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SCATTERING OF ELECTRONS BY NUCLEI ACCORDING TO THE α-PARTICLE MODEL

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Elastic and inelastic (including excitation of rotational levels) scattering of high-energy electrons by Be^9, C^{12}, and O^{16} nuclei is considered on the basis of the α-particle model. The calculated differential cross sections for elastic scattering on these nuclei, and also the inelastic scattering with excitation of the 2.43- and 6.8-Mev levels in Be and the 4.43-Mev level in C^{12} are in good agreement with the experiments. It is also shown that elastic scattering on C^{12} with excitation of the 9.61-Mev level can be explained within the framework of the model under consideration if one ascribes spin and parity 3^- to this level.

1. INTRODUCTION

A number of experimental facts obtained in the last few years indicate the correctness of the description, in the case of several nuclei at least, in terms of the α-particle model. Thus, for example, Baz' noted[1] that the curve of neutron separation energies has, as function of atomic number A, a sawtoothed shape in the region $4 \leq A \leq 40$, such that each nucleus of type $A = 4n$ corresponds to a maximum in this curve, and each nucleus of type $A = 4n + 1$, to a minimum.

In experiments[2] on capture of high-energy neutrons, rather detailed information about the momentum distribution of nucleons in He^4, Be^9, and C^{12} was obtained. These experiments succeeded in separating off the contribution in the distribution for Be^9 coming from the weakly-bound ($E_B \cong 1.5$ Mev) external nucleon, so as to give the distribution for the Be^8 core. All three of the distributions obtained were very similar; this is easy to explain, using the α-particle model, since this model leads directly to identical distributions, aside from effects from the relatively slow movement of α particles in these nuclei.

Results of experiments on (p, α) reactions by high-energy protons (see reference 3; references to other work on the same subject are given there) are also of great interest. It turned out that particular features of this process gave a direct indication of the existence of strongly-correlated α-particle groups in both light and heavy nuclei.

In connection with further study of the existence of strong α correlations in nuclei, it is of interest to consider other experiments from this point of view. Experiments on elastic and inelastic scattering of high-energy electrons by nuclei would contribute greatly to this.

In the present work, expressions are obtained on the basis of the α-particle model for elastic and inelastic scattering of electrons, and experimental data relating to Be^9, C^{12}, and O^{16} is analyzed.

2. CROSS SECTIONS FOR ELASTIC AND INELASTIC SCATTERING OF ELECTRONS BY Be^9, C^{12}, AND O^{16}

In connection with the basic assumption that a class of nuclei with a clear α-particle structure exists, we shall assume that the target nucleus can be represented by a system of α particles, the positions of which are fixed relative to each other. This system can begin to rotate as a whole because of the action of the field of the incident electron. The rotational states will, as usual, be characterized by quantum numbers I and K (I is the angular momentum of the rotational state, K the projection on the axis of maximum symmetry of the system considered). The problem consists in calculating $\sigma_{IK}(\theta)$, the cross section for the process in which the electron is scattered through an angle θ and the nucleus goes into the rotational state I, K. Here, the only transitions considered are those in which the internal structure of the α particle is preserved.

Calculations in the Born approximation, which is adequate for treating the scattering of electrons by light nuclei, can be carried through in all of the cases considered below. The cross section for elastic scattering on α particles, $\sigma_\alpha(\theta)$, enters, of course, as a multiplicative factor into the expression for the derived cross section. It can be written as $\sigma_\alpha(\theta) = \sigma_\alpha^C(\theta) F_\alpha(q)$, where σ_α^C is the cross section for Coulomb scattering by a point α particle. According to the data of Hofstadter et al.,[4] the form-factor for the α particle is $F_\alpha(q) = \exp(-q^2a^2/6)$ with a mean-square radius $a = 1.61$ f.

Be^9. The simplest α-particle nucleus, Be^8, is unstable; however, our considerations can be applied to Be^9, which can be viewed as consisting of two α particles and an external neutron. The latter is a neutral particle, and does not affect the scattering directly. However, the presence of the neutron influences the rotational functions of the nucleus and, in this way, the results of interaction of the electron with the nucleus.

The spin of Be^9 is $I_0 = 3/2$; therefore it follows that we should take $K_0 = 3/2$. Excited states will be characterized by the quantum numbers $K = K_0 = 3/2$ and $I = 5/2, 7/2$, etc. Denoting the distance between α particles by 2d, we obtain the following expressions for the cross sections for elastic and inelastic scattering with excitation of the $I = 5/2$ and $I = 7/2$ levels:

$$\sigma_{3/2,\,3/2}(\theta) = 4\sigma_\alpha(\theta)\,[j_0^2(qd) + j_2^2(qd)], \tag{1}$$

$$\sigma_{5/2,\,3/2}(\theta) = \frac{72}{7}\sigma_\alpha(\theta)\,[j_2^2(qd) + \frac{1}{6} j_4^2(qd)], \tag{2}$$

$$\sigma_{7/2,\,3/2}(\theta) = \frac{40}{7}\sigma_\alpha(\theta)\,[j_2^2(qd) + \frac{9}{5} j_4^2(qd)], \tag{3}$$

where j_l is the spherical Bessel function and q is the momentum transfer. For electron energies below ~ 200 Mev, the terms involving j_4 in Eqs. (2) and (3) are small, and, therefore, we can write, with good accuracy,

$$\sigma_{5/2,\,3/2}(\theta) = \frac{9}{5}\sigma_{7/2,\,3/2}(\theta). \tag{4}$$

We consider now the corresponding experimental data. In the work of McIntyre et al.,[5] the angular distributions of elastic and inelastic scattering, with excitation of the 2.5- and 6.8-Mev levels of Be^9, were measured at angles of 60 — 90° with 190-Mev electrons. The 2.5-Mev level appears[6] to have spin and parity $5/2^-$; the 6.8-Mev level must be $7/2^-$ if it is to be in the same rotational band. Such an assumption is confirmed by the ratio of energies, 6.8 : 2.5 = 2.7, which is near to the value of 12 : 5 = 2.4 obtained from the formula

$$E_I = (\hbar^2/2J)\,[I(I+1) - I_0(I_0+1)].$$

Comparison of calculated and experimental data is shown in Fig. 1. In so far as absolute values were not measured, the choice of the general scale is made by setting the calculated value

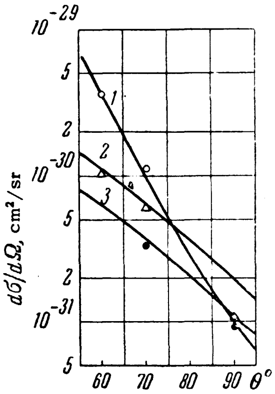


FIG. 1. Differential cross sections for elastic and inelastic scattering of 190-Mev electrons by beryllium: 1 – calculated curve for elastic scattering; 2 – inelastic scattering with excitation of the 2.5-Mev level; 3 – calculated curve for inelastic scattering with excitation of the 6.8-Mev level; o, Δ, ● – experimental values for the corresponding processes.

equal to the experimental point for elastic scattering at $\theta = 60°$. The best value for d turned out to be d = 1.9 f. We see that there is good agreement with experiment. A noticeable divergence occurs only in the case of curve 2 for $\theta = 90°$. The preliminary character of the experimental data makes it difficult to decide what importance to attribute to this difference.

If we choose d = 1.9 f, we obtain a = 2.48 f for the mean-square radius of Be^9, in reasonable agreement with the value $a = 2.2 \pm 0.2$ f given by Hofstadter in his last review.[7] For the intrinsic quadrupole moment we obtain $Q_0 = 8d^2 = 0.29$ barn, from which

$$Q = Q_0 I_0 (2I_0 - 1)/(I_0+1)(2I_0+3) = 0.058 \text{ barn}.$$

Unfortunately, no reliable data on the quadrupole moment of Be^9 exists. In references 8 and 9, rough estimates give $|Q| = 0.02$ barn.

Thus, the existing experimental data on Be^9 does not contradict the proposed treatment; however, it would be desirable to have more detailed and accurate experimental points for both the elastic and inelastic scattering of electrons by this nucleus.

C^{12}. We will consider the carbon nucleus as consisting of three α particles, which form an isosceles triangle. The axis of maximum symmetry is, clearly, a line going through the center of the triangle and perpendicular to its plane.

Since the symmetry axis of this system is not of infinite order, not only rotational states with $K = 0$ can exist, but also those with $K > 0$. Invariance of this system under rotations through $2\pi/3$ limits the acceptable values of K to $K = 3n$, where n is an integer. The parity Π of the rotational levels is connected with K by the relation $\Pi = (-1)^K$. The energy of the rotational level is determined by the formula

$$E_K^I = \frac{\hbar^2}{2J_1} I(I+1) + \frac{\hbar^2}{2}\left(\frac{1}{J_2} - \frac{1}{J_1}\right) K^2, \tag{5}$$

where J_1 is the moment of inertia relative to the two-fold symmetry axis and J_2 is the moment of inertia relative to the three-fold symmetry axis..

The levels of carbon have been analyzed with the α-particle model by Glassgold and Galonsky.[10] The carbon levels 0^+ ($E = 0$) and 2^+ ($E = 4.43$ Mev) can be related to the rotational band with $K = 0$; then the 3^- level (first level in the band with $K = 3$) is obtained at 5.53 Mev. Since this 5.53-Mev level has not been observed to date, this conclusion was considered by these authors to be a grave defect of this theory.

It should be noted, however, that this conclusion is based on a classical calculation of the moment of inertia: the α particles were considered to be points, so that $J_1 = J_2/2$. It is known from the study of the rotational spectra of heavy nuclei that J_{rigid} is often several times larger than the effective moment of inertia J. The situation may be similar in our case, leading to a strong decrease in J_2 compared to its classical value. Then the 3^- level would appear substantially higher. One can, in particular, assume this level to be the 9.61-Mev level.

The existing experimental data do not contradict this assumption. From the fact that the 9.61-Mev level decays into Be^8 and α particles, it follows that the spin and parity of this level should be either both even or both odd. It is clear that this requirement is satisfied by the assignment 3^-. The stripping reaction $B^{11}(d, n)$ with excitation of the 3^- level of C^{12} should give the value $l = 2$. This value was obtained in reference 11. It should be noted that these data, while not contradicting the value 3^-, also are consistent with the value 1^-.

The following expressions are obtained for the cross sections for elastic scattering and excitation of 2^+ and 3^- levels:

$$\sigma_{00}(\theta) = 9\sigma_\alpha(\theta)\, j_0^2(x), \quad \sigma_{20}(\theta) = \tfrac{45}{4}\sigma_\alpha(\theta)\,[3j_1(x)/x - j_0(x)]^2,$$
$$\sigma_{33}(\theta) = \tfrac{315}{16}\sigma_\alpha(\theta)\, j_3^2(x), \qquad x = 2qd/\sqrt{3}. \tag{6}$$

Comparison of cross sections calculated from Eq. (6) with the experimental data of Fregeau are shown in Fig. 2, for an electron energy of 187 Mev. The best value of the parameter d was d = 1.5. We see that the calculated curves for the elastic scattering and for the inelastic scattering with excitation of the 4.43-Mev level agree well with experiment. The curve describing the inelastic scattering with excitation of the 9.61-Mev level is in generally satisfactory agreement with experiment. It diverges in the small-angle ($\theta \leq 50°$) region; however, the experimental data in this region are not reliable. This can be seen both

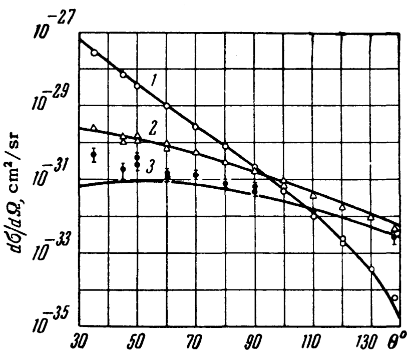
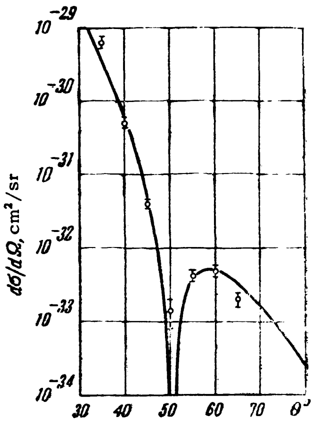


FIG. 2. Differential cross sections for elastic and inelastic scattering of 187-Mev electrons on carbon: 1 – calculated curve for elastic scattering; 2 – inelastic scattering with excitation of the 4.43-Mev level; 3 – inelastic scattering with excitation of the 9.61-Mev level; o, ▲, ● – experimental values for the corresponding processes.

from the irregular trend of the experimental points for θ = 35, 45, and 50°, and from the form of the energy spectra at small angles given in reference 12. The data on the elastic scattering of 420 Mev electrons is compared with calculations (which used the same value d = 1.5 f) on Fig. 3.

FIG. 3. Calculated curve and experimental values of the differential cross section for elastic scattering of 420-Mev electrons by carbon.



We see that the agreement between theory and experiment is good even for large values of the momentum transfer q. It should be noted that the vanishing of the cross section at $\theta = 51°$ comes from the incorrectness of the Born approximation in the region of a diffraction minimum — an accurate calculation should lead to a nonzero value of the cross section.

O^{16}. Dennison[13] was successful in showing that the position and characteristics of many of the levels of the oxygen atom could be explained by application of the α-particle model to this nucleus. If we consider this nucleus to be composed of α particles forming a tetrahedron, we obtain the following expression for the elastic-scattering cross section:

$$\sigma_{00}(\theta) = 16\,\sigma_\alpha(\theta)\, j_0^2\,(\sqrt{3/2}\; qd). \tag{7}$$

Comparison with the experiment of Hofstadter[7] (see Fig. 4) shows that the theoretical curve reproduces the experimental data well (using d = = 1.6 f) aside from the region of the diffraction minimum, where the Born approximation is inapplicable.

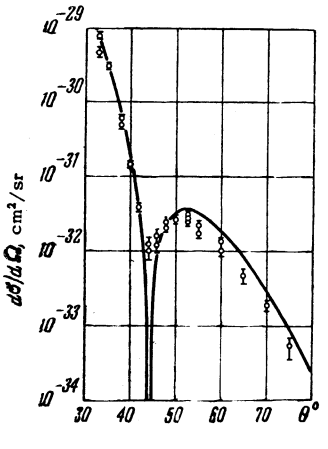


FIG. 4. Calculated curve and experimental values of the differential cross section for elastic scattering of 420-Mev electrons by oxygen.

3. CONCLUDING REMARKS

All in all, one can assert that the α-particle model is successful in explaining the existing experimental data on the scattering of electrons by Be^9, C^{12}, and O^{16}. Further experiments are necessary for definitive conclusions about the applicability of the α-particle model to these nuclei. In particular, a reliable experimental determination of the spin and parity of the 9.61-Mev level in C^{12} would be of great interest, since the existence of a 3^- state in nuclei consisting of three α particles is a very characteristic feature of such a model. It would also be important to establish the spin and parity of the 6.8-Mev level in Be^9.

In the preceding, we applied the terminology of the α-particle model in its most primitive form. However, it should be emphasized that the results obtained are independent of a number of assumptions characteristic of the primitive α-particle model. In particular, the α-particle groups entering into the composition of the nuclei considered do not have to be considered to be true α particles, preserving their individuality in the nucleus. For the considerations given here, only the type of density distribution is of basic importance. Thus, the symmetry D_{3h} assumed for C^{12} completely determines the rotational spectrum of this nucleus (0^+, 2^+, 3^-, etc.) and it is clear that to determine the rotational spectrum it is not necessary to resort to arguments connected with α particles that obey Bose statistics. From a mathematical point of view, this is connected with the fact that the group D_{3h} is isomorphous with the group formed by rearrangements of the isosceles triangle. Thus, the proposed considerations do not exclude the possibility of exchange of nucleons between the α-particle groups. From this point of view, the root mean square radius of the α-particle group a should be considered to be a parameter entering into the theory, and the result that the value of a obtained is close to the root mean square radius of the α particle should indicate that the deviation of the mean density in the α-particle group from that of the density of the free α particle is small.

It should be noted that the theoretical interpretation of the data on the scattering of electrons by the nuclei considered has been treated in a number of articles. Thus, Morpurgo[14] calculated the form-factor for the excitation of the 4.43-Mev state in C^{12}, using shell-model functions, in both the L-S and j-j coupling. His results gave too small an inelastic cross section (by a factor of two in the L-S coupling, and of six in the j-j coupling). Calculations for Be^9 and C^{12} were also carried out in intermediate coupling in the shell model,[15,16] but no satisfactory agreement with experiment was obtained there. Ferrell and Visscher[17] have shown that the data on C^{12} could be explained by adding an appropriate amount of the state corresponding to collective motion of the nucleus.

In the light of these results, the results obtained here appear to deserve attention, since, on the basis of a simple model, they give a very unambiguous (in so far as the data on elastic scattering already completely determine the parameters of the model) explanation of the experimental data on both elastic and inelastic scattering of several nuclei.

It is interesting to follow up how several of the conclusions of the α-particle model agree with conclusions of the shell model. It is well known that for p-shell nuclei, the shell model leads to a charge density

$$\rho(r) = \frac{2}{\pi^{3/2} a_0^3 (2 + 3\alpha)} \left(1 + \alpha \frac{r^2}{a_0^2}\right) \exp\left(-\frac{r^2}{a_0^2}\right), \qquad (8)$$

where $\alpha = (Z-2)/3$ and a_0 is a parameter. It is also well known[7] that this distribution fits well the data on elastic scattering of electrons by C^{12} ($a_0 = 1.64$) and O^{16} ($a_0 = 1.77$).

It is of interest to compare this distribution with that obtained by averaging the α-particle density over all orientations of the nucleus. The elastic scattering on the α-particle nucleus is obviously the same as that on a spherical one possessing just that density. A straightforward calculation leads to the following result for this density:

$$\rho(r) = \sqrt{\frac{3}{8\pi^3}}\,\frac{1}{abr}\exp\left[-\frac{3}{2a^2}(r^2+b^2)\right]\sinh\frac{3rb}{a^2}, \quad (9)$$

where a is the root mean square radius of the α particle (a = 1.61 f), and b is the distance from the center of gravity of the nucleus to the center of the α particle ($b = 2d/\sqrt{3}$ for C^{12} and $b = \sqrt{3/2}\,d$ for O^{16}).

The distributions calculated from Eqs. (8) and (9) for C^{12} and O^{16} are given in Fig. 5. We see that the distributions given by the shell model and the α-particle model coincide almost completely.

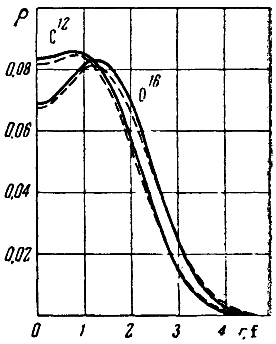


FIG. 5. Shape of the charge distribution (proton charge/f^3) in C^{12} and O^{16}, calculated from the α-particle model (solid curves) and from the shell model model (dashed curves).

Mention should be made in this connection of the work by Perring and Skyrme,[18] who have shown that it is possible here to construct wave functions for the ground states of Be^8, C^{12}, and O^{16} which, when antisymmetrized, turn out to be identical with the wave functions of the shell model.

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