resonance scattering. Then the problem contains no dimensionless parameters. From dimensional considerations it follows that in this region

$$E_1 \sim U_1 \sim \overline{E} \sim E_f \sim \omega^{2/3}. \tag{13}$$

The formula (11) for E_1 is in agreement with this requirement. But then the correction to E_1 because of the influence of a third neutron on the wave functions of a given pair is also proportional to E_f, i.e., depends on the same power of the density and can differ from E_f and E_1 only by a numerical coefficient. This case is not like the usual one; in the Fermi gas at absolute zero with resonance scattering one cannot expand in a series of powers of the density.

We have not found the corrections for the interactions of three and more particles; it is quite possible that they will change the sign of E_1 in the region $\omega > \omega_0$. We know that $E_1 > 0$ for $\omega < \omega_0$. On the other hand, for values of ω approaching the density of ordinary nuclei it is to be expected that the energy will lie above that calculated from the resonance S scattering.* Therefore, if from an exact solution of the problem of the Fermi gas with resonance interaction it is found that $E_1 > 0$, this will mean that the existence of nuclei composed of neutrons only is impossible.

We note that the expression (11) for E_1 found by using the pair interaction is not the mathematical expectation of the energy, calculated with the unperturbed functions of the problem without interaction (otherwise we could assert that the true E_1 could only be lower than that so found); in the calculation of the interaction the change of the wave functions was taken into account from the very start (see beginning of Sec. 2). Actually the calculation of the energy of the pair interaction includes within itself the change of the wave function at the origin. We recall that $\rho(0)$ is the density that would exist in the absence of interaction; in the presence of the interaction we get for small r

$$\Psi \sim r^{-1}, \quad \rho = {}^1/_4\, \pi^2 (h^2/pr)^2 \rho(0).$$

It is obvious that the change of the density and the wave function (and consequently also of the momentum spectrum) affects the interaction of the pair under consideration with other particles. We note that with a finite change of the total energy in this way of treating the pair interaction the mathematical expectations of the kinetic and potential energies are infinite and of opposite signs.

Resonance scattering with a singular potential that is nonvanishing in a small region gives in the limit zero interaction in the first order, second order, and so on, in perturbation theory; a finite result is given only by the sum of an infinite number of terms (for details see reference 15). The expression for E_1 given above is not the first approximation of perturbation theory for a Fermi gas with pair interaction between the neutrons. E_1 is the result of including in a definite way a chosen infinite succession of the terms of the perturbation-theory series, and therefore it is not clear what is the sign of the correction to E_1. The assertion of Yang and Lee[16] that not only in a Bose gas, but also in a Fermi gas any attraction always leads to a condensation seems not to be well founded.

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*By the method described above we would get for nuclear matter consisting of equal numbers of neutrons and protons, with the Coulomb interaction neglected, the result $U_1 = -4\overline{E}$; for the ordinary nuclear density this would give a binding energy $\sim$ 60 Mev, many times the experimental value.

[1] P. É. Nemirovskiĭ, JETP **36**, 889 (1959), Soviet Phys. JETP **9**, 627 (1959).

[2] P. É. Nemirovskiĭ, JETP **33**, 746 (1957), Soviet Phys. JETP **6**, 573 (1958).

[3] A. I. Baz', Атомная энергия (Atomic Energy) **6**, 571 (1959).

[4] N. Jarmie and M. G. Silbert, Phys. Rev. Letters **3**, 50 (1959).

[5] D. Kurath, Phys. Rev. **88**, 804 (1952).

[6] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).

[7] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

[8] D. M. Van Patter and W. Whaling, Revs. Modern Phys. **26**, 402 (1954).

[9] C. H. Blanchard and R. G. Winter, Phys. Rev. **107**, 774 (1957).

[10] Ya. B. Zel'dovich, JETP **38**, 278 (1960), Soviet Phys. JETP **11**, 202 (1960).

[11] J. A. Wheeler, in Niels Bohr and the Development of Physics (Russian Translation), IIL, 1958.

[12] H. Bethe and P. Morrison, Elementary Nuclear Theory (Russian Translation) IIL, 1958.

[13] L. D. Landau and E. M. Lifshitz, Статистическая физика (Statistical Physics), Gostekhizdat, 1951.

[14] K. Huang and C. N. Yang, Phys. Rev. **105**, 767 (1957).

[15] Ya. B. Zel'dovich, JETP **38**, 278 (1960), Soviet Phys. JETP **11**, 202 (1960).

[16] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1119 (1957).

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