RENORMALIZATION IN PARITY-NONCONSERVATION THEORY

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A method is proposed for the renormalization of mass, charge, and wave functions in the parity-nonconservation theory. The method is checked in the case in which the "three-Γ approximation" equation is used for the vertex part.

THE renormalization program (for a theory which is invariant with respect to time reversal) should be somewhat different in parity-nonconservation theory than in ordinary theory, for the reason that the radiative corrections based on an interaction which does not conserve parity contains terms of a type different (pseudoscalar) than those contained in the free Lagrangian. Renormalization in parity-nonconservation theory was considered by d'Espagnat and Prentki[1] and Sekine.[2] The former did not study the problem sufficiently fully, and the latter reached a conclusion that a systematic renormalization program was impossible, which is erroneous from our point of view. It is therefore appropriate to consider this question again.

In the presence of parity nonconservation, the free equations can, generally speaking, have a form that differs from the ordinary. Therefore, we shall first (in Sec. 1) consider the properties of free equations in the presence of parity nonconservation. In Sec. 2 the renormalization program will be given, and in Sec. 3 it will be shown how this program is realized in a concrete example — the calculation of the asymptotic behavior of the Green's function and the vertex parts for large p^2.

1. THE EQUATIONS OF MOTION OF FREE PARTICLES IN PARITY NONCONSERVATION

In parity nonconservation, the equations of motion of free particles generally differ from the form usually taken. At first glance, it can be shown that in such forms of parity violation in which time (combined) parity is not conserved, the observed effects appear in the propagation of a free particle in a vacuum, for example, the rotation of the plane of polarization of light. Actually however, because of nonconservation of parity, no observed effects can arise in the propagation of the free particle in a vacuum. Physically, this is almost obvious, inasmuch as the motion of the free particle is determined by its momentum and the projection of its spin on a certain axis, which do not change by virtue of the laws of conservation, regardless of whether parity is conserved or not.

In the case of bosons with spin zero or one, the hypothesis of parity nonconservation does not change the equations of motion, as can be seen directly from the free Lagrangian. In fact, for bosons with spin zero one can construct only the scalars $(\partial\varphi/\partial x_\nu)^2$ and φ^2 from the field φ and its first derivatives; it is not possible to construct pseudoscalars, so that the ordinary expression for the Lagrangian density

$$L = -\tfrac{1}{2}[(\partial\varphi/\partial x_\nu)^2 + \mu^2\varphi^2] \tag{1.1}$$

does not change upon the consideration of parity nonconservation. For bosons with spin one, for example, for the electromagnetic field A_μ, in addition to the scalar $F_{\mu\nu}^2$ ($F_{\mu\nu} = \partial A_\nu/\partial x_\mu - \partial A_\mu/\partial x_\nu$) which is quadratic over the field, there is also the pseudoscalar $\epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$. However, the addition of this pseudoscalar to the Lagrangian

$$L = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{4}\lambda\varepsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma} \tag{1.2}$$

does not change the equations of motion, inasmuch as the second term in (1.2) reduces to a total derivative after differentiation by parts.

For particles with spin one-half, the most general expression for the Lagrangian density of free particles in the presence of parity nonconservation differs from the ordinary, which corresponds to the Dirac equation, and has the form*

$$L = \bar{\psi}[\hat{p}(1+\lambda\gamma_5) - m - i\mu\gamma_5]\psi,$$
$$\hat{p} = i\gamma_\mu\partial/\partial x_\mu \equiv i(\beta\partial/\partial t + \beta\alpha\nabla), \tag{1.3}$$

Here, because of the Hermite character of the Lagrangian, the constants λ and μ should be real. The equation for the ψ function

$$[\hat{p}(1+\lambda\gamma_5) - m - i\mu\gamma_5]\psi = 0, \tag{1.4}$$

*The following notation is employed: $\gamma_\mu = \{\beta, \beta\alpha\}$, $\gamma_5 = -i\gamma_1\gamma_2\gamma_3\gamma_4$, $\gamma_5^2 = 1$, $\hat{a} = a_\mu\gamma_\mu = a_0\beta - \beta\alpha a$.

follows from (1.3); upon squaring this equation, we easily find the connection between energy and momentum:

$$p^2 = (m^2 + \mu^2)/(1 - \lambda^2) \text{ or } E^2 = \mathbf{p}^2 + (m^2 + \mu^2)/(1 - \lambda^2). \quad (1.5)$$

Thus, account of parity nonconservation leads only to appearance of the effective mass $m_{eff} = \sqrt{(m^2 + \mu^2)/(1 - \lambda^2)}$.

It is important to note that (1.5) has meaning only for $|\lambda| < 1$. We shall see below that if the terms which do not conserve parity in the free equation appear only because of the interaction, and the ordinary Dirac equation holds for "bare" particles, then $|\lambda|$ actually does not exceed unity.

Invariance relative to time reversal or charge conjugation imposes definite limitations on the general form of the Lagrangian (1.3). It is easy to prove that in the case of T-invariant theory, $\mu = 0$, while in the case of C-invariant theory, on the other hand, $\lambda = 0$. Inasmuch as T-invariant theory possesses the greater interest, we shall consider this case in more detail.*

The free Lagrangian in the T-invariant theory has the form

$$L = \bar{\psi}\,[\hat{p}(1 + \lambda\gamma_5) - m]\,\psi, \quad (1.6)$$

$$p^2 = m^2/(1 - \lambda^2), \qquad |\lambda| < 1. \quad (1.7)$$

In order to carry out the second quantization, we determine the canonically conjugate momentum

$$\pi = \partial L/\partial(\partial\psi/\partial t) = -i\bar{\psi}\beta(1 + \lambda\gamma_5) \quad (1.8)$$

and construct the Hamiltonian

$$H = \pi\partial\psi/\partial t - L = -i\bar{\psi}\boldsymbol{\gamma}\nabla(1 + \lambda\gamma_5)\psi + m\bar{\psi}\psi, \qquad H = H^+. \quad (1.9)$$

The anticommutation relations for canonically conjugate variables taken at the same instant of time can be determined in the following fashion:

$$\{\psi_\alpha(\mathbf{r}, t), \pi_\beta(\mathbf{r}', t)\} = i\delta_{\alpha\beta}\,\delta(\mathbf{r} - \mathbf{r}'). \quad (1.10)$$

Substituting (1.8) in (1.10), we find the value of the anticommutator of the functions ψ and $\bar{\psi}$ at the same instant of time:

$$\{\psi_\alpha(\mathbf{r}, t), \bar{\psi}_\beta(\mathbf{r}', t)\} = (1 - \lambda^2)^{-1}[\gamma_4(1 + \lambda\gamma_5)]_{\alpha\beta}\,\delta(\mathbf{r} - \mathbf{r}'). \quad (1.11)$$

The remaining anticommutators vanish as usual:

$$\{\psi_\alpha, \psi_\beta\} = \{\bar{\psi}_\alpha, \bar{\psi}_\beta\} = 0. \quad (1.12)$$

In accord with the ordinary rules of quantum mechanics, the time dependence of the ψ operator can be obtained from the equation

$$-i\partial\psi/\partial t = \int d^3x'\,[H(x'), \psi(x)]. \quad (1.13)$$

Computing the commutator in (1.13) by means of (1.11), we obtain the equation

$$[\hat{p}(1 + \lambda\gamma_5) - m]\,\psi \equiv i\gamma_\mu(1 + \lambda\gamma_5)\,\partial\psi/\partial x_\mu - m\psi = 0, \quad (1.14)$$

as was to be expected. It is interesting to note that if we rewrite (1.14) in the form of the Schrödinger equation

$$i\partial\psi/\partial t = \mathscr{H}\psi, \quad (1.15)$$

then the corresponding Hamiltonian

$$\mathscr{H} = \boldsymbol{\alpha}\mathbf{p} + \beta m(1 + \lambda\gamma_5)/(1 - \lambda^2) \quad (1.16)$$

becomes non-Hermitian. Of course, this is not any defect of the theory, inasmuch as the Hamiltonian (1.9) in second-quantization theory was Hermitian, and the eigenvalues of the energy were real [which also follows directly from (1.7)].

In the method of quantization of (1.10) and (1.11), used by us, the anticommutator $\{\psi_\alpha(\mathbf{r}, t), \bar{\psi}_\beta(\mathbf{r}', t)\}$ is an integral of the motion. It is easy to demonstrate this by calculating the derivative of the anticommutator with respect to time by means of (1.15), (1.16), and (1.11). In the case of any other method of quantization, for example, when just γ_4 appears on the right-hand side of (1.11), the anticommutator will no longer be an integral of the motion.

Equation (1.14) and the anticommutation relations (1.11) do not have the usual form in T-invariant theory with parity nonconservation. One can show, however, that all the physical consequences for the motion of a free particle (or for the motion of a particle in an electromagnetic field) are the same as in the theory with parity conservation. For this purpose, we carry out a transformation of the ψ functions:

$$\psi = S\psi', \qquad \bar{\psi} = \bar{\psi}'\tilde{S}, \qquad \tilde{S} = \beta S^+\beta \quad (1.17)$$

and put the matrix S in the form

$$S = A + B\gamma_5, \quad (1.18)$$

where A and B are real numbers. We substitute (1.17) in (1.6), and require that there be no terms $\hat{p}\gamma_5$ in the new expression for the Lagrangian, and that the coefficient of $\hat{p}$ be equal to unity. This leads to the following set of equations for the coeffi-

*In C-invariant theory, the term with the time derivative $\partial\psi/\partial t$ does not change as a result of parity violation. Therefore, the commutation relations remain the same as in ordinary theory, and additional problems do not appear in carrying out the renormalization program.

cients A and B:

$$(A^2+B^2)\lambda+2AB=0, \qquad A^2+B^2+2AB\lambda=1, \quad (1.19)$$

the solution of which has the form

$$A=\tfrac{1}{2}[(1+\lambda)^{-1/2}+(1-\lambda)^{-1/2}],$$
$$B=\tfrac{1}{2}[(1+\lambda)^{-1/2}-(1-\lambda)^{-1/2}]. \quad (1.20)$$

For such values of A and B, the Lagrangian (1.6) is expressed in terms of $\overline{\psi}'$ and ψ', and takes the form

$$L=\overline{\psi}'(\hat{p}-m/\sqrt{1-\lambda^2})\psi', \quad (1.21)$$

where the anticommutation relations (1.11) reduce to the usual:

$$\{\psi'_\alpha(\mathbf{r},t),\ \overline{\psi}'_\beta(\mathbf{r}',t)\}=(\gamma_4)_{\alpha\beta}\,\delta(\mathbf{r}-\mathbf{r}'). \quad (1.22)$$

In the presence of an electromagnetic field the operator $\hat{p}$ should be replaced by $\hat{p}-e\hat{A}$ in the Lagrangian (1.6) because of gauge invariance. It is clear that the transformation (1.17) will hold in this case also and lead to (1.21) with the substitution of $\hat{p}-e\hat{A}$ for $\hat{p}$. We thus see that for the case of the motion of a free particle, or of a particle in an electromagnetic field, the entire effect of the introduction of the parity-nonconserving term $\hat{p}\gamma_5$ into the Lagrangian leads, after transformation of the ψ functions, to the appearance of an effective mass $m_{eff}=m/\sqrt{1-\lambda^2}$. Inasmuch as the mass of the particles is determined from experiment, it is physically impossible to distinguish between the Lagrangians (1.6) and (1.21).

Some interest attaches to the problem of how the equations change upon introduction of the pseudoscalar term $\lambda\hat{p}\gamma_5$ in the presence of interaction with other fields. In order to make this clear, it suffices to consider how the five covariant expressions formed from ψ functions, scalar, pseudoscalar, tensor, vector, and axial vector, change under the transformation (1.17). Making use of (1.16) and (1.20), it is easy to prove that the scalar, pseudoscalar, and tensor change in the transformation (1.17) according to the law

$$\overline{\psi}O_j\psi=\overline{\psi}'O_j\psi'/\sqrt{1-\lambda^2}, \qquad O_j=S,T,P, \quad (1.23)$$

while the vector and axial vector change as

$$\overline{\psi}O_j\psi=\overline{\psi}'O_j(1-\lambda\gamma_5)\psi'/(1-\lambda^2), \qquad O_j=V,A. \quad (1.24)$$

Equations (1.23) and (1.24) do not have much meaning, because if one considers that the term $\lambda\hat{p}\gamma_5$ arises because of an interaction which does not conserve parity, then the same interaction produces corrections to the vertex part of the interaction under consideration. For the interactions S, T, and P it follows from the T invariance that the general form of the interaction operator cannot contain the matrix γ_5. For vector interactions the correction should have the same form as in electrodynamics, i.e., $1+\lambda\gamma_5$ and, consequently, should compensate the factor $(1-\lambda\gamma_5)/(1-\lambda^2)$ in (1.24). It can be shown that for axial interaction the ratio of the parity-nonconservation term (in the correction to the vertex part) to the parity-conservation term is the same as in the vector interaction, i.e., the correction will also have the form $1+\lambda\gamma_5$ and will be compensated. The latter statements (relative to V and A interactions) hold only for vertex parts with free ends, while for vertex parts with $p^2\neq m^2_{phys}$, terms not conserving parity of course remain.

One must also make the following observation. The transformation (1.17) is not unitary. Therefore, in the physical interpretation, it is necessary to assume that the physical particles are described by the functions ψ' (and not by ψ) so that, for example, for a particle at rest, ψ' (and not ψ) has the form $\psi'=\begin{pmatrix} v \\ 0 \end{pmatrix}$, where v is a two-component spinor.

2. RENORMALIZATION IN PARITY-NONCONSERVATION THEORY

As a parity-nonconservation interaction, we consider the interaction of a charged boson* possessing spin zero with a fermion field having the two states X and N. (The interaction in the decay $\Lambda^0\to N+\pi$ serves as an example of such interaction.) We shall write down the total Lagrangian in the form

$$L=\overline{\psi}_X(\hat{p}-m_{0X})\psi_X+\overline{\psi}_N(\hat{p}-m_{0N})\psi_N+\varphi^+(p^2-\mu_0^2)\varphi$$
$$+g_0\overline{\psi}_X(\alpha+\beta\gamma_5)\psi_N\varphi+\text{Herm. Conj.} \quad (2.1)$$

where m_{0X}, m_{0N}, μ_0 are the bare masses of the corresponding particles, g_0 is the bare charge, $\alpha^2+\beta^2=1$, α and β are real.

As usual (see, for example, reference 3), it is convenient to carry out the renormalization of the mass and of the ψ functions of X and N by considering the equation for the corresponding Green's function which, for example, has the following form for N:

$$\{\hat{p}-m_{0N}-M(\hat{p})\}\,G_N(\hat{p})=1; \quad (2.2)$$

*These considerations can be carried through in a quite similar fashion for the case of interaction with a neutral boson field. Of course, it is necessary to bear in mind that in this case N and X cannot be one and the same particle, inasmuch as this would lead to the vanishing of terms which do not conserve parity because of the T invariance and the Hermitian character of the Hamiltonian.

$M(\hat{p})$ is the mass operator:

$$M(\hat{p}) = \frac{g_0^2}{4\pi^3 i} \int d^4k (\alpha - \beta\gamma_5)$$
$$\times G_X(\hat{p} + \hat{k}) \Gamma^+(\hat{p}, \hat{p} + \hat{k}; \hat{k}) D(k^2); \quad (2.3)$$

$\Gamma^+(\hat{p}, \hat{p}+\hat{k}; \hat{k})$ is the exact vertex part corresponding to the interaction $\bar{\psi}_X(\alpha + \beta\gamma_5)\psi_N\varphi$ (Γ^- corresponds to the Hermitian-adjoint interaction $\bar{\psi}_N(\alpha - \beta\gamma_5)\psi_X\varphi^+$); $D(k^2)$ is the Green's function of the meson.

In T-invariance theory, the general expression for the mass operator should have the form

$$M(\hat{p}) = \hat{p}M_1(p^2) + \hat{p}\gamma_5 M_2(p^2) + M_3(p^2), \quad (2.4)$$

where M_1, M_2, M_3 are scalar functions of p^2, and the equation for the Green's function (2.2) is written as

$$\{\hat{p} - m_{0N} - \hat{p}M_1(p^2) - \hat{p}\gamma_5 M_2(p^2) - M_3(p^2)\} G_N = 1. \quad (2.5)$$

We transform (2.5) in the following manner:

$$\{\hat{p}[1 - M_1(m_{\text{phys}}^2)] - \hat{p}\gamma_5 M_2(m_{\text{phys}}^2) - m_{0N} - M_3(m_{\text{phys}}^2)$$
$$- \hat{p}[M_1(p^2) - M_1(m_{\text{phys}}^2)] - \hat{p}\gamma_5[M_2(p^2) - M_2(m_{\text{phys}}^2)]$$
$$- [M_3(p^2) - M_3(m_{\text{phys}}^2)]\} G_N = 1, \quad (2.6)$$

where m_{phys} is the physical mass. It is seen from (2.6) that for $p^2 \to m_{\text{phys}}^2$, the equation for the Green's function G_N has the form

$$Z_2^{-1}\{\hat{p}(1 + \lambda\gamma_5) - m\} G_N = 1; \quad (2.7)$$
$$Z_2^{-1} = 1 - M_1(m_{\text{phys}}^2),$$
$$m = [m_{0N} + M_3(m_{\text{phys}}^2)] Z_2, \; \lambda = -Z_2 M_2(m_{\text{phys}}^2), \quad (2.7')$$

that is (with accuracy up to the factor Z_2^{-1}), the form of the equation for the Green's function corresponding to the free equation (1.14).

By studying the free equation (1.14), we came to the conclusion that in the transition to physical particles described by the ordinary Dirac equation (with the observed mass $m_{\text{phys}} = m/\sqrt{1-\lambda^2}$) it is necessary to make the transformation (1.17). Since the Green's function is defined in terms of the T-product as

$$G_{\alpha\beta}(x, x') = \langle 0 | T(\psi_\alpha(x), \bar{\psi}_\beta(x')) | 0 \rangle, \quad (2.8)$$

then the transformation of the ψ functions (1.17) produces the following transformation of the Green's function:

$$G = SG'\tilde{S}. \quad (2.9)$$

We now make another numerical renormalization of the ψ functions: $\psi' = Z^{1/2}\psi_R$ and the Green's function: $G' = Z_2 G_R$. Then (2.6) takes the form

$$\Big\{\hat{p} - m_{\text{phys}} - \hat{p}\frac{1 - \lambda\gamma_5}{1 - \lambda^2} Z_2 [M_1(p^2) - M_1(m_{\text{phys}}^2)]$$
$$- \hat{p}Z_2 \frac{\gamma_5 - \lambda}{1 - \lambda^2}[M_2(p^2) - M_2(m_{\text{phys}}^2)]$$
$$- (Z_2/\sqrt{1-\lambda^2})[M_3(p^2) - M_3(m_{\text{phys}}^2)]\Big\} G_R = 1. \quad (2.10)$$

For $p^2 \to m_{\text{phys}}^2$, the renormalized Green's function is $G_R \to (\hat{p} - m_{\text{phys}})^{-1}$, i.e., it has the form of the Green's function of a free particle with mass m_{phys}.

Renormalization of the mass and of the ψ functions for the second fermion takes place in precisely the same fashion. Renormalization of the mass and of the wave equations of the boson is carried out just as in ordinary theory, inasmuch as parity nonconservation does not lead to the appearance of terms of a new type in the expression for the polarization operator.

There still remains the charge renormalization. For this purpose, we consider the exact expression for the interaction energy [$a(p^2, (p-k)^2, k^2)$ and $b(p^2, (p-k)^2, k^2)$ are certain functions of p^2, $(p-k)^2$, k^2]:

$$g_0\bar{\psi}_X(\hat{p})[a(p^2, (p-k)^2, k^2)$$
$$+ \gamma_5 b(p^2, (p-k)^2, k^2)]\psi_N(\hat{p} - \hat{k})\varphi(k^2), \quad (2.11)$$

for $p^2 \to m_X^2$, $(p-k)^2 \to m_N^2$ and $k^2 \to \mu^2$ (m_X, m_N, μ are the physical masses). Then, in accordance with the definition of physical charge, we should have

$$g_0\bar{\psi}_X(m_X)[a(m_X^2, m_N^2, \mu^2) + b(m_X^2, m_N^2, \mu^2)\gamma_5]\psi_N(m_N)\varphi(\mu^2)$$
$$= g\bar{\psi}_{XR}(m_X)(\alpha_R + \beta_R\gamma_5)\psi_{NR}(m_N)\varphi_R(\mu),$$
$$\alpha_R^2 + \beta_R^2 = 1. \quad (2.12)$$

The renormalized ψ functions are connected with the usual relations

$$\psi_X = Z_{2X}^{1/2}(A_X - B_X\gamma_5)\psi_{XR},$$
$$\psi_N = Z_{2N}^{1/2}(A_N + B_N\gamma_5)\psi_{NR}, \; \varphi = Z_3^{1/2}\varphi_R. \quad (2.13)$$

We note that for equal masses of N and X, there will be $Z_{2X} = Z_{2N}$, $A_N = A_X$, $B_N = B_X$ and $\lambda_N = -\lambda_X$. This circumstance follows directly from a comparison of the equations of the mass operators (2.3) for N and X. Substituting (2.13) and (2.12), we find the relation between the renormalized and non-renormalized charges (and also the expression for the values of α_R and β_R) at $m_X = m_N$:

$$g^2 = g_0^2 Z_2^2 \tilde{Z}_1^{-2} Z_3, \quad \alpha_R = \tilde{Z}_1(a - b\lambda)/(1 - \lambda^2),$$
$$\beta_R = \tilde{Z}_1(b - a\lambda)/(1 - \lambda^2);$$
$$\tilde{Z}_1 = (1 - \lambda^2)/\sqrt{(a^2 + b^2)(1 + \lambda^2) - 4ab\lambda}. \quad (2.14)$$

The expression for the renormalization of the charge (2.14) is materially simplified in the case of the two-component theory. Then, as can be proved (see the following section), $a = b \equiv Z_1^{-1}/\sqrt{2}$, $\lambda = 1 - Z_2$ and, in place of (2.14), we have

$$g^2 = g_0^2 Z_2^2 Z_1^{-2} Z_3 (1+\lambda)^{-2} = g_0^2 Z_2^2 Z_1^{-2} Z_3 (2 - Z_2)^{-2}. \quad (2.14')$$

The exact vertex part (for $p^2 = m^2$) has the form

$$\Gamma^+ = Z_1^{-1}(1+\gamma_5)/\sqrt{2},$$

so that Z_1^{-1} has the meaning of an ordinary renormalized constant in the renormalization of the vertex part.

The remaining part of the proof of the renormalizability of the theory (in particular, the elimination of the so-called "b divergences"), inasmuch as it is not connected with the specific parity nonconservation, can be carried out in the same fashion as in the usual theory.[4-6]

The renormalization that we have carried out will be valid only in the case in which the quantity λ, determined in accord with (2.7), is less than unity in absolute value, $|\lambda| \leq 1$. We now shall show that this is actually the case. For this purpose, we generalize the spectral decomposition of Lehmann-Kallen to the case of noconservation of spatial parity under the preservation of time (combination) parity (see also reference 2): just as in reference 7, we consider the function

$$G_{\alpha\beta}^{(+)}(x, x') = \langle 0 | \psi_{\alpha R}(x) \overline{\psi}_{\beta R}(x') | 0 \rangle, \; G_{\alpha\beta}^{(-)}(x, x') = \langle 0 | \overline{\psi}_{\beta R}(x') \psi_{\alpha R}(x) | 0 \rangle \quad (2.15)$$

and the anticommutator

$$G_{\alpha\beta}^{(a)}(x, x') = \langle 0 | \{\psi_{\alpha R}(x), \overline{\psi}_{\beta R}(x')\} | 0 \rangle. \quad (2.16)$$

in addition to the Feynman Green's function (2.8). (As is well known, the Lehmann-Kallen relations are written down for renormalized functions.) Making use of CP invariance, it is easy to prove that the function $G^{(-)}$ can be expressed in terms of $G^{(+)}$ by means of the equality (C is the charge-conjugation matrix)

$$G^{(-)T}(-\mathbf{r}', t'; -\mathbf{r}, t) = -C\gamma_4 G^{(+)}(-\mathbf{r}, t; -\mathbf{r}', t')\gamma_4 C^{-1} \quad (2.17)$$

and, consequently, all the vacuum averages, including the Feynman Green's function and the anticommutator, are expressed in terms of $G^{(+)}$.

According to Lehmann,[7] the function $G^{(+)}$ can be written as

$$\langle 0 | \psi_{\alpha R}(x) \overline{\psi}_{\beta R}(x') | 0 \rangle = \sum_p \langle 0 | \psi_{\alpha R}(x) | \Phi_p \rangle \langle \Phi_p | \overline{\psi}_{\beta R}(x') | 0 \rangle = \sum_p C_p^\alpha \overline{C}_p^\beta \exp\{-ip(x - x')\}, \quad (2.18)$$

where Φ_p is the state with the 4-momentum p. From a consideration of the relativistic invariance and the CP invariance, the general expression for $\sum C_p^\alpha \overline{C}_p^\beta$ (the summation is taken over all states with 4-momentum p) should have the form

$$(2\pi)^3 \sum_p C_p^\alpha \overline{C}_p^\beta = (\hat{p}_{\alpha\beta} + \delta_{\alpha\beta}\sqrt{p^2})\rho_1(p^2) - (\hat{p}\gamma_5)_{\alpha\beta}\rho_3(p^2) - \delta_{\alpha\beta}\rho_2(p^2) = \{(\hat{p}[1 - f(p^2)\gamma_5])_{\alpha\beta} + \delta_{\alpha\beta}\sqrt{p^2}\}\rho_1(p^2) - \delta_{\alpha\beta}\rho_2(p^2);$$
$$f(p^2) = \rho_3(p^2)/\rho_1(p^2), \quad (2.19)$$

where ρ_1, ρ_2, ρ_3 are real functions. Substituting (2.19) in (2.18), we obtain the general expression for the function $G^{(+)}$, and consequently, for the arbitrary Green's function:

$$G^{(\ )}(x) = \int_0^\infty \{[\hat{p}(1 - f(\varkappa^2)\gamma_5) + \varkappa]\rho_1(\varkappa^2) - \rho_2(\varkappa^2)\}\Delta^{(\ )}(x, \varkappa^2)\, d\varkappa^2; \quad (2.20)$$

Here $\Delta^{(\ )}(x, \kappa^2)$ are the vacuum functions for the boson field with mass κ.

In the momentum representation, Eq. (2.20) is written, for example, for the Feynman function, as

$$G_F(\hat{p}) = \int_0^\infty \frac{[\hat{p}(1 - f(\varkappa^2)\gamma_5) + \varkappa]\rho_1(\varkappa^2) - \rho_2(\varkappa^2)}{p^2 - \varkappa^2 + i\varepsilon}\, d\varkappa^2. \quad (2.21)$$

For the rest of the functions, the rule for bypassing the poles $p = \pm\kappa$ should be chosen in similar fashion.

The functions $f(\kappa^2)$, $\rho_1(\kappa^2)$, $\rho_2(\kappa^2)$ entering into (2.19) – (2.21) satisfy the inequalities

$$\rho_1(\varkappa^2) \geqslant 0, \; 0 \leqslant \rho_2(\varkappa^2) \leqslant 2\varkappa\rho_1(\varkappa^2), \; f^2(\varkappa^2) \leqslant 1. \quad (2.22)$$

The first of the inequalities (2.22) is obtained immediately after multiplying (2.19) by $(\gamma_4)_{\beta\alpha}$ and summing over α. In order to obtain the other inequalities, it suffices to compute the sum

$$\sum_\alpha h_\alpha h_\alpha^* = p_0\{\rho_1(p^2)(\alpha - p)^2 + p^2 x[x - 2f(p^2)] + 2\alpha\rho_2(p^2)\} \geqslant 0; \quad (2.23)$$

$$h_\alpha = \{\gamma_4[\hat{p}(1 + x\gamma_5) - \alpha]\}_{\alpha\gamma} C_p^\gamma, \quad (2.24)$$

x and α are arbitrary real numbers. The second inequality of (2.22) follows from (2.23) for $x = 0$ and $\alpha = \alpha_{\min} = (-\rho_2 + p\rho_1)/\rho_1$, while for

$\alpha = 0$, and $x = x_{min} = f$, we obtain the third.*

A stable particle with physical mass m corresponds to the appearance of a δ function in $\rho_1(\kappa^2)$: $\rho_1(\kappa^2) = \delta(\kappa^2 - m^2)$. Here, it must be true that $\rho_2(m^2) = 0$ and $f(m^2) = 0$. The latter condition follows from the fact that the operators entering into the determination of the vacuum functions are unrenormalized physical quantities which satisfy the Dirac equation (with physical mass m).

By means of the inequalities (2.22), we can now prove the inequality $|\lambda| \leq 1$ which is of interest to us. For this purpose, let us write down the expression for the Lagrangian (2.1) in unrenormalized functions

$$L = Z_{2X}(1-\lambda_X^2)^{-1}\bar{\psi}_{XR}\,[\hat{p}(1-\lambda_X\gamma_5) - \sqrt{1-\lambda_X^2}m_{0X}]\,\psi_{XR} + Z_{2N}(1-\lambda_N^2)^{-1}\bar{\psi}_{NR}\,[\hat{p}(1-\lambda_N\gamma_5) - \sqrt{1-\lambda_N^2}m_{0N}]\,\psi_{NR} + Z_3\varphi_R^+(p^2-\mu_0^2)\varphi_R + Z_1 g\bar{\psi}_{XR}(A_X + B_X\gamma_5) \times (\alpha + \beta\gamma_5)(A_N + B_N\gamma_5)\psi_{NR}. \quad (2.25)$$

The anticommutator of the ψ functions, taken at the same instant of time, will, in accord with (1.11), be equal to

$$\{\psi_{\alpha R}(\mathbf{r}, t), \bar{\psi}_{\beta R}(\mathbf{r}', t)\} = Z_2^{-1}[\gamma_4(1-\lambda\gamma_5)]_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}') \quad (2.26)$$

(the index X or N is omitted). On the other hand, the expression

$$\{\psi_{\alpha R}(\mathbf{r}, t), \bar{\psi}_{\beta R}(\mathbf{r}', t)\} = \int_0^\infty [\gamma_4(1-f(\varkappa^2)\gamma_5)]_{\alpha\beta}\rho_1(\varkappa^2)\,d\varkappa^2\delta(\mathbf{r}-\mathbf{r}'). \quad (2.27)$$

follows from (2.20) for the same anticommutator.

Comparing (2.26) and (2.27), we have

$$Z_2^{-1} = \int_0^\infty \rho_1(\varkappa^2)\,d\varkappa^2;$$

$$Z_2^{-1}\lambda = \int_0^\infty f(\varkappa^2)\rho_1(\varkappa^2)\,d\varkappa^2, \quad (2.28)$$

$$\lambda = \int_0^\infty f(\varkappa^2)\rho_1(\varkappa^2)\,d\varkappa^2 \Big/ \int_0^\infty \rho_1(\varkappa^2)\,d\varkappa^2. \quad (2.29)$$

Since $\rho_1 \geq 0$ and has the component $\delta(\kappa^2 - m^2)$, then, as follows from (2.28), $Z_2^{-1} \geq 1$ and

$$0 \leqslant Z_2 \leqslant 1,$$

i.e., we obtain the usual inequality for Z_2. From (2.29) and the inequalities $\rho_1 \geq 0$ and $f^2 \leq 1$, it follows directly that

$$|\lambda| \leqslant 1.$$

The inequality $|\lambda| \leq 1$ is proved in general form and the renormalization of the δ functions (2.13) carried out above is justified in the same fashion.

Let us now sum up. Renormalization of the mass and charge in the case of a T-invariant parity-nonconserving interaction can be carried out in the same fashion as renormalization in the case of parity conservation. Here, the equation for the renormalized ψ functions in the case of free motion is the usual Dirac equation with physical mass. However the free Lagrangian (2.25) expressed in these ψ functions and the Poisson brackets (2.26) differ from the usual in the presence of terms which do not conserve parity.

3. THE ASYMPTOTIC FORM OF THE GREEN'S FUNCTION AND THE EFFECTIVE CHARGE IN A SERIES WITH WEAK PARITY-NONCONSERVING INTERACTION

For the investigation of the asymptotic behavior (at large p^2) of the Green's function in weak coupling theory, Landau[8] proposed to use the so-called "three-Γ approximation" in which all the terms of order $[g^2 \ln(p^2/m^2)]^n$ are kept in the equations for the Green's functions and the vertex part, while terms of the order $g^{2m}[\ln(p^2/m^2)]^n$ $(m > n)$ are discarded. Solution of such equations for the case of electrodynamics[8] and pseudoscalar theory[9,10] has led, however, to unfavorable results, inasmuch as the Green's functions that are obtained have nonphysical poles at a certain value $p^2 = p_{cr}^2$ and the three-Γ approximation proves inapplicable beyond the pole $(p^2 > p_{cr}^2)$ where the weak coupling becomes effectively strong. This result led Landau and Pomeranchuk[11-13] to conclude that the renormalized charged vanished, so that the theory was unsuitable.

It is to ascertain whether the difficulty that has been pointed out persists in the case of interactions that do not preserve parity, i.e., whether the solutions of the equations in the three-Γ approximation will contain nonphysical poles. By solving this equation, one will be able to test on a concrete example the method of renormalization set up above.

We choose a formal model, namely a scalar symmetric theory, in which both the fermions X and N have isotopic spin one-half, while the meson

*Interesting consequences may arise if $f(\kappa^2) \to \pm 1$ for $\kappa^2 \to \infty$. In this case, the general proof of Lehmann that the exact Green's functions should disappear as $p^2 \to \infty$ no more rapidly than the free ones will evidently not have adequate rigor in the analysis of the divergences, inasmuch as in the calculation of the polarization operator for parity conservation, the interaction of terms containing a higher divergence can vanish as a result of the renormalization of the operators $1 + f\gamma_5$ or $1 - f\gamma_5$ [see Eq. (3.20)].

has isotopic spin one. We shall assume that the fermions enter into the interaction with two components of their wave functions, so that the Hamiltonian of the interaction is written as

$$H_I = g_0\bar{\psi}_X(1+\gamma_5)\tau_i\psi_N\varphi_i + g_0\bar{\psi}_N(1-\gamma_5)\tau_i\psi_X\varphi_i. \quad (3.1)$$

Inasmuch as it is necessary for us to test our scheme of renormalization, we shall solve the corresponding set of equations directly, not using the renormalization group.[14,15] The set of equations for the Green's function and the vertex parts will have the form (see reference 10)

$$\{\hat{p} - m_0 + 3\frac{g_0^2}{4\pi^3 i}\int d^4k(1-\gamma_5)G_X(\hat{p}-\hat{k}) \times\Gamma^+(\hat{p}-\hat{k},\ \hat{p};\ \hat{k})D(k^2)\}G_N(\hat{p}) = 1, \quad (3.2)$$

$$\{\hat{p} - m_0 + 3\frac{g_0^2}{4\pi^3 i}\int d^4k(1+\gamma_5)G_N(\hat{p}-\hat{k}) \times\Gamma^-(\hat{p}-\hat{k},\ \hat{p};\ \hat{k})D(k^2)\}G_X(\hat{p}) = 1, \quad (3.2')$$

$$\{k^2 - \mu_0^2 - 2\frac{g_0^2}{4\pi^3 i}\int d^4p Sp[(1+\gamma_5)G_N(\hat{p}) \times\Gamma^-(\hat{p},\ \hat{p}+\hat{k};\ \hat{k})G_X(\hat{p}+\hat{k})]\}D(k^2) = 1, \quad (3.3)$$

$$\Gamma^+(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}) = 1+\gamma_5 + \frac{g_0^2}{4\pi^3 i}\int d^4q\Gamma^+(\hat{p},\ \hat{p}-\hat{q};-\hat{q}) \times G_N(\hat{p}-\hat{q})\Gamma^-(\hat{p}-\hat{q},\ \hat{p}-\hat{q}-\hat{k};\ -\hat{k}) \times G_X(\hat{p}-\hat{k}-\hat{q})\Gamma^+(\hat{p}-\hat{q}-\hat{k},\ \hat{p}-\hat{k};\ \hat{q})D(q^2), \quad (3.4)$$

$$\Gamma^-(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}) = 1-\gamma_5 + \frac{g_0^2}{4\pi^3 i}\int d^4q\Gamma^-(\hat{p},\ \hat{p}-\hat{q};\ -\hat{q}) \times G_X(\hat{p}-\hat{q})\Gamma^+(\hat{p}-\hat{q},\ \hat{p}-\hat{q}-\hat{k};\ -\hat{k}) \times G_N(\hat{p}-\hat{q}-\hat{k})\Gamma^-(\hat{p}-\hat{q}-\hat{k},\ \hat{p}-\hat{k};\ \hat{q})D(q^2). \quad (3.4')$$

Here Γ^+ and Γ^- are vertex parts corresponding to the interaction $\bar{\psi}_X(1+\gamma_5)\tau_i\psi_N\varphi_i$, $\bar{\psi}_N(1-\gamma_5) \times \tau_i\psi_X\varphi_i$, m_0 and μ_0 are the bare masses (we consider the masses of N and X to be equal).

In the two-component theory, the exact vertex part Γ^+ should be proportional to $1+\gamma_5$, while Γ^- is proportional to $1-\gamma_5$:

$$\Gamma^+(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}) = (1+\gamma_5)s^+(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}),$$
$$\Gamma^-(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}) = (1-\gamma_5)s^-(\hat{p},\ \hat{p}-\hat{k};\ -\hat{k}), \quad (3.5)$$

where s^+ and s^- are operators which do not contain the matrix γ_5 (but only $\hat{p}$ and $\hat{k}$). This proof follows directly from perturbation theory. In fact, any diagram for the vertex part Γ^+ contains the factor $1+\gamma_5$ in its left angle. Therefore, if the rest of the expression is written in the form

$$(1+\gamma_5)M + (1-\gamma_5)N,$$

where M, N are operators which do not contain γ_5, then the terms with N makes no contribution to Γ^+.

Being interested in the asymptotic behavior of the Green's functions, we can throw away terms with the mass and seek solutions of Eqs. (3.2) — (3.4′) in the form[8-10]

$$G_N(\hat{p}) = (\hat{p}/p^2)[1+f_N(p^2)\gamma_5]F_N(p^2),$$
$$G_X(\hat{p}) = (\hat{p}/p^2)[1+f_X(p^2)\gamma_5]F_X(p^2), \quad D(k^2) = \varphi(k^2)/k^2,$$
$$s^\pm(\hat{p}-\hat{k},\ \hat{p};\ -\hat{k}) = s_0^\pm[(p-k)^2,\ p^2,\ k^2] - \hat{k}\hat{p}/(k^2+p^2)s_1^\pm[(p-k)^2,\ p^2,\ k^2]; \quad (3.6)$$

we consider the functions $F_N(p^2)$, $F_X(p^2)$, $\varphi(k^2)$, s_0, s_1 to be slowly changing functions of their arguments.

If we set $f_N = -f_X \equiv f$, $s^+ = s^- \equiv s$, $F_N = F_X \equiv F$, then Eqs. (3.2′) and (3.4′) coincide with Eqs. (3.2) and (3.4), respectively. Thus, it is necessary to solve the following set of equations:

$$\left\{\hat{p} + 6\frac{g_0^2}{4\pi^3 i}(1-\gamma_5)\int\frac{d^4k}{k^2}[1-f((p-k)^2)]\frac{\hat{p}-\hat{k}}{(p-k)^2} \times F[(p-k)^2]\,s(\hat{p}-\hat{k},\ \hat{p};\ \hat{k})\varphi(k^2)\right\} \times(\hat{p}/p^2)[1+\gamma_5 f(p^2)]F(p^2) = 1, \quad (3.7)$$

$$\left\{k^2 - 4\frac{g_0^2}{4\pi^3 i}\int\frac{d^4p}{p^2}[1-f(p^2)]F(p^2) \times[1-f((p+k)^2)]F((p+k)^2) \times \mathrm{Sp}[\hat{p}s(\hat{p},\ \hat{p}+\hat{k};\ \hat{k})(\hat{p}+\hat{k})/(p+k)^2]\right\}\varphi(k^2)/k^2 = 1, \quad (3.8)$$

$$s(\hat{p},\ \hat{p}-\hat{k};\ \hat{k}) = 1 + 4\frac{g_0^2}{4\pi^3 i}\int\frac{d^4q}{q^2}\frac{\hat{p}-\hat{q}}{(p-q)^2}\frac{\hat{p}-\hat{q}-\hat{k}}{(p-q-k)^2} \times[1-f((p-q)^2)][1-f((p-q-k)^2) \times F((p-q)^2)F((p-q-k)^2)s(\hat{p},\ \hat{p}-\hat{q};\ -\hat{q}) \times s(\hat{p}-\hat{q},\ \hat{p}-\hat{q}-\hat{k};\ -\hat{k}) \times s(\hat{p}-\hat{q}-\hat{k},\ \hat{p}-\hat{k};\ \hat{q})\varphi(q^2). \quad (3.9)$$

Multiplying (3.7) by $1-\gamma_5$ and $1+\gamma_5$, successively, we can convert this equation to a set of two equations

$$\left\{\hat{p} + 12\frac{g_0^2}{4\pi^3 i}\int\frac{d^4k}{k^2}\frac{\hat{p}-\hat{k}}{(p-k)^2}[1-f((p-k)^2)]F((p-k)^2) \times s(\hat{p}-\hat{k},\ \hat{p};\ \hat{k})\varphi(k^2)\right\}(\hat{p}/p^2)[1-f(p^2)]F(p^2) = 1, \quad (3.10)$$

$$[1+f(p^2)]F(p^2) = 1. \quad (3.11)$$

Comparing Eqs. (3.8) — (3.10) with the corresponding equations in scalar theory with parity conservation,* it is easy to ascertain that, except for the difference in the coefficients for g_0^2, the

*The equations in scalar theory (for large p^2) have the same form and the same values of the coefficients as in the equations of pseudoscalar theory. Therefore, we can, for example, compare (3.8) – (3.10) with the corresponding formulas of the work of Galanin and others.[10]

only change arising as a result of parity nonconservation reduces to the appearance of the function $[1-f(p^2)]F(p^2)$ in place of the function $F(p^2)$. Therefore, all the discussions carried out previously[9,10] remain valid, and we can immediately write down the set of differential equations for the functions $\Phi(p^2) = [1-f(p^2)]F(p^2)$, $\varphi(p^2)$ and $s_0(p^2)$ (which are analogous to Eqs. (50) — (52) of reference 10):

$$d\Phi(\xi)/d\xi = 6\varkappa s_0^2(\xi)\Phi^3(\xi)\varphi(\xi),$$
$$d\varphi(\xi)/d\xi = 8\varkappa s_0^2(\xi)\Phi^2(\xi)\varphi^2(\xi),$$
$$ds_0(\xi)/d\xi = -4\varkappa s_0^3(\xi)\Phi^2(\xi)\varphi(\xi). \quad (3.12)$$

Here $\kappa = g_0^2/4\pi$; $\xi = \ln(p^2/\Lambda^2)$, Λ is the cutoff momentum; we shall consider Φ, φ and s_0 to be unrenormalized. The values of the coefficients on the right-hand side of (3.12) are changed in comparison with the coefficients in Eqs. (50) — (52) of reference 10 in correspondence with the change of coefficients in the formulas for G, Γ, and D (there is an additional factor of four in the formulas for G and Γ, and of two in the formula for D). Solutions of Eqs. (3.12) corresponding to the initial conditions for $\xi = 0$, $\Phi = \varphi = s_0 = 1$ have the form

$$\Phi(\xi) = Q^{-1/2}, \quad \varphi(\xi) = Q^{-2/3}, \quad s_0(\xi) = Q^{1/3},$$
$$Q = 1 - 12\varkappa\xi. \quad (3.13)$$

The functions $F(\xi)$ and $f(\xi)$ can be found from (3.13) and (3.11), and are seen to be equal to

$$F = (1+Q^{1/2})/2Q^{1/2}, \quad f = (Q^{1/2}-1)/(Q^{1/2}+1). \quad (3.14)$$

In order to carry out charge renormalization on the basis of Eq. (2.14′), it is necessary to express the values of Z_1, Z_2, Z_3 and λ in terms of the values of the functions F, φ, s_0, s. From the renormalization conditions for the functions $\varphi(\xi)$ and $s_0(\xi)$ we have $\varphi = Z_3\varphi_R$, $s_0 = Z_1^{-1}s_{0R}$ and the relations

$$Z_3 = \varphi(\mu^2/\Lambda^2) \approx Q_\Lambda^{2/3}, \quad Z_1^{-1} = s_0(m^2/\Lambda^2) = Q_\Lambda^{-1/3},$$
$$Q_\Lambda^{-1} = 1 + 12(g_0^2/4\pi)\ln(\Lambda^2/m^2). \quad (3.15)$$

In order to express Z_2 and λ in terms of F and f, we write the equation for the Green's function of the N particle in the form (2.7) in the vicinity of $p^2 = m^2$:

$$Z_2^{-1}\hat{p}(1+\lambda\gamma_5)(\hat{p}/p^2)[1+f(m^2/\Lambda^2)\gamma_5]F(m^2/\Lambda^2) = 1. \quad (3.16)$$

We then find

$$\lambda = f(m^2/\Lambda^2),$$

$$Z_2 = F(m^2/\Lambda^2)(1-\lambda^2)$$
$$= F(m^2/\Lambda^2)[1-f^2(m^2/\Lambda^2)] = 1 - f(m^2/\Lambda^2), \quad (3.17)$$

and Eq. (2.14′) for the coupling between the bare and the renormalized charges takes the form

$$g^2 = g_0^2 Z_1^{-2}Z_2^2Z_3(1+\lambda)^2$$
$$= g_0^2 s_0^2\left(\frac{m^2}{\Lambda^2}\right)F^2\left(\frac{m^2}{\Lambda^2}\right)\varphi\left(\frac{m^2}{\Lambda^2}\right)\left[1-f^2\left(\frac{m^2}{\Lambda^2}\right)\right]$$
$$= g_0^2 s_0^2\left(\frac{m^2}{\Lambda^2}\right)\Phi^2\left(\frac{m^2}{\Lambda^2}\right)\varphi\left(\frac{m^2}{\Lambda^2}\right). \quad (3.18)$$

Substituting (3.13) in (3.16), we have

$$g^2 = g_0^2[1+3(g_0^2/\pi)\ln(\Lambda^2/m^2)]^{-1}. \quad (3.19)$$

The renormalized functions are expressed in terms of the experimental charge in the following fashion:

$$\Phi_R = Q_\Lambda^{-1/2}\Phi = Q_R^{-1/2}, \quad \varphi_R = Q_R^{-2/3}, \quad s_{0R} = Q_R^{1/3},$$
$$G_{NR} = \frac{\hat{p}}{p^2}\frac{F(p^2)}{F(m^2)}[1-f(p^2)\lambda + (f-\lambda)\gamma_5]\frac{1}{1-\lambda^2}$$
$$= \frac{\hat{p}}{p^2}\frac{1}{2}Q_R^{-1/2}[1+Q_R^{1/2}-\gamma_5(1-Q_R^{1/2})],$$
$$\frac{F(p^2)}{F(m^2)} = \frac{Q_\Lambda^{1/2}+Q_R^{1/2}}{(Q_\Lambda^{1/2}+1)Q_R^{1/2}}, \quad f(p^2) = -\frac{Q_\Lambda^{1/2}-Q_R^{1/2}}{Q_\Lambda^{1/2}+Q_R^{1/2}},$$
$$\lambda = -\frac{Q_\Lambda^{1/2}-1}{Q_\Lambda^{1/2}+1},$$
$$Q = Q_R/Q_\Lambda, \quad Q_R = 1 - 12(g^2/4\pi)\ln(p^2/m^2),$$
$$Q_\Lambda = 1 - 12(g^2/4\pi)\ln(\Lambda^2/m^2). \quad (3.20)$$

It is not difficult to see that f and λ actually satisfy the conditions $f^2 \leq 1$, $\lambda^2 \leq 1$. The value of the cutoff limit comes from the expression for the renormalized G, Γ, D. In the same way, we have shown that one can systematically and without contradiction carry out the renormalization in the three-Γ approximation in the theory with parity nonconservation.

We shall now analyze the results obtained. First of all, it is evident that the difficulty from the vanishing of the renormalized charge remains in the theory under consideration with parity nonconservation: the renormalized Green's functions have a nonphysical singularity. However, if it can be so expressed, the situation becomes more favorable. The fact is that if we denote the coefficients on the right side of Eqs. (3.12) by a, b and c, respectively, then the presence or absence of a nonphysical pole will be determined by the sign of the quantity $2(a+c)+b$ (there is no pole for $2(a+c)+b<0$). According to the theorem of Lehmann, a and b must be positive. Therefore, the only possibility of removal of the pole is connected with the presence of a large negative coefficient c. In pseudoscalar symmetrical theory with parity conservation, $a = 3/2$, $b = 4$, $c = -1$. Introduction of parity nonconservation changes the coefficients and they become equal to $a = 6$, $b = 8$, $c = -4$, i.e., the coefficients a and c increase by a factor of four while b increases by only a factor of two. Thus consideration of parity nonconservation increases the relative role of the coefficient c.

We note that the introduction of additional fields can improve the situation materially. Thus, for example, if we assume the presence of a scalar meson χ with isotopic spin 0 in addition to the pseudoscalar meson π with isotopic spin 1, then in parity nonconservation the corresponding coefficients are shown to be $a = 2$, $b_\pi = b_\chi = 4$, $c_\pi = c_\chi = -2$ and the value of $2(a + c_\pi) + b_\pi$, or of what is equal to it, $2(a + b_\chi) + c_\chi$ approaches 0 even more closely.

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