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QUANTUM OSCILLATIONS OF THE PHOTOELECTRIC YIELD OF METALS IN A MAGNETIC FIELD

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The quantum oscillations of the volume (external) photoelectric effect in metals in a magnetic field are investigated in the ultraviolet region of the spectrum for electrons obeying an arbitrary dispersion law. The expression for the photocurrent contains, besides oscillations of the de Haas-van Alphen type, some new terms which are also characteristic of other optical phenomena. It is shown that a study of the photocurrent oscillations and the photoelectron energy distribution function permits one, in principle, to determine the shape of the Fermi surface as well as the arrangement of the separate electron groups in the reciprocal lattice and also to determine the shape of the isoenergetic surfaces lying below the Fermi surface. The calculation is carried out for the case where the magnetic field is perpendicular to the surface. The influence of collisions between photoelectrons prior to exit from the metal on the oscillations of the photoelectron yield is investigated.

1. INTRODUCTION

THE investigations of recent years, starting with the work of Lifshitz, Kosevich, and Pogorelov,[1] have brought out that it is very important to study the quantum oscillation effects if one of the basic problems of the electron theory of metals, the determination of the dispersion law for the conduction electrons in actual metals, is to be solved.*

The best known of these effects are the de Haas-van Alphen effect (oscillations of the magnetic susceptibility of metals in a varying magnetic field) and the related Shubnikov-de Haas effect (oscillations of the electric resistance). A careful experimental study of the de Haas-van Alphen effect carried out by Verkin, Lazarev, and co-workers in the U.S.S.R.[3] and by Shoenberg (cf. reference 4 and others) in England has already led to the determination of the shape of the Fermi surface in a number of actual cases.

In the present paper we consider the external photoelectric effect in metals in a magnetic field in the ultraviolet region of the spectrum. The magnetic field is directed perpendicular to the surface, so that the electrons reach the anode freely. Under these conditions the photoelectron current contains, besides the a component which changes monotonically with H, several oscillatory components. One of these is due to the same oscillations of the number of states of the electrons in the magnetic field which give rise to the de Haas-van Alphen effect. The periods are in this case determined by the extremal[1] (or limiting[5]) cross sections of the Fermi surface.

The second component is due to the periodic dependence of the transition matrix element on the magnetic field. It has nothing to do with the number of states and represents a new oscillatory effect which is common to all optical phenomena connected with transitions between bands. The study of this effect allows us, in principle, to determine the arrangement of separate regions of the Fermi surface in the reciprocal lattice. This possibility of determining the location of the separate groups, which did not exist previously, is due to the presence of a new parameter in the optical effects: the frequency of light. The third component represents the interference effect.

Another new piece of information which may be obtained from photoelectric experiments (measuring the energy distribution of the emitted electrons) is the shape of the isoenergetic surfaces lying below the Fermi surface. Finally, it is an interesting feature of photoelectric experiments that they permit us, in principle, to study in relatively weak fields large electron groups, whose oscillations are washed out under the usual conditions on account of the domain structure.[1] In photoelectric experiments one can use a narrow light beam which takes in only one domain.

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*The study of the resonance effects serves the same purpose (cf. Azbel' and Kaner[2]).

In conclusion we note the important circumstance that the effect under consideration, just as the de Haas-van Alphen effect, depends practically only on the dispersion law and not on its genesis (the wave functions).

2. THE VOLUME PHOTOEFFECT IN A METAL

It is known that the absorption of light by a conduction electron near the surface, as a result of which the electron leaves the metal (surface photoeffect[6,7]), is possible only if the energy conservation law is fulfilled, whereas the extra momentum can be transferred to the surface. This effect occurs for the frequencies $\omega \geq \omega_g$, where $\omega_g = W/\hbar$ (W is the work function).

For a light frequency ω which surpasses the second photoelectric threshold, $\omega \geq \omega_g'$ the so-called volume photoeffect becomes possible,[6,8,9] in which the electron first goes from the conduction band to a higher lying band and then leaves the metal. The photoelectrons are thus not excited in a thin surface layer, but in a volume of metal of thickness δ (δ is the penetration depth of the light in the metal, $\delta \gg a$). The yield of the volume photoeffect is therefore in general much greater than the yield of the surface effect.

Usually the maxima of the spectral distribution curves of the yield for these two types of the photoeffect do not overlap, since ω_g, as a rule, lies in the visible or near the ultraviolet region, while ω_g' lies in the more remote ultraviolet.* In the following we shall consider the photoeffect in the ultraviolet region of the spectrum, assuming that it can be separated from the surface effect. The energy of the electrons in this case is sufficiently large, so that one can neglect the reflection of the electrons from the surface potential barrier and from the distortions of the periodic potential near the surface. The calculation of the current of emitted electrons reduces then to the calculation of the current towards the surface inside the metal.

All oscillatory effects are observed only at very low temperatures and in pure metals, and the penetration depth of light in the metal, δ, is small ($\delta \sim 10^{-5}$ to 10^{-6} cm). Collisions of the photoelectrons with the phonons (or impurities) do not play any role (besides, they do not change the energy of the electron to any appreciable extent). The collisions of the photoelectron with the other electrons lead, however, to a great loss of energy. As is known (see, for example, reference 13), these losses can be divided into losses due to the excitation of plasma oscillations and losses due to collisions with separate screened electrons.

The free path length for the first interaction (the collective one) is equal to a few lattice constants; however, the excitation of plasma oscillations is possible only after the photoelectron has reached a certain minimal energy E_p. From the experimental data on the characteristic losses of electrons[13,14] it is known that this energy is equal to 20 to 25 ev for many metals. We can therefore choose a metal in which the photoelectrons do not excite plasma oscillations.

As to the losses of the second type (the individual losses), it appears that the corresponding free path length is very great on account of the effective screening. According to reference 15, this length is 100 or more times greater than the lattice constant for slow electrons (photoelectrons) in alkali metals. The calculation of l for the screened Coulomb interaction[13,16,17] of free electrons with account of the exclusion principle gives a value which is several times smaller. However, the band structure of the spectrum either forbids or lowers the probability for a number of transitions which are possible for free electrons, which leads to an increase in l. In the case where the photoelectron with the wave vector $\mathbf{k}_1$ remains in the upper band after the collision while the conduction electron stays in the lower band, the time of free flight can be shown[21] to be equal to

$$\tau = \tau_B\,(15\pi/8)(\alpha/k_1)^4\,D^{-2}.$$

Here $\tau_B = \hbar^3/me^4$ is the characteristic Bohr time, α^{-1} is the screening radius (according to the estimate of references 6 and 13, $\alpha^{-1} \gtrsim 10^{-8}$ cm), and D^{-2} is a factor which depends on the wave functions in both bands, $D^2 < 1$. If $\mathbf{k}_1$ is small, for example, $|\mathbf{k}_1| = \alpha/4$ (here $D^2 = 1/3$), $\tau \approx 10^4\,\tau_B$, i.e., the time of flight is large. If both electrons appear in the lower band after the collision, which is possible if k_1 and the width of the forbidden band are small, then

$$\tau = \tau_B\,(3\pi\,/\,8)\,(a^2 + \lambda)^2\,k_0^{-3}\lambda^{-1/2}D^{-2},$$

where λ is the width of the conduction band (multiplied by $m/\hbar^2$). In this case D^2 is always small (for example, of order 10^{-2} to 10^{-4}), so that τ is large again.

The collisions lead to a decrease of the oscillation amplitude [according to reference 18, by a factor $\exp(h/\tau\mu H)$] without change in the period. Since τ depends critically on the band structure of the metal, it may occur that oscillations of the

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*This is not true for all metals.[10] Contrary to previous acceptance,[6,7] it has recently been suggested[9,11,12] that the photoeffect in alkali metals in the region 500 to 300 mμ is also predominantly a volume effect.

photocurrent are observable in one metal and inobservable in another. It is evidently convenient to conduct the experiments in metals with large $\mu = e\hbar/m^*c$ in strong fields ($H \sim 10^5$ oersted). Here the energy of the photoelectron must be below the level of the discrete (plasma) losses.

In conclusion we note that the ultraviolet radiation of the metal causes not only transitions from the conduction band to the higher lying band, but also transitions from the narrow bands below the conduction band into states immediately below the barrier.[19] The two kinds of photoelectrons are easily separated since they have completely different energies after leaving the metal.

3. CALCULATION OF THE PHOTOCURRENT

The Hamiltonian of the electron in a metal placed in a magnetic field can be written in the form

$$\hat{H} = \hat{H}^{(0)} + \hat{V},$$

where

$$\hat{H}^{(0)} = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}^0\right)^2 + V_p(\mathbf{x}); \qquad (1)$$

$$\hat{V} = -\frac{e}{mc}\mathbf{A}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}^0\right). \qquad (2)$$

Here $V(\mathbf{x})$ is the periodic potential; $\mathbf{A}^0(-Hy, 0, 0)$ is the vector potential of the constant magnetic field $H = H_z$; $\mathbf{A}$ is the vector potential of the light wave, and is equal to

$$\mathbf{A} = (cF/i\omega)\,\mathbf{a}\exp\{i\varkappa\mathbf{x} - i\omega t\}$$

(F is the amplitude of the electric vector; $\mathbf{a}$ is the polarization vector; $|\mathbf{a}| = 1$).

To solve the Schrödinger equation $\hat{H}\Psi = i\hbar\,\partial\Psi/\partial t$ we must first find the eigenfunctions of the unperturbed problem. Using the results of a previous paper,[20] we write down the orthonormal eigenfunctions of the electron obeying an arbitrary dispersion law in the magnetic field:*

$$\Psi_{k_1nk_3}(\mathbf{x}) = \frac{1}{\sqrt{L}}e^{ik_1x}\frac{1}{\sqrt{L}}e^{ik_3z} \times \int g_n(k_2, k_3)\exp\{ik_2(y + \alpha_0^2k_1)\}\,v\left(k_1 + \frac{y}{\alpha_0^2}, k_2, k_3; x, y, z\right)dk_2, \qquad (3)$$

where $v(\mathbf{k}; \mathbf{x})$ is a periodic Bloch function normalized in the "region of periodicity" of volume L^3 according to

$$L^{-3}\int |v|^2\,d\tau = 1.$$

In formula (3) $g_n(k_2, k_3)$ is the eigenfunction of the electron in the magnetic field in the $\mathbf{k}$ representation. It is given by

$$g_n(k_2, k_3) = \frac{\hbar A}{\pi\sqrt{m}}\left|\frac{\partial E}{\partial\varkappa_1}\right|^{-1/2}\exp\{-i\alpha_0^2\varkappa_{10}(k_2 - k_{20})\}\cos\left(\alpha_0^2S_n(k_2) - \frac{\pi}{4}\right). \qquad (4)$$

The trajectory of the electron in the reciprocal space (Fig. 1) is given by the intersection of the isoenergetic surface $E(\mathbf{k}) = E$ with the plane $k_3 = \text{const}$; correspondingly, the equation of the trajectory $\varkappa_1 = \varkappa_1(k_2)$ is found from the relation $E(\varkappa_1, k_2, k_3) = E$; $\alpha_0 = (\hbar c/eH)^{1/2}$ is the radius of the lowest quantum mechanical orbit of the free electron, $a/\alpha_0 \ll 1$ (a is the lattice constant); the meaning of the area $S_n(k_2)$ is clear from Fig. 1.

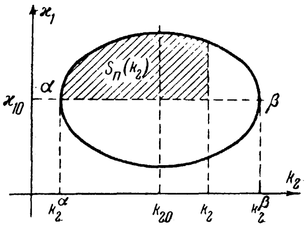

FIG. 1

For the sake of generality, we consider an arbitrary group of electrons whose center ($\varkappa_{10}$, k_{20}) does not coincide with the origin of a cell of the reciprocal lattice. The dimensionless constant A is found from the normalization condition:

$$2\pi\int |g_n(k_2, k_3)|^2\,dk_2 = 1.$$

The corresponding wave function for the total Hamiltonian $\hat{H}$ is sought in the form

$$\Psi(\mathbf{x}, t) = \psi_{k_1nk_3}(\mathbf{x})\exp\{-iE_n(k_3)t/\hbar\} + \Phi(\mathbf{x}, t),$$

where $\Phi(\mathbf{x}, t)$ is the perturbation term. The expansion of Φ in terms of the complete orthonormal system of functions has the form

$$\Phi(\mathbf{x}, t) = \sum_{n'}\int dk_1'dk_3'\,C_{k_1'n'k_3'}\,\psi'_{k_1'n'k_3'}(\mathbf{x})\exp\left\{-\frac{iE'_{n'}(k_3')\,t}{\hbar}\right\}$$

[the prime characterizes functions of the electron in the upper band; $v'(\mathbf{k}; \mathbf{x})$ is a Bloch function for the upper band; $g'_{n'}(k_2, k_3)$ has a form analogous to (4)]. The usual perturbation theory leads to the following expression for the coefficients $C_{k_1'n'k_3'}$:

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*We neglect the effect of the boundary of the metal. The wave function for the finite metal differs from the wave function (3) by an additional exponentially decreasing (at a distance ~ a from the boundary) term. Since in the calculation of the transition probability we are interested in the overlap integral of the eigenfunctions of the initial and final states ψ and ψ' (cf. below), the exponentially decreasing term can be neglected (this cannot be done in the calculation of the surface effect, where the basic integral without account of the boundary reduces to zero in virtue of the orthonormality).

$$C_{k_1' n' k_3'} = \left(\frac{L}{2\pi}\right)^2 \frac{\pi e F}{m\omega} M_{k_1 n k_3}^{k_1' n' k_3'} \delta(E' - E - \hbar\omega), \qquad (5)$$

where the matrix element for the transition is equal to

$$M_{k_1 n k_3}^{k_1' n' k_3'} = \int \psi^{*\prime}_{k_1' n' k_3'}(\mathbf{x})\, \mathbf{a} \left(\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}^0\right) \psi_{k_1 n k_3}(\mathbf{x}) d\tau$$

(in this formula we neglect the factor $e^{i\boldsymbol{\kappa}\cdot\mathbf{x}}$, since the wave vector of the photon κ is much smaller than the wave vector of the electron $\mathbf{k}$: $\kappa \ll k$). With the help of formula (3) we write this expression in the form

$$M_{k_1 n k_3}^{k_1' n' k_3'} = \int g^{*\prime}_{n'}(k_2', k_3')\, \chi(\mathbf{k}', \mathbf{k})\, g_n(k_2, k_3)\, dk_2\, dk_2', \qquad (6)$$

where

$$\chi(\mathbf{k}', \mathbf{k}) = \frac{\hbar L}{a} \delta_{k_1 k_1'} \delta_{k_3 k_3'} \sum_{m=-\infty}^{\infty} \sigma_m(k_2, k_3)\, \delta_{k_2', k_2 + ma/\alpha_0^2}. \qquad (7)$$

Here the σ_m are certain dimensionless coefficients (which depend weakly on the magnetic field) which can be expressed in terms of the Bloch functions of the conduction band and the higher lying band. Substituting (7) in (6) we obtain the matrix element for the transition in the form

$$M_{k_1 n k_3}^{k_1' n k_3'} = \frac{2\pi\hbar}{a} S_{nn'}(k_3)\, \delta_{k_1 k_1'}\, \delta_{k_3 k_3'}, \qquad (8)$$

where

$$S_{nn'}(k_3) = \sum_m \int g^{*\prime}_{n'}(k_2 + ma/\alpha_0^2, k_3)\, \sigma_m(k_2, k_3)\, g_n(k_2, k_3) dk_2.$$

Using the condition $a/\alpha_0 \ll 1$ and formula (4) for the eigenfunctions $g_n(k_2, k_3)$ we transform $S_{nn'}$ to the form

$$S_{nn'}(k_3) = \int g^{*\prime}_{n'}(k_2, k_3)\, \sigma(k_2, k_3)\, g_n(k_2, k_3)\, dk_2, \qquad (9)$$

where

$$\sigma(k_2, k_3) = \sum_m \sigma_m(k_2, k_3) \exp\{ima\, \varkappa_1(k_2)\}$$

is a quantity which depends smoothly on the magnetic field. As was mentioned already, we must know the exact eigenfunctions of the electron in the metal (Bloch functions) in order to calculate $\sigma(k_2, k_3)$. However, a number of effects in the magnetic field, in particular, the oscillations of the photoelectric current with the magnetic field, do not depend on the actual form of these functions. These oscillations are determined by the functions $g_n(k_2, k_3)$ and $g'_{n'}(k_2, k_3)$, which depend only on the dispersion law. The exact wave functions are needed only for the calculation of the amplitude of the oscillations. But since σ is a dimensionless quantity of order unity, one can estimate the order of magnitude of the amplitudes.

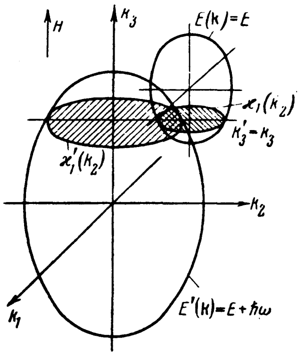

FIG. 2

Thus the oscillatory properties of the transition depend on the dispersion law for the electrons in the metal and not on its genesis.

It is seen from formulas (5) and (8) that the transitions considered above obey the following selection rules:

$$k_3' = k_3, \quad E'_{n'}(k_3') = E_n(k_3) + \hbar\omega$$

(the condition $k_1' = k_1$ implies, moreover, that the "center of rotation" does not change during the transition). The trajectories of the electron in the initial $[\kappa_1(k_2)]$ and final $[\kappa_1'(k_2)]$ states are shown in Fig. 2. Here $E(\mathbf{k})$ and $E'(\mathbf{k})$ determine the dispersion law for the electron in the conduction band and the upper band, respectively. Since the functions $g_n(k_2, k_3)$ decrease very fast (exponentially) outside the region of classical motion of the electron (see reference 20), the quantities $S_{nn'}(k_3)$ and the matrix element for the transition will be different from zero only if the trajectories $\kappa_1(k_2)$ and $\kappa_1'(k_2)$ intersect each other (Fig. 2). Otherwise the matrix element for the transition is negligibly small, and the photon yield from this group of electrons is zero.

Knowing the perturbed wave function, we can find the current of emitted electrons according to the known quantum mechanical formula

$$j_z(n, k_3) = \frac{ie\hbar}{2m}\left(\Phi \frac{\partial \Phi^*}{\partial z} - \Phi^* \frac{\partial \Phi}{\partial z}\right);$$

$j_z(n, k_3)$ represents the current corresponding to a given initial state n, k_3. To find the total current J_z we must sum this quantity over all electrons in the metal, which gives

$$J_z = \frac{L^3}{2\pi^2\alpha_0^2} \sum_n \int \frac{1}{\exp\{(E - \zeta)/\theta\} + 1} \bar{j}_z(n, k_3) dk_3, \qquad (10)$$

where $\bar{j}_z(n, k_3)$ denotes the average value of the current density. It is easily seen that the function $\psi_{k_1 n k_3}(\mathbf{x})$ [formula (3)] corresponds to the current

$$\breve{j}_z = (e/\hbar L^3)\, \partial E_n(k_3)/\partial k_3.$$

We calculate the current $\bar{j}_z(n, k_3)$ with the help of (5) and (8) and substitute it in formula (10). Taking account of $\delta_{k_1 k_1'}$, $\delta[E_{n'}'(k_3') - E_n(k_3) - \hbar\omega]$, and $\delta_{k_3 k_3'} = (2\pi/L)\,\delta(k_3 - k_3')$ in the integration over dk_1', dk_3', and dk_3, respectively, we obtain

$$J_z = \frac{\pi e^3 \hbar F^2 L}{m^2 a^2 \omega^2 \alpha_0^2} \sum_{n,n'} (e^{(E-\zeta)/\theta} + 1)^{-1} |S_{nn'}(k_3)|^2 \left|\frac{\partial H_{nn'}}{\partial k_3}\right|^{-1},$$

$$H_{nn'}(k_3) = E_{n'}'(k_3') - E_n(k_3) - \hbar\omega. \tag{11}$$

Up to this point we have not taken into account the attenuation of the light wave in the metal, as a consequence of which the photocurrent appeared to be proportional to the thickness of the cathode L. It can be shown that the consideration of the damping leads to the replacement of the quantity L in formula (11) by δ, the penetration depth of the field in the metal (we assume $\delta \gg a$). Then the photocurrent per unit area of the surface ceases to depend on the thickness of the cathode, if $L > \delta$.

To change the summation in formula (11) to an integration, we use the Poisson summation formula:

$$\sum_{n=0}^{\infty} f(n) = \frac{1}{2} f(0) + \sum_{p=-\infty}^{\infty} \int_0^{\infty} e^{2\pi i p n} f(n)\, dn,$$

and apply it to the summation over n and n′. We also introduce the following notations. For the calculation of the quantities $S_{nn'}$ we must use the method of steepest descent, and it will turn out (see below) that $S_{nn'}$ has in the general case the form

$$|S_{nn'}(k_3)|^2 = \left(\frac{a}{\alpha_0}\right)^2 \sum_{\alpha} \vartheta_{nn'}^{(\alpha)}(k_3) \cos(\alpha_0^2 \sigma_\alpha(E, k_3) + \gamma_\alpha), \tag{12}$$

where the $\vartheta_{nn'}^{(\alpha)}$ are certain coefficients which depend smoothly on the magnetic field, and the $\sigma_\alpha(E, k_3)$ are areas which determine the oscillations of the matrix element of the transition; the γ_α are certain phases, $\sigma_0 = 0$, $\gamma_0 = 0$; $\vartheta_{nn'}^{(0)}$ represents the non-oscillatory term.

In this way we transform (11) to the form

$$J_z = \sum_{\alpha} \sum_{p,q=-\infty}^{\infty} e^{-i\pi p - i\pi q} \int dk_3 \int dS \frac{\Lambda^{(\alpha)}(S, k_3)}{\exp\{(E-\zeta)/\theta\}+1} \exp\{ip\alpha_0^2 S\}$$
$$\times \exp\{iq\alpha_0^2 S'(E, k_3)\} \cos[\alpha_0^2 \sigma_\alpha(E, k_3) + \gamma_\alpha], \tag{13}$$

where

$$\Lambda^{(\alpha)} = \frac{e^3 \hbar F^2 \delta}{4\pi m^2 \omega^2} \left(\frac{\partial S'}{\partial k_3}\right)_S \vartheta_{nn'}^{(\alpha)}(k_3) \left|\frac{\partial H_{nn'}}{\partial k_3}\right|^{-1} \tag{14}$$

is a quantity which depends smoothly on the magnetic field. J_z in formula (13) represents the photocurrent from the given electron group. The total current is obtained by summing over all groups in the conduction band.

Changing the integration over dS in formula (13) to an integration over the energy, with the help of the formula for the semiclassical quantization of the energy levels of the electron in the magnetic field,[1,20]

$$S(E, k_3) = 2\pi(n + \tfrac{1}{2})/\alpha_0^2,$$

we can write the photocurrent J_z in the form of a sum:

$$J_z = J_z^0 + J_z^1 + J_z^2 + J_z^{12}, \tag{15}$$

$$J_z^0 = \int_0^{\infty} \frac{dE}{\exp\{(E-\zeta)/\theta\}+1} \int dk_3 \frac{\partial S}{\partial E} \Lambda^{(0)}(E, k_3), \tag{16}$$

$$J_z^1 = 2\,\mathrm{Re} \sum_{p=1}^{\infty} e^{-i\pi p} \int_0^{\infty} \frac{dE}{\exp\{(E-\zeta)/\theta\}+1} \int dk_3 \frac{\partial S}{\partial E} \Lambda^{(0)}(E, k_3)$$
$$\times \exp\{ip\,\alpha_0^2 S(E, k_3)\}, \tag{17}$$

$$J_z^2 = \mathrm{Re} \sum_{\alpha \neq 0} \int_0^{\infty} \frac{dE}{\exp\{(E-\zeta)/\theta\}+1} \int dk_3 \frac{\partial S}{\partial E} \Lambda^{(\alpha)}(E, k_3)$$
$$\times \exp\{i\alpha_0^2 \sigma_\alpha(E, k_3) + i\gamma_\alpha\}. \tag{18}$$

Here J_z^0 is the non-oscillatory part of the current; J_z^1 is the oscillatory part of the current due to the periodic dependence of the number of states on the magnetic field (analogous to the de Haas-van Alphen effect); J_z^2 is the oscillatory part of the current due to the periodic dependence of the matrix element of the transition on the magnetic field; J_z^{12} is the oscillatory part of the current corresponding to the interference of parts 1 and 2, which we shall not write down explicitly.

In the case of the volume photoeffect we therefore have to do with two types of oscillations of the photoyield with the magnetic field: 1) oscillations connected with the number of states (J_z^1), analogous to the de Haas-van Alphen and Shubnikov-de Haas effects etc., and 2) oscillations connected with the matrix element for the transition (J_z^2), which are characteristic for all optical phenomena (volume photoeffect, absorption of light in metals, etc.) related to transitions between different energy bands.

4. THE OSCILLATORY PARTS OF THE CURRENT

1. The oscillatory part of the current connected with the number of states is given by formula (17). The integrals in (17) can be evaluated by the method of steepest descent, so that the final answer will contain the extremal Fermi cross sections $S_m(\zeta)$, just as in the case of the de Haas-van Alphen effect.[1] We find

$$J_z^1 = \frac{2\sqrt{2\pi}}{\alpha_0^3} \Lambda^{(0)}(\zeta, k_{3m}) \left|\frac{\partial^2 S}{\partial k_3^2}\right|_m^{-1/2} \sum_{p=1}^{\infty} p^{-1/2} \frac{z}{\mathrm{sh}\, pz} \sin\left[p\alpha_0^2 S_m(\zeta) - \pi p \pm \frac{\pi}{4}\right], \tag{19}$$

where

$$z = 2\pi^2\theta/\mu^* H,\ \mu^* = e\hbar/m^* c,\ m^* = (\hbar^2/2\pi)\,\partial S_m/\partial\zeta. \tag{20}$$

In the limiting cases of weak and strong magnetic fields ($z \gg 1$ and $z \ll 1$) we have:

for $z \gg 1$,

$$J_z^1 = \frac{2(2\pi)^{3/2}\theta}{\alpha_0}\Lambda^{(0)}(\zeta, k_{3m})\frac{\partial S_m}{\partial\zeta}\left|\frac{\partial^2 S}{\partial k_3^2}\right|_m^{-1/2}\exp\left\{-2\pi^2\frac{\theta}{\mu^* H}\right\}$$
$$\times\sin\left[\alpha_0^2 S_m(\zeta) - \pi \pm \frac{\pi}{4}\right]; \tag{21}$$

for $z \ll 1$,

$$J_z^1 = \frac{2(2\pi)^{1/2}}{\alpha_0^3}\Lambda^{(0)}(\zeta, k_{3m})\left|\frac{\partial^2 S}{\partial k_3^2}\right|_m^{-1/2}\sum_{p=1}^{\infty}p^{-3/2}$$
$$\times\sin\left[p\alpha_0^2 S_m(\zeta) - \pi p \pm \frac{\pi}{4}\right]. \tag{22}$$

Let us estimate the relative magnitude of the oscillatory terms as compared with the non-oscillatory term J_z^0, assuming that by order of magnitude

$$J_z^0 = \int_0^\zeta dE\int dk_3\frac{\partial S}{\partial E}\Lambda^{(0)}(E, k_3) \sim \zeta\bar{\Lambda}^{(0)}\frac{2\pi m}{\hbar^2}\left(\frac{2m\zeta}{\hbar^2}\right)^{1/2}.$$

We find

$$J_z^1/J_z^0 \sim \frac{4\pi}{\sqrt{2}}\left(\frac{\mu H}{\zeta}\right)^{1/2}\frac{\theta}{\zeta}\frac{m^*}{m}\bar{\Lambda}_s^{(0)}/\bar{\Lambda}^{(0)},\ z \gg 1; \tag{23}$$

$$J_z^1/J_z^0 \sim \frac{1}{\pi\sqrt{2}}\left(\frac{\mu H}{\zeta}\right)^{3/2}\bar{\Lambda}_s^{(0)}/\bar{\Lambda}^{(0)},\ z \ll 1. \tag{24}$$

Here $\bar{\Lambda}_s^{(0)}$ is the average value of $\Lambda^{(0)}(E, k_3)$ for the given (s-th) group, and $\bar{\Lambda}^{(0)}$ is the analogous quantity for the basic group; m is the mass of the free electron; $\mu = e\hbar/mc$ [here the oscillatory part of the current J_z^1 refers to the given electron group (which is small, for example), while the non-oscillatory part J_z^0 refers to the basic (large) group].

For an estimate of the ratio $\Lambda_s^{(0)}/\Lambda^{(0)}$ we assume that approximately

$$\bar{\Lambda}_s^{(0)}/\bar{\Lambda}^{(0)} \sim \bar{\vartheta}_{nn's}^{(0)}/\bar{\vartheta}_{nn'}^{(0)}$$

[cf. formula (14)], which leads to the very crude estimate

$$\bar{\Lambda}_s^{(0)}/\bar{\Lambda}^{(0)} \sim (n_0/n)^{1/3},$$

where n_0 is the density of the electrons of the basic group, and n that of the given group.

2. The oscillatory part of the current connected with the matrix element of the transition is given by formula (18). The calculation of J_z^2 is analogous to that of J_z^1; as a result we have

$$J_z^2 = \frac{(2\pi)^{1/2}}{\alpha_0^3}\sum_{\alpha\neq 0}\Lambda^{(\alpha)}(\zeta, k_{3m})\left|\frac{\partial^2\sigma_\alpha}{\partial k_3^2}\right|^{-1/2}\frac{\partial S(\zeta, k_{3m}^\alpha)/\partial\zeta}{\partial\sigma_\alpha^m/\partial\zeta}\frac{z_\alpha}{\sinh z_\alpha}$$
$$\times\sin\left[\alpha_0^2\sigma_\alpha^m(\zeta) + \gamma_\alpha \pm \frac{\pi}{4}\right], \tag{25}$$

where

$$z_\alpha = 2\pi^2\theta/\mu_\alpha^* H,$$
$$\mu_\alpha^* = e\hbar/m_\alpha^* c,\quad m_\alpha^* = (\hbar^2/2\pi)\partial\sigma_\alpha^m/\partial\zeta \tag{26}$$

[$\sigma_\alpha^m(\zeta)$ is the extremal value of the area σ_α; $k_3 = k_{3m}^\alpha$ is the extremal point]. In the limiting cases $z_\alpha \gg 1$ and $z_\alpha \ll 1$ formula (25) gives:

for $z_\alpha \gg 1$

$$J_z^2 = \frac{(2\pi)^{3/2}\theta}{\alpha_0}\sum_{\alpha\neq 0}\Lambda^{(\alpha)}(\zeta, k_{3m}^\alpha)\frac{\partial S(\zeta, k_{3m}^\alpha)}{\partial\zeta}\left|\frac{\partial^2\sigma_\alpha}{\partial k_3^2}\right|_m^{-1/2}$$
$$\times\exp\left\{-2\pi^2\frac{\theta}{\mu_\alpha^* H}\right\}\sin\left[\alpha_0^2\sigma_\alpha^m(\zeta) + \gamma_\alpha \pm \frac{\pi}{4}\right]; \tag{27}$$

for $z_\alpha \ll 1$

$$J_z^2 = \frac{(2\pi)^{1/2}}{\alpha_0^3}\sum_{\alpha\neq 0}\Lambda^{(\alpha)}(\zeta, k_{3m}^\alpha)\left|\frac{\partial^2\sigma_\alpha}{\partial k_3^2}\right|_m^{-1/2}\frac{\partial S(\zeta, k_{3m}^\alpha)/\partial\zeta}{\partial\sigma_\alpha^m/\partial\zeta}$$
$$\times\sin\left[\alpha_0^2\sigma_\alpha^m(\zeta) + \gamma_\alpha \pm \frac{\pi}{4}\right]. \tag{28}$$

The period of the oscillations is in this case equal to

$$\Delta(1/H) = 2\pi e/\hbar c\sigma_\alpha^m(\zeta). \tag{29}$$

The order of magnitude of the oscillation amplitudes is estimated in the same way as under point 1. We have

$$z_\alpha \gg 1:\quad J_z^2/J_z^0 \sim \frac{2\pi}{\sqrt{2}}g_\alpha\left(\frac{\mu H}{\zeta}\right)^{1/2}\frac{\theta}{\zeta}\frac{m^*}{m}\bar{\Lambda}_s^{(\alpha)}/\bar{\Lambda}^{(0)}, \tag{30}$$

$$z_\alpha \ll 1:\quad J_z^2/J_z^0 \sim \frac{g_\alpha}{2\pi\sqrt{2}}\left(\frac{\mu H}{\zeta}\right)^{3/2}\frac{m^*}{m_\alpha^*}\bar{\Lambda}_s^{(\alpha)}/\bar{\Lambda}^{(0)}, \tag{31}$$

where $g_\alpha = (1/2\pi)\,|\partial^2\sigma_\alpha/\partial k_3^2|_m^{-1/2}$. As before, $\bar{\Lambda}_s^{(\alpha)}/\bar{\Lambda}^{(0)} \sim (n_0/n)^{1/3}$. The meaning of the quantities $\sigma_\alpha^m(\zeta)$, m_α^*, and g_α will be explained below (point 4).

3. The oscillatory component due to interference J_z^{12} has in general a very complicated form. It contains oscillatory terms of the type

$$\sin\left[\alpha_0^2(pS + qS' \pm \sigma_\alpha) + \delta\right]$$

(and a few others). However, if we are interested only in low frequency oscillations corresponding to small groups and in weak fields ($z \gg 1$), we can restrict ourselves to the term $\sin[\alpha_0^2(S - \sigma_\alpha) + \delta]$. The quantity $S - \sigma_\alpha = \sigma'_\alpha$ represents the area complementary to σ_α (see Fig. 3).

4. For the calculation of the amplitudes $\vartheta_{nn'}^{(\alpha)}(k_3)$ and the areas $\sigma_\alpha(E, k_3)$ we turn to the formulas (12) and (9). The integral (9) is computed by the method of steepest descent. The saddle points are given by the intersections of the trajectory of the electron in the initial and final bands, t_1 and t_2. $\sigma_\alpha(E, k_3)$ represents the shaded area in Fig. 3 [the final answer contains the extremal Fermi value of this area $\sigma_\alpha^m(\zeta)$].

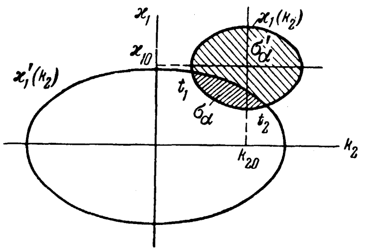

FIG. 3

As already mentioned, it is necessary for the occurence of a photoyield from the given (for example, small) electron group that the isoenergetic surfaces $E(\mathbf{k}) = \zeta$ and $E'(\mathbf{k}) = \zeta + \hbar\omega$ intersect. The low frequency oscillations corresponding to a small electron group (J_z^1 and J_z^2) will therefore be confined to some narrow interval of frequencies,

$$\omega_0 - \Delta\omega \leqslant \omega \leqslant \omega + \Delta\omega_0,$$

where, in order of magnitude, $\Delta\omega/\omega_0 \sim 2(n/n_0)^{1/3}$ (see Fig. 4; the isoenergetic surfaces of the upper band are schematically represented by spheres). The position of the center of this interval, ω_0, is determined by the arrangement of the small electron group in the reciprocal lattice.

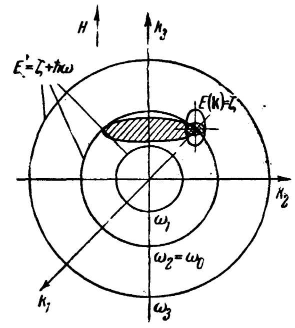

FIG. 4

In the investigation of the oscillations of the photoelectric yield (and also of the coefficient of the absorption of light and of some other optical quantities) we can therefore not only determine the shape of the Fermi surface, as in the oscillation effects of the type of the de Haas-van Alphen effect, but also the location of the centers of the groups.

Let us consider the spectral distribution of the oscillation frequencies $\Omega = 2\pi/\Delta(1/H)$:

$$\Omega(\omega) = (\hbar c/e)\,\sigma_\alpha^m(\zeta, \omega). \tag{32}$$

The function $\Omega(\omega)$ has the characteristic behavior shown in Fig. 5, where the $\omega_0^{(k)}$ correspond to the centers of the groups which alternately intersect the surface $E' = \zeta + \hbar\omega$ as the frequency changes. Near the limits of the frequency interval in which the photoyield of the given group occurs, $\sigma_\alpha^m(\zeta, \omega)$, $m_\alpha^*(\omega)$, and $g_\alpha(\omega)$ become very small, so that the oscillation amplitude increases [see formulas (30) and (31)]. Away from the limits of this interval we have, in order of magnitude, $\sigma_\alpha^m \lesssim S_m$, $m_\alpha^* \leq m^*$, and $g_\alpha \sim 1$.

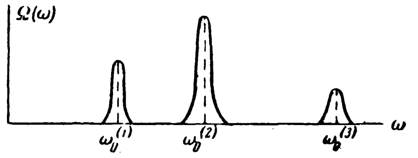

FIG. 5

5. ENERGY DISTRIBUTION OF THE PHOTOELECTRONS

To find the energy distribution of the emitted electrons $Z(E) = dJ_z/dE$ we turn to the formulas (15) to (18). We have

$$Z(E) = Z^0(E) + Z_{\text{osc}}^1(E) + Z_{\text{osc}}^2(E) + Z_{\text{osc}}^{12}(E);$$

$$Z^0(E) = \frac{1}{\exp\{(E_1 - \zeta)/\theta\} + 1} \int \frac{\partial S}{\partial E_1} \Lambda^{(0)}(E_1, k_3)\, dk_3, \tag{33}$$

$$Z_{\text{osc}}^1(E) = \frac{1}{\exp\{(E_1 - \zeta)/\theta\} + 1} \times 2\,\mathrm{Re} \sum_{p=1}^{\infty} e^{-i\pi p} \int \frac{\partial S}{\partial E_1} \Lambda^{(0)}(E_1, k_3) \exp\{ip\alpha_0^2 S(E_1, k_3)\}\, dk_3, \tag{34}$$

$$Z_{\text{osc}}^2 = \frac{1}{\exp\{(E_1 - \zeta)/\theta\} + 1}\,\mathrm{Re} \sum_{\alpha \neq 0} \int \frac{\partial S}{\partial E_1} \Lambda^{(\alpha)}(E_1, k_3) \times \exp\{i\alpha_0^2 \sigma_\alpha(E_1, k_3) + i\gamma_\alpha\}\, dk_3, \tag{35}$$

where $E_1 = E - \hbar\omega$ is the energy of the electron before the transition. Here $Z^0(E)$ is the nonoscillatory part of the distribution function; $Z_{\text{osc}}^1(E)$ is the oscillatory part of the energy distribution function of the photoelectrons connected with the number of states; $Z_{\text{osc}}^2(E)$ is that part connected with the matrix element of the transition.

The calculation of the integrals (34) and (35) by the method of steepest descent leads to the expressions

$$Z_{\text{osc}}^1(E) = \frac{1}{\exp\{(E_1 - \zeta)/\theta\} + 1} \frac{2(2\pi)^{1/2}}{\alpha_0} \Lambda^{(0)}(E_1, k_{3m}) \left|\frac{\partial^2 S}{\partial k_3^2}\right|_m^{-1/2} \frac{\partial S_m}{\partial E_1} \times \sum_{p=1}^{\infty} p^{-1/2} \cos\left[p\alpha_0^2 S_m(E_1) - \pi p \pm \frac{\pi}{4}\right], \tag{36}$$

$$Z_{\text{osc}}^2(E) = \frac{1}{\exp\{(E_1 - \zeta)/\theta\} + 1} \frac{(2\pi)^{1/2}}{\alpha_0} \times \sum_{\alpha \neq 0} \Lambda^{(\alpha)}(E_1, k_{3m}^\alpha) \frac{\partial S(E_1, k_{3m}^\alpha)}{\partial E_1} \left|\frac{\partial^2 \sigma_\alpha}{\partial k_3^2}\right|_m^{-1/2} \cos\left[\alpha_0^2 \sigma_\alpha^m(E_1) + \gamma_\alpha \pm \frac{\pi}{4}\right]. \tag{37}$$

In contrast to the saturation current J_z, the oscillations of the energy distribution function of the photoelectrons are not determined by the extremal Fermi cross sections $S_m(\zeta)$, $\sigma_\alpha^m(\zeta)$, but by the extremal cross sections $S_m(E_1)$, $\sigma_\alpha^m(E_1)$ corresponding to the given energy $E_1 = E - \hbar\omega$.

The investigation of the oscillations of the energy distribution function of the photoelectrons with the magnetic field permits us, in principle, to determine any arbitrary isoenergetic surface besides the Fermi surface $E(\mathbf{k}) = \zeta$.

Let us now consider the function

$$J_z(E) = \int_0^E Z(E)\,dE.$$

This function is connected with the current-voltage characteristic of the photoelement J(V), i.e., the dependence of the photocurrent on the voltage V at the anode of the photoelement, through the relation $J(V) = J_z - J_z(-eV)$ (here $J_z = J_z(\infty)$ is the saturation current). With the help of formulas (36) and (37) we obtain

$$J_z^1 = \frac{2(2\pi)^{1/2}}{\alpha_0^3}\Lambda^{(0)}(E_1, k_{3m})\left|\frac{\partial^2 S}{\partial k_3^2}\right|_m^{-1/2}\sum_{p=1}^{\infty} p^{-3/2}\sin\left[p\alpha_0^2 S_m(E_1) - \pi p \pm \frac{\pi}{4}\right], \tag{38}$$

$$J_z^2 = \frac{(2\pi)^{1/2}}{\alpha_0^3}\sum_{\alpha\neq 0}\Lambda^{(\alpha)}(E_1, k_{3m}^\alpha)\left|\frac{\partial^2\sigma_\alpha}{\partial k_3^2}\right|_m^{-1/2}\frac{\partial S(E_1, k_{3m}^\alpha)/\partial E_1}{\partial\sigma_\alpha^m/\partial E_1} \times\sin\left[\alpha_0^2\sigma_\alpha^m(E_1) + \gamma_\alpha \pm \frac{\pi}{4}\right]. \tag{39}$$

The order of magnitude of the amplitudes of the oscillatory terms are estimated in analogy to what we did before (Sec. 3, point 4):

$$J_z^1(E)/J_z^0(E) \sim (1/\pi\sqrt{2})(\mu H/E_1)^{3/2}\bar{\Lambda}_s^{(0)}/\bar{\Lambda}^{(0)}, \tag{40}$$

$$J_z^2(E)/J_z^0(E) \sim (g_\alpha/2\pi\sqrt{2})(\mu H/E_1)^{3/2}(m^*/m_\alpha^*)\bar{\Lambda}_s^{(\alpha)}/\bar{\Lambda}^{(0)}. \tag{41}$$

The $J_z(E)$ oscillate with the magnetic field with the same frequencies as Z(E). However, the function $J_z(E)$ not only oscillates for variations of the magnetic field, but also for variations of E. This leads to oscillations of the current-voltage characteristic of the photoelement J(V) as a function of the voltage at the anode V. The period of these oscillations is equal to

$$\Delta E = 2\pi\alpha_0^{-2}\Big/\frac{\partial S_m}{\partial E_1} = \mu^* H. \tag{42}$$

Correspondingly

$$\Delta V = \mu H\gamma/e, \tag{43}$$

where $\gamma = m/m^*$ for $J_{\text{osc}}^1(V)$ and $\gamma = m/m_\alpha^*$ for $J_{\text{osc}}^2(V)$. For $H = 10^4$ we have $\Delta V = 10^{-4}\gamma V$.

The instability of the potential at the anode, δV, the non-nonochromatic nature of the light, $\delta\omega$, and collisions of the photoelectrons before leaving the metal can lead to a smoothing out of the oscillations of the current-voltage curve. In order for these oscillations to be observable, the following conditions must be fulfilled:

$$e\delta V \ll \mu^* H; \quad \hbar\delta\omega \ll \mu^* H.$$

We already mentioned (see Sec. 2) that the collisions of photoelectrons with the conduction electrons may give rise to a lowering of the amplitude of the oscillations of the total photocurrent. In the investigation of the oscillations of the energy distribution of the photoelectrons another effect will superpose itself on this effect: the "intermingling" of the electrons as a consequence of the collisions.

The experimentally observed oscillations can be enhanced by applying a variable saw-tooth voltage on the anode of the photoelement and including a resonant circuit tuned to the frequency $\nu = fV_0/\Delta V$, where f and V_0 are the frequency and the amplitude of the sawtooth voltage.

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232