POLARIZATION EFFECTS IN THE SCATTERING OF ELECTRONS BY DEUTERONS

G. V. FROLOV

Radium Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 8, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 1148-1152 (April, 1960)

The differential cross section and the change of polarization of the electrons are calculated for the process of disintegration of polarized deuterons by polarized electrons, with the electromagnetic form factors of the nucleons taken into account. An expression is also obtained for the polarization of recoil deuterons from the elastic scattering of polarized electrons by unpolarized deuterons.

In a previous paper[1] the differential cross section and the change of polarization of the electrons were calculated for the elastic scattering of polarized electrons by polarized deuterons. In the present paper we consider the differential cross section and the change of the polarization of the electrons in the disintegration of polarized deuterons by polarized electrons, and also obtain formulas for the polarization of the recoil deuterons from the elastic scattering of polarized electrons by unpolarized deuterons. The deuterons are assumed to be nonrelativistic, and the admixture of D wave in the ground state of the deuteron is neglected.

Treating the interaction of an electron with a deuteron as the interaction with the bound neutron and proton, as was done in reference 1, we have for the matrix element of the interaction:

$$S_{if} = e^2 q^{-2} \langle \chi_f | S | \chi_i \rangle. \tag{1}$$

Here $q = p_1 - p_2$ is the change of the four-momentum of the electron, χ_f and χ_i are the spin functions of the final and initial states of the deuteron,

$$S = A + \mathbf{B}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2 + \mathbf{C}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)/2,$$

$$A = I_1(g_4 - i\mathbf{g}\mathbf{q}/2M) - \mathbf{I}_2\mathbf{g}/2M, \qquad \mathbf{B} = I_3[\mathbf{q}\times\mathbf{g}],$$

$$\mathbf{C} = I_4[\mathbf{q}\times\mathbf{g}], \qquad g_\mu = (\bar{u}_2\gamma_\mu u_1), \tag{2}$$

u_2 and u_1 are spinors for the electron,

$$I_1 = \int \varphi^*_{\varkappa t}(a_p e^{i\mathbf{qr}/2} + a_n e^{-i\mathbf{qr}/2})\varphi_d\, d\mathbf{r},$$

$$\mathbf{I}_2 = 2\int \varphi^*_{\varkappa t}(a_p e^{i\mathbf{qr}/2} - a_n e^{-i\mathbf{qr}/2})\nabla\varphi_d\, d\mathbf{r},$$

$$I_3 = \frac{1}{2M}\int \varphi^*_{\varkappa t}[(a_p + b_p)e^{i\mathbf{qr}/2} + (a_n + b_n)e^{-i\mathbf{qr}/2}]\varphi_d\, d\mathbf{r},$$

$$I_4 = \frac{1}{2M}\int \varphi^*_{\varkappa s}[(a_p + b_p)e^{i\mathbf{qr}/2} - (a_n + b_n)e^{-i\mathbf{qr}/2}]\varphi_d\, d\mathbf{r},$$

a_p, a_n and b_p, b_n are the electric and magnetic form factors of the proton and neutron,[2] $\mathbf{r}$ is the relative coordinate of the nucleons, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the spin operators of the proton and neutron, $\varphi_{\kappa t}$ and $\varphi_{\kappa s}$ are the coordinate functions of the triplet and singlet states of the two nucleons, φ_d is the coordinate function of the deuteron, and M is the mass of a nucleon.

The initial state is described by a density matrix which is the direct product of the density matrices of the electron and deuteron:

$$\rho_0 = \rho_e \times \rho_d,$$

where

$$\rho_e = \tfrac{1}{2}(1 + i\gamma_5\hat{\zeta}_1)\eta^{(+)}(p_1),$$

$$\rho_d = \tfrac{1}{4}[1 + \tfrac{1}{3}\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 + \boldsymbol{\alpha}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) + \beta_{im}(\sigma_{1i}\sigma_{2m} + \sigma_{2i}\sigma_{1m})],$$

$\eta^{(+)}(p) = (i\hat{p} - m)\gamma_4/2\epsilon$ is the projection operator that selects positive-energy states. The four-vector $\zeta_\mu = (\boldsymbol{\zeta}, \zeta_4)$ characterizes the polarization of the electron.

In the rest system $\zeta_\mu = (\boldsymbol{\zeta}^0, 0)$; in an arbitrary reference system

$$\boldsymbol{\zeta} = \boldsymbol{\zeta}^0_t + \boldsymbol{\zeta}^0_l \varepsilon/m, \qquad \zeta_4 = i\,\mathbf{p}\boldsymbol{\zeta}^{(0)}/m,$$

where $\boldsymbol{\zeta}^0_t$ and $\boldsymbol{\zeta}^0_l$ are the transverse and longitudinal components of the vector $\boldsymbol{\zeta}^0$; $\mathbf{p}$ is the momentum, ϵ the energy, and m the mass of the electron; $\boldsymbol{\alpha}$ and β_{im} characterize the polarization of the deuteron; $\beta_{im} = \beta_{mi}$; $\mathrm{Sp}\,\beta_{im} = \beta_{ii} = 0$. In the present case the concept of polarization also includes what is ordinarily called alignment.

THE DIFFERENTIAL CROSS SECTION FOR INELASTIC SCATTERING

The differential cross section for disintegration of the deuteron which corresponds to the relative momentum κ of the nucleons in the final state has the following form:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{e^2}{4\pi}\right)^2 \frac{\mathrm{Sp}\{\eta^{(+)}(p_2)\, S\rho_0 S^+\}}{[1 + \xi\sin^2(\vartheta/2)]\,\varepsilon_1^2\sin^4(\vartheta/2)}\frac{d\boldsymbol{\varkappa}}{(2\pi)^3}. \tag{3}$$

Here ϑ is the angle of scattering of the electron, and $\xi = \epsilon_1/M$.

For a prescribed scattering angle the energy spectrum of the electrons has a sharp peak. Therefore for the range of energies that makes the main contribution to the cross section, q does not change to any important extent. Regarding q as constant, one can easily carry out the integration over κ, using in an approximate way the completeness of the functions $\varphi_{\kappa s}$ and $\varphi_{\kappa t}$. Performing also the summation over the spin variables of the nucleons and the electron, we have

$$\int \frac{d\varkappa}{(2\pi)^3} \mathrm{Sp}\,\{\eta^{(+)}(p_2)\; S\rho_0 S^+\} = \overline{|I_1|^2}\cos^2\frac{\vartheta}{2} - \frac{m\overline{I_1 I_3}}{\varepsilon_1\varepsilon_2} - [(\boldsymbol{\alpha}\boldsymbol{\zeta})\,\mathbf{q}^2 - (\boldsymbol{\alpha}\mathbf{q})(\boldsymbol{\zeta}\mathbf{q})] + \frac{\mathbf{q}^2}{3}\left(1 + \sin^2\frac{\vartheta}{2}\right)(2\overline{|I_3|^2} + \overline{|I_4|^2})$$
$$+ \frac{im}{2\varepsilon_1\varepsilon_2}(\overline{|I_3|^2} + \overline{|I_4|^2})(\boldsymbol{\alpha}\mathbf{q})[\zeta_4\mathbf{q}^2 - i(\varepsilon_1 - \varepsilon_2)(\boldsymbol{\zeta}\mathbf{q})] + \frac{(\overline{|I_3|^2} - \overline{|I_4|^2})}{\varepsilon_1\varepsilon_2}\beta_{lm}\left\{\varepsilon_1\varepsilon_2\cos^2\frac{\vartheta}{2}(p_{1l}p_{2m} + p_{2l}p_{1m})\right.$$
$$\left. - \varepsilon_2\left(\varepsilon_2 + \varepsilon_1\sin^2\frac{\vartheta}{2}\right)p_{1l}p_{1m} - \varepsilon_1\left(\varepsilon_1 + \varepsilon_2\sin^2\frac{\vartheta}{2}\right)p_{2l}p_{2m}\right\}.$$

Here

$$\overline{|I_1|^2} = \int\frac{d\varkappa}{(2\pi)^3}|I_1|^2 = a_p^2 + a_n^2 + 2a_p a_n f(2q) - (a_p + a_n)^2 f^2(q), \qquad \overline{I_1 I_3} = \int\frac{d\varkappa}{(2\pi)^3}\mathrm{Re}\,(I_1 I_3^*) = \frac{1}{2M}\{a_p(a_p + b_p) + a_n(a_n + b_n)$$
$$+ [a_p(a_n + b_n) + a_n(a_p + b_p)]\,f(2q) - (a_p + a_n)(a_p + a_n + b_p + b_n)\,f^2(q)\},$$
$$\overline{|I_3|^2} = (2M)^{-2}\{(a_p + b_p)^2 + (a_n + b_n)^2 + 2(a_p + b_p)(a_n + b_n)\,f(2q) - (a_p + a_n + b_p + b_n)^2 f^2(q)\},$$
$$\overline{|I_4|^2} = (2M)^{-2}\{(a_p + b_p)^2 + (a_n + b_n)^2 - 2(a_p + b_p)(a_n + b_n)\,f(2q)\}. \tag{4}$$

The form factor of the deuteron is

$$f(q) = \int|\varphi_d|^2 e^{i\mathbf{q}\mathbf{r}/2}\,d\mathbf{r} = \int_0^\infty u^2\frac{\sin(qr/2)}{qr/2}\,dr,$$

where $\varphi_d = u/(4\pi)^{1/2}r$. In particular, if $\psi_d = (\gamma/2\pi)^{1/2}e^{-\gamma r}/r$, the form factor is

$$f(q) = \frac{4\gamma}{q}\tan^{-1}\frac{q}{4\gamma}.$$

From these expressions it can be seen that the difference $\overline{|I_3|^2} - \overline{|I_4|^2}$ involves only terms containing $f^2(q)$ and $f(2q)$. These quantities are small for the changes of momentum that are of interest here, and therefore the contribution to the cross section from the "tensor" polarization is also small.

The final expression for the differential cross section for disintegration accompanied by scattering of the electron by the angle ϑ can be conveniently written with the following choice of coordinate axes:

$$\mathbf{k} = \mathbf{p}_1/|\mathbf{p}_1|, \qquad \mathbf{n} = [\mathbf{p}_1\times\mathbf{p}_2]/|[\mathbf{p}_1\times\mathbf{p}_2]|, \qquad \mathbf{l} = [\mathbf{k}\times\mathbf{n}]. \tag{5}$$

We get

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0(1 - N/N_0), \tag{6}$$

where

$$N_0 = \overline{|I_1|^2} + \frac{\mathbf{q}^2}{3}\left(1 + 2\tan^2\frac{\vartheta}{2}\right)(2\overline{|I_3|^2} + \overline{|I_4|^2}), \qquad N = 2\overline{I_1 I_3}(\boldsymbol{\zeta}_1^0\mathbf{k})\tan\frac{\vartheta}{2}[(\boldsymbol{\alpha}\mathbf{k})\,\varepsilon_2\sin\vartheta - (\boldsymbol{\alpha}\mathbf{l})(\varepsilon_1 - \varepsilon_2\cos\vartheta)]$$
$$+ (\overline{|I_3|^2} + \overline{|I_4|^2})\tan\frac{\vartheta}{2}(\boldsymbol{\zeta}_1^0\mathbf{k})(\varepsilon_1 + \varepsilon_2)[(\boldsymbol{\alpha}\mathbf{k})(\varepsilon_1 - \varepsilon_2\cos\vartheta) + (\boldsymbol{\alpha}\mathbf{l})\,\varepsilon_2\sin\vartheta]$$
$$+ (\overline{|I_3|^2} - \overline{|I_4|^2})\sin^2\frac{\vartheta}{2}\beta_{im}\left\{k_i k_m\left(1 + \tan^2\frac{\vartheta}{2}\right)(\varepsilon_1^2 + 2\varepsilon_1\varepsilon_2 + \varepsilon_2^2\cos^2\vartheta) + 4l_i l_m\varepsilon_2\left(\varepsilon_1 + \varepsilon_2\sin^2\frac{\vartheta}{2}\right)\right.$$
$$\left. + 4l_i k_m\varepsilon_2\tan\frac{\vartheta}{2}(\varepsilon_1 - \varepsilon_2\cos\vartheta)\right\}. \tag{7}$$

The differential cross section for unpolarized particles is

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{4}\left(\frac{e^2}{4\pi}\right)^2\frac{N_0\cos^2(\vartheta/2)}{[1 + \xi\sin^2(\vartheta/2)]\,\varepsilon_1^2\sin^4(\vartheta/2)}. \tag{8}$$

It has been assumed everywhere in the calculations that

$$\varepsilon_1 \gg m, \qquad \vartheta \gg m/\varepsilon_1.$$

The formula (8) goes over into the expression obtained by Jankus[3] for point nucleons if in the expression for N_0 we neglect $f(2q)$ and in the term with q^2 we also neglect $f^2(q)$.

From the expressions that have been obtained it can be seen that the differential cross section does not depend on the transverse component of the polarization of the electrons. If the electrons are not polarized, only the last term remains in (7). In this case the cross section differs little from $(d\sigma/d\Omega)_0$.

THE CHANGE OF POLARIZATION OF THE ELECTRONS IN INELASTIC SCATTERING

The basic expression for the final polarization $\boldsymbol{\zeta}_2$ of the electrons from disintegration of polarized deuterons by electrons of polarization $\boldsymbol{\zeta}_1$ is as follows:

$$\zeta_2 = \frac{i(\varepsilon_2/m)\,\mathrm{Sp}\,\{\gamma\gamma_4\gamma_5\eta^{(+)}(p_2)\,Sp_0S^+\eta^{(+)}(p_2)\}d\varkappa}{\mathrm{Sp}\,\{\eta^{(+)}(p_2)\,Sp_0S^+\}\,d\varkappa}. \qquad (9)$$

To determine the polarization of those electrons that have been scattered through the angle ϑ, independent of the final state of the nucleons, we must carry out an integration over $\varkappa$. This is done in just the same way as before.

We present the result:

$$\zeta_2^0 = \mathbf{k}\,[C_{11}(\zeta_1^0\mathbf{k}) + C_{12}(\zeta_1^0\mathbf{n}) + C_{13}(\zeta_1^0\mathbf{l}) + D_{11}(\boldsymbol{\alpha}\mathbf{k}) + D_{13}(\boldsymbol{\alpha}\mathbf{l})] + \mathbf{n}\,[C_{22}(\zeta_1^0\mathbf{n}) + C_{23}(\zeta_1^0\mathbf{l})] + \mathbf{l}\,[C_{31}(\zeta_1^0\mathbf{k}) + C_{32}(\zeta_1^0\mathbf{n}) + C_{33}(\zeta_1^0\mathbf{l}) + D_{31}(\boldsymbol{\alpha}\mathbf{k}) + D_{33}(\boldsymbol{\alpha}\mathbf{l})]. \qquad (10)$$

The coefficients appearing in the formula (10) have the following values:

$$C_{11} = \frac{\cos\vartheta}{N_0 - N}\Big\{\overline{|I_1|^2} + \frac{q^2}{3}(2\overline{|I_3|^2} + \overline{|I_4|^2})\Big(1 + 2\tan^2\frac{\vartheta}{2}\Big) - 2\varepsilon_2\tan^2\frac{\vartheta}{2}(\overline{|I_3|^2} - \overline{|I_4|^2})\,\beta_{im}\Big[l_ik_m\sin\vartheta\,(\varepsilon_1 - \varepsilon_2\cos\vartheta) + 2k_ik_m\Big(\varepsilon_1 - \frac{1}{4}\varepsilon_2\sin^2\vartheta\Big) + 2l_il_m\cos^2\frac{\vartheta}{2}\Big(\varepsilon_1 + \varepsilon_2\sin^2\frac{\vartheta}{2}\Big)\Big]\Big\},$$

$$C_{22} = \frac{1}{N_0 - N}\Big\{\overline{|I_1|^2} + \frac{q^2}{3}(2\overline{|I_3|^2} + \overline{|I_4|^2}) - 2\varepsilon_2\tan^2\frac{\vartheta}{2}(\overline{|I_3|^2} - \overline{|I_4|^2})\,\beta_{im}\Big[2k_ik_m\Big(\varepsilon_1 + \frac{1}{4}\varepsilon_2\sin^2\vartheta\Big) + 2l_il_m\Big[\varepsilon_1\Big(1 + \sin^2\frac{\vartheta}{2}\Big) - \frac{1}{4}\varepsilon_2\sin^2\vartheta\Big] - l_ik_m\sin\vartheta\,(\varepsilon_1 - \varepsilon_2\cos\vartheta)\Big]\Big\},$$

$$C_{23} = \frac{2\varepsilon_2\tan(\vartheta/2)\,(\overline{|I_3|^2} - \overline{|I_4|^2})\,\beta_{im}}{N_0 - N}\Big\{2n_il_m\Big[\varepsilon_1\Big(1 + \sin^2\frac{\vartheta}{2}\Big) - \varepsilon_2\cos^2\frac{\vartheta}{2}\Big] - n_ik_m\sin\vartheta\,(\varepsilon_1 + \varepsilon_2)\Big\},$$

$$D_{11} = -\frac{\cos\vartheta\tan(\vartheta/2)}{N_0 - N}\Big\{2\overline{I_1I_3}\varepsilon_2\sin\vartheta + (\overline{|I_3|^2} + \overline{|I_4|^2})\times\tan\frac{\vartheta}{2}(\varepsilon_1 + \varepsilon_2)(\varepsilon_1 - \varepsilon_2\cos\vartheta)\Big\},$$

$$D_{13} = \frac{2\cos\vartheta\tan(\vartheta/2)}{N_0 - N}\{\overline{I_1I_3}(\varepsilon_1 - \varepsilon_2\cos\vartheta) - (\overline{|I_3|^2} + \overline{|I_4|^2})\times\sin^2\frac{\vartheta}{2}\varepsilon_2(\varepsilon_1 + \varepsilon_2)\};$$

$$C_{13} = C_{22}\sin\vartheta, \quad C_{12} = -C_{23}\sin\vartheta, \quad C_{31} = -C_{11}\tan\vartheta,$$

$$D_{33} = -D_{13}\tan\vartheta, \quad C_{33} = C_{22}\cos\vartheta, \quad C_{32} = -C_{23}\cos\vartheta,$$

$$D_{31} = -D_{11}\tan\vartheta.$$

From these expressions it can be seen that the "tensor" polarization of the deuteron gives a small contribution to the polarization of the scattered electron. It is more convenient to express the final polarization in the coordinate system connected with the scattered electron:

$$\zeta_{2l} = (\zeta_2\mathbf{p}_2)/|\mathbf{p}_2|, \qquad \zeta_{2t}^{\perp} = (\zeta_2\mathbf{n}), \qquad \zeta_{2t}^{\parallel} = (\zeta_2[\mathbf{p}_2\times\mathbf{n}])/|\mathbf{p}_2|;$$

ζ_{2l} is the longitudinal component; $\zeta_{2t}^{\perp}$ is the transverse component perpendicular to the plane of the reaction; $\zeta_{2t}^{\parallel}$ is the transverse component in the plane of the reaction. For the incident electron

$$\zeta_{1l} = (\zeta_1\mathbf{k}), \qquad \zeta_{1t}^{\perp} = (\zeta_1\mathbf{n}), \qquad \zeta_{1t}^{\parallel} = (\zeta_1\mathbf{l}).$$

In this notation the result can be written in the following way:

$$\zeta_{2l}^0 = [C_{11}\zeta_{1l}^0 + (\boldsymbol{\alpha}\mathbf{k})\,D_{11} + (\boldsymbol{\alpha}\mathbf{l})\,D_{13}]/\cos\vartheta,$$
$$\zeta_{2t}^{0\perp} = C_{22}\zeta_{1t}^{0\perp} + C_{23}\zeta_{1t}^{0\parallel},$$
$$\zeta_{2t}^{0\parallel} = C_{22}\zeta_{1t}^{0\parallel} - C_{23}\zeta_{1t}^{0\perp}.$$

It can be seen that in the scattering the longitudinal and transverse components change independently of each other. If the deuteron is originally unpolarized, we get

$$\zeta_{2l}^0 = \zeta_{1l}^0,$$
$$\zeta_{2t}^{0\parallel} = \zeta_{1t}^{0\parallel}\{1 - 2q^2\tan^2(\vartheta/2)\,(2\overline{|I_3|^2} + \overline{|I_4|^2})/3N_0\},$$
$$\zeta_{2t}^{0\perp} = \zeta_{1t}^{0\perp}\{1 - 2q^2\tan^2(\vartheta/2)\,(2\overline{|I_3|^2} + \overline{|I_4|^2})/3N_0\}.$$

In this case the longitudinal component of the polarization of the electron does not change in the scattering, and both transverse components change in the same way. If the electron is originally unpolarized, we have

$$\zeta_{2l}^0 = [(\boldsymbol{\alpha}\mathbf{k})\,D_{11} + (\boldsymbol{\alpha}\mathbf{l})\,D_{13}]/\cos\vartheta, \qquad \zeta_{2t}^{0\perp} = \zeta_{2t}^{0\parallel} = 0.$$

Longitudinally polarized electrons are produced.

Comparison of these expressions with experimental data can give additional information about the form factors of nucleons.

POLARIZATION OF THE RECOIL DEUTERONS FROM ELASTIC SCATTERING

The following expressions are obtained for the quantities α and β_{im} that characterize the polarization of the recoil deuterons from elastic scattering of polarized electrons by unpolarized deuterons:

$$(\boldsymbol{\alpha}\mathbf{k}) = -\frac{4\varepsilon_2\sin^2(\vartheta/2)\,(a_p + a_n + b_p + b_n)}{3MR_0}(\zeta_1^0\mathbf{k})\Big\{a_p + a_n - \frac{\varepsilon_1(a_p + a_n + b_p + b_n)}{4M}\Big(2\tan^2\frac{\vartheta}{2} + \xi\sin^2\frac{\vartheta}{2}\Big)\Big\},$$

$$(\boldsymbol{\alpha}\mathbf{l}) = \frac{2\varepsilon_2\sin^2(\vartheta/2)\tan(\vartheta/2)\,(a_p + a_n + b_p + b_n)}{3MR_0}(\zeta_1^0\mathbf{k})\Big\{(a_p + a_n)\times(2 + \xi) + \frac{\varepsilon_1(a_p + a_n + b_p + b_n)}{2M}\Big(2 - \xi\sin^2\frac{\vartheta}{2}\Big)\Big\},$$

$$\beta_{im}k_ik_m = \frac{q^2(a_p + a_n + b_p + b_n)^2}{12M^2R_0}\Big[\sin^2\frac{\vartheta}{2} - \frac{1}{3}\Big(1 + 2\tan^2\frac{\vartheta}{2}\Big) - \xi\sin^4\frac{\vartheta}{2}\Big],$$

$$\beta_{im}l_il_m = \frac{q^2(a_p + a_n + b_p + b_n)^2}{12M^2R_0}\Big[\frac{1}{3}\Big(\tan^2\frac{\vartheta}{2} - 1\Big) - \sin^2\frac{\vartheta}{2} + \xi\sin^4\frac{\vartheta}{2}\Big],$$

$$\beta_{im}l_ik_m = -\frac{q^2(a_p + a_n + b_p + b_n)^2}{24M^2R_0}\tan\frac{\vartheta}{2}\sin^2\frac{\vartheta}{2}(2 + \xi\cos\vartheta),$$

$$\beta_{im}n_in_m = -\beta_{im}k_ik_m - \beta_{im}l_il_m.$$

Here

$$R_0 = (a_p + a_n)^2 - \frac{q^2(a_p + a_n + b_p + b_n)^2}{6M^2}\Big(1 - \frac{2}{\cos^2(\vartheta/2)}\Big).$$

The other notations are the same as before.

From these expressions it can be seen that the average value $\frac{1}{2}\alpha$ of the spin of the recoil deuteron lies in the plane of the scattering. α is zero if the incident electrons are unpolarized, but the quantities β_{im} are different from zero even in this case.

Therefore experiments to determine the "tensor" polarization of the recoil deuterons are in a certain sense the simplest of the polarization experiments on the elastic scattering of electrons by deuterons.

[1] G. V. Frolov, JETP **37**, 522 (1959), Soviet Phys. JETP **10**, 369 (1960).

[2] Akhiezer, Rozentsveĭg, and Shmushkevich, JETP **33**, 765 (1957), Soviet Phys. JETP **6**, 588 (1958).

[3] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).

Translated by W. H. Furry
225