MOMENTS OF INERTIA OF ODD ATOMIC NUCLEI

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An expression is derived for the moment of inertia of odd nuclei, with the effect of pair correlation taken into account. The theory is compared with the experiments.

It is well known that in the mass number regions $150 < A < 190$ and $A > 225$ the atomic nuclei are deformed and have rotational levels in addition to single-particle levels. Experiments show that the moments of inertia of odd nuclei are larger than those of neighboring even nuclei on the average by 10 — 20% in units of the moment of inertia for a solid, and in some cases by as much as 30 and even 60% (the ground state of Dy^{161}).

In a number of papers[1] the formula for the moments of inertia of even and odd nuclei was derived and the difference between them was calculated on the basis of the single-particle model,[2] but without taking consistently into account effects due to pair correlations. The inclusion of pair correlations substantially reduces the moment of inertia of the nucleus in comparison to the moment of a solid.[3,4]

In this work use is made of the Green's function technique for a finite system with an odd number of particles.[5] For the calculation of the moment of inertia we make use of the method developed by Migdal[3] for even-even nuclei.

The moment of inertia J is determined by the expression

$$J = \langle M^x \rangle / \Omega, \tag{1}$$

where

$$\langle M^x \rangle = \sum_{\lambda\lambda'} M^x_{\lambda\lambda'} \rho_{\lambda'\lambda}$$

is the average value of the projection of the angular momentum of the system on the x axis, which is perpendicular to the symmetry axis z of the nucleus; $\rho_{\lambda\lambda'}$ is the density matrix, and Ω is the angular velocity whose direction coincides with the x axis. In a previous paper by the authors[5] the density matrix $\rho_{\lambda\lambda'}$ (Eq. 25) for an odd system was calculated for a statistical perturbation. In the case of a rotation of the system the perturbation of the Hamiltonian, as was shown by Migdal,[3] is of the form

$$\hat{V} = -\hat{V}^* = \hat{M}^x \Omega M / M_{\text{eff}},$$

where M is the nucleon mass, M_{eff} is the effective mass of the quasiparticle and for Δ' it is convenient to introduce the notation $\Delta' = if(\mathbf{r})\,\Omega M/M_{\text{eff}}$.

If we use the density matrix $\rho_{\lambda\lambda'}$ obtained in reference 5 to evaluate the expression (1) we obtain

$$J = \sum_{\lambda\lambda'} \frac{(E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} - \Delta^2)\,|M^x_{\lambda\lambda'}|^2 - \Delta \dot{M}^x_{\lambda\lambda'} f_{\lambda'\lambda}}{2E_\lambda E_{\lambda'} (E_\lambda + E_{\lambda'})} \frac{M}{M_{\text{eff}}} + \sum_{\lambda}{}' \frac{2(E^2_{\lambda_0} + \varepsilon_\lambda \varepsilon_{\lambda_0} + \Delta^2)\,|M^x_{\lambda\lambda_0}|^2 + \Delta(\dot{M}_{\lambda\lambda_0} f_{\lambda_0\lambda} + \dot{M}_{\lambda_0\lambda} f_{\lambda\lambda_0})}{E_{\lambda_0}(\varepsilon^2_\lambda - \varepsilon^2_{\lambda_0})}, \tag{2}$$

where $E_\lambda = \sqrt{\epsilon^2_\lambda + \Delta^2}$, ϵ_λ is the single-particle energy measured from the chemical potential ϵ_0, and λ_0 is the state occupied by the odd particle. The prime in the single summation over λ indicates that the state with energy $\epsilon_\lambda = \epsilon_{\lambda_0}$ should be omitted. For a nucleus with N nucleons, Δ is determined from the expression

$$\Delta(N) = \tfrac{1}{2}\,|2E_0(N) - E_0(N+1) - E_0(N-1)|, \tag{3}$$

where $E_0(N)$ denotes the energy of the ground state of the system. The integral equation for f takes the form

$$\sum_{\lambda\lambda'} \varphi_\lambda \varphi^*_{\lambda'} \frac{(\varepsilon_\lambda - \varepsilon_{\lambda'})^2 f_{\lambda\lambda'} + 2\Delta \dot{M}^x_{\lambda\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \sum_{\lambda(\varepsilon_\lambda = \varepsilon_{\lambda_0})} \varepsilon_{\lambda_0} E^{-2}_{\lambda_0} (\varphi_\lambda \varphi^*_{\lambda_0} f_{\lambda\lambda_0} - \varphi_{\lambda_0} \varphi^*_\lambda f_{\lambda_0\lambda}) + \sum_\lambda{}' \frac{\varphi_\lambda \varphi^*_{\lambda_0}(2E_{\lambda_0} - \varepsilon_\lambda + \varepsilon_{\lambda_0})\,[(\varepsilon_\lambda - \varepsilon_{\lambda_0})^2 f_{\lambda\lambda_0} + 2\Delta \dot{M}^x_{\lambda\lambda_0}]}{E_{\lambda_0}(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)(\varepsilon_{\lambda_0} - \varepsilon_\lambda)} - \sum_\lambda{}' \frac{\varphi_{\lambda_0} \varphi^*_\lambda (2E_{\lambda_0} - \varepsilon_{\lambda_0} + \varepsilon_\lambda)\,[(\varepsilon_{\lambda_0} - \varepsilon_\lambda)^2 f_{\lambda_0\lambda} + 2\Delta \dot{M}^x_{\lambda_0\lambda}]}{E_{\lambda_0}(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)(\varepsilon_{\lambda_0} - \varepsilon_\lambda)} = 0. \tag{4}$$

The first term in this expression corresponds to the equation for f for an even-even nucleus, and the first term in (2) corresponds to the moment of inertia of such a nucleus.[3]

We write f as $f = f^0 + f'$, where f^0 denotes the solution of the integral equation for an even-even nucleus. Insofar as Eq. (4) differs from the integral equation for f for an even-even nucleus by the addition of a single summation over λ the modification of its solution will be quasiclassically small in comparison with f^0 (see Appendix). Therefore in the single summation over λ we may replace f by f^0. Then expression (2) becomes

$$J = J_e(\varkappa) - \sum_{\lambda\lambda'} \frac{\Delta \dot{M}^x_{\lambda\lambda'} f'_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \frac{M}{M_{\text{eff}}} + \sum_\lambda{}' \frac{2(E^2_{\lambda_0} + \varepsilon_\lambda \varepsilon_{\lambda_0} + \Delta^2) | M^x_{\lambda\lambda_0}|^2 + \Delta(\dot{M}^x_{\lambda\lambda_0} f^0_{\lambda_0\lambda} + \dot{M}^x_{\lambda_0\lambda} f^0_{\lambda\lambda_0})}{E_{\lambda_0}(\varepsilon^2_\lambda - \varepsilon^2_{\lambda_0})} \times \frac{M}{M_{\text{eff}}}, \quad (5)$$

$$J_e(\varkappa) = \sum_{\lambda\lambda'} \frac{(E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} - \Delta^2) | M^x_{\lambda\lambda'}|^2 - \Delta \dot{M}^x_{\lambda\lambda'} f^0_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}. \quad (6)$$

Here $\kappa = \beta\omega_0/2\Delta$, the parameter on which the moment of inertia of an even-even nucleus depends,[3] is evaluated for values of Δ and β corresponding to the odd nucleus. From the integral equation (4) we obtain the following equation for f':

$$\sum_{\lambda\lambda'} \varphi_\lambda(\mathbf{r})\varphi^*_{\lambda'}(\mathbf{r}) \frac{(\varepsilon_\lambda - \varepsilon_{\lambda'})^2 f'_{\lambda\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + \frac{\varepsilon_{\lambda_0}}{E^2_{\lambda_0}} \sum_{\substack{\lambda \\ (\varepsilon_\lambda = \varepsilon_{\lambda_0})}} (\varphi_\lambda \varphi^*_{\lambda_0} f^0_{\lambda\lambda_0} - \varphi_{\lambda_0}\varphi^*_\lambda f^0_{\lambda_0\lambda})$$
$$+ \sum_\lambda{}' \frac{\varphi_\lambda \varphi^*_{\lambda_0}(2E_{\lambda_0} - \varepsilon_\lambda + \varepsilon_{\lambda_0})[(\varepsilon_\lambda - \varepsilon_{\lambda_0})^2 f^0_{\lambda\lambda_0} + 2\Delta \dot{M}^x_{\lambda\lambda_0}]}{E_{\lambda_0}(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)(\varepsilon_{\lambda_0} - \varepsilon_\lambda)} - \sum_\lambda{}' \frac{\varphi_{\lambda_0}\varphi^*_\lambda(2E_{\lambda_0} - \varepsilon_{\lambda_0} + \varepsilon_\lambda)[(\varepsilon_{\lambda_0} - \varepsilon_\lambda)^2 f^0_{\lambda_0\lambda} + 2\Delta\dot{M}^x_{\lambda_0\lambda}]}{E_{\lambda_0}(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)(\varepsilon_{\lambda_0} - \varepsilon_\lambda)} = 0 \quad (7)$$

and the following equation for f^0:

$$\sum_{\lambda\lambda'} \varphi_\lambda(\mathbf{r})\varphi^*_{\lambda'}(\mathbf{r})[2\Delta \dot{M}^x_{\lambda\lambda'} + (\varepsilon_\lambda - \varepsilon_{\lambda'})^2 f^0_{\lambda\lambda'}]/2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'}) = 0. \quad (8)$$

Multiplying Eq. (7) by f^0 and Eq. (8) by f' and integrating over $\mathbf{r}$ we obtain respectively

$$\sum_{\lambda\lambda'} \frac{(\varepsilon_\lambda - \varepsilon_{\lambda'})^2 f'_{\lambda\lambda'} f^0_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} + 2\Delta \sum_\lambda \frac{(\dot{M}^x_{\lambda\lambda_0} f^0_{\lambda_0\lambda} - \dot{M}^x_{\lambda_0\lambda} f^0_{\lambda\lambda_0})}{(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)(\varepsilon_{\lambda_0} - \varepsilon_\lambda)} + \sum_\lambda{}' \frac{2\Delta(\dot{M}^x_{\lambda\lambda_0} f^0_{\lambda\lambda_0} + f^0_{\lambda_0\lambda}\dot{M}^x_{\lambda_0\lambda})}{E_{\lambda_0}(\varepsilon^2_{\lambda_0} - \varepsilon^2_\lambda)} + \frac{2}{E_{\lambda_0}} \sum_\lambda{}' \frac{|f^0_{\lambda\lambda_0}|^2(\varepsilon_{\lambda_0} - \varepsilon_\lambda)}{\varepsilon_{\lambda_0} + \varepsilon_\lambda} = 0, \quad (9)$$

$$\sum_{\lambda\lambda'} [2\Delta \dot{M}^x_{\lambda\lambda'} f'_{\lambda'\lambda} + (\varepsilon_\lambda - \varepsilon_{\lambda'})^2 f^0_{\lambda\lambda'} f'_{\lambda'\lambda}]/2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'}) = 0. \quad (10)$$

It follows from Eq. (8) that the matrix $f^0_{\lambda\lambda'}$ has the same symmetry properties as the matrix $\dot{M}^x_{\lambda\lambda'}$. Therefore $\dot{M}^x_{\lambda\lambda_0} f^0_{\lambda_0\lambda} - \dot{M}^x_{\lambda_0\lambda} f^0_{\lambda\lambda_0} = 0$. Eliminating f' from the expression for the moment of inertia by making use of Eqs. (9) and (10) we find:

$$J = J_e(\varkappa) + \frac{M}{M_{\text{eff}}} \sum_\lambda{}' \frac{2(E^2_{\lambda_0} + \varepsilon_\lambda\varepsilon_{\lambda_0} + \Delta^2)|M^x_{\lambda\lambda_0}|^2 + 2\Delta(f^0_{\lambda\lambda_0}\dot{M}^x_{\lambda_0\lambda} + f^0_{\lambda_0\lambda}\dot{M}^x_{\lambda\lambda_0}) + (\varepsilon_\lambda - \varepsilon_{\lambda_0})^2 |f^0_{\lambda\lambda_0}|^2}{E_{\lambda_0}(\varepsilon^2_\lambda - \varepsilon^2_{\lambda_0})} \quad (11)$$

In this manner it is seen that J is independent of f' and there is no need for solving Eq. (7).

A solution of the integral equation (8) for f^0 can be expressed in an analytic form only for an oscillator potential. In what follows we evaluate terms containing f^0 for such a potential. According to Migdal,[3] f^0 has the form

$$f^0(\mathbf{r}) = -(g_1 + g_2)\dot{M}^x(\mathbf{r})/2\Delta(g_1\nu_1^2 + g_2\nu_2^2),$$
$$g(\nu) = \frac{\sinh^{-1}\nu}{\nu\sqrt{1+\nu^2}}, \quad g_1 = g(\nu_1), \quad g_2 = g(\nu_2),$$
$$\nu_1 = \frac{(\omega_z - \omega_y)}{2\Delta}, \quad \nu_2 = \frac{(\omega_z + \omega_y)}{2\Delta}, \quad (12)$$

where ω_z, $\omega_x = \omega_y$ are the corresponding frequencies. By substituting (12) into (11) and setting $\epsilon_{\lambda_0} = 0$ we reduce (11) to the form

$$J = J_e(\varkappa) + \sum_\lambda{}' \frac{|\dot{M}^x_{\lambda\lambda_0}|^2}{4\Delta^3}\left[\frac{1}{\nu^2_\lambda} - \frac{g_1 + g_2}{g_1\nu_1^2 + g_2\nu_2^2}\right]^2 \frac{M}{M_{\text{eff}}}, \quad (13)$$

where $\nu_\lambda = \epsilon_\lambda/2\Delta$.

In the quasiclassical approximation the nonvanishing matrix elements of $\dot{M}^x$ are equal in the representation (n_x, n_y, n_z). In the representation $(N, n_z, \Lambda, \Omega)$, where N is the principal quantum number, and Λ and Ω are the projections of the orbital and total angular momenta on the z axis, the quasiclassical equality of matrix elements will also hold provided that the degeneracy of energy levels in this representation is taken into account and the effective expressions $|\dot{M}^x_{\lambda\lambda_0}|^2_{\text{eff}}$ are used, corresponding to the two possible transitions to the degenerate level. Then Eq. (13) becomes

$$J_0 = J_e(\varkappa_0) + \frac{(N - n_z)n_z}{\Delta}\nu_1^2\left[\frac{1}{\nu_1^4} + \frac{1}{\nu_2^4} - \frac{2(g_1 + g_2)}{(g_1\nu_1^2 + g_2\nu_2^2)} \times \left(\frac{1}{\nu_1^2} + \frac{1}{\nu_2^2}\right) + \frac{2(g_1 + g_2)^2}{(g_1\nu_1^2 + g_2\nu_2^2)^2}\right]\frac{M}{M_{\text{eff}}}, \quad (14)$$

Insofar as M differs little from M_{eff} we set in what follows $M/M_{eff} \approx 1$.

For deformed nuclei $\nu_1 \sim 1$, $g_1 \sim 0.6$, $\nu_2 \sim 10$, and $g_2 \sim 0.03$. It follows from Eq. (14) that terms connected with f^0 can be as large as 25% of the first term. However the dependence of these terms on ν_1 is much weaker than that of the first term. In addition, it follows from Eq. (14) that the summation over λ is important only within the one shell which contains the state λ_0, because the principal contribution to J_0 comes from terms corresponding to the minimal energy ϵ_λ.

Let us consider the difference between the moments of inertia of an odd nucleus and the neighboring even-even nucleus:

$$\delta J = J_o - J_e = J_e(\varkappa_o) - J_e(\varkappa_e) + \sum_\lambda \frac{|M_{\lambda\lambda_0}|^2}{4\Delta^3}\left[\frac{1}{\nu_\lambda^2} - \frac{g_1+g_2}{g_1\nu_1^2+g_2\nu_2^2}\right]^2 \quad (15)$$

The last term in Eq. (15) is always positive. For an oscillator potential $\kappa = \hbar\omega_0\beta/2\Delta$. Since $\Delta_e > \Delta_o$, and β does not vary much between neighboring nuclei, we have in a majority of cases $\kappa_o > \kappa_e$ and $\delta J > 0$, in agreement with experiment. Let us estimate the dependence of the main term in δJ on the number of particles in the system, A. Since

$$\sum_\lambda{}' |M^x_{\lambda\lambda_0}|^2 \sim A^{2/3},\ \varepsilon_\lambda \sim \Delta \sim \varepsilon_0 A^{-2/3},$$

it follows that $\delta J \sim A^{4/3}/\epsilon_0$. The moment of inertia of a solid goes as $J_s \sim A^{5/3}/\epsilon_0$ and therefore

$$\delta J / J_s \sim A^{-1/3}. \quad (16)$$

The experimentally observed sharp jumps in δJ correspond to those cases when the odd particle occupies states which in the limit of small deformations go over into states with a large orbital angular momentum l. The dependence on l is particularly clear in the example of a rectangular potential well, when

$$\varepsilon_\lambda + \varepsilon_0 = \varepsilon^0_\lambda + \beta(m^2/l^2 - 1/3)\varepsilon^0_\lambda,$$

where ϵ^0_λ is measured from the bottom of the well. In that case the main term in Eq. (15) has the form

$$\delta J \approx [\Delta l^4 / 2(\beta\varepsilon^0_{\lambda_0})^2][l^2/m^2 - 1].$$

This formula illustrates the strong dependence of the moment of inertia on the state occupied by the odd particle.

We discuss next the limiting value of J. In the limit as $\Delta \to 0$ $(\nu \to \infty)$, according to Migdal,[3] $J_e(\kappa) \to J_s$, and the second term in Eq. (14) tends to zero. This is understandable since in the case of a solid $\delta J/J_s \sim 1/A$ and terms of this order in A are not included in our formula.

Let us discuss the limitations on the permissible values of the parameters β and Δ imposed by the criterion of applicability of perturbation theory $\rho'_{\lambda\lambda'} \ll 1$. We assume that $\Delta \ll \epsilon_0 A^{-1/3} \sim \omega_0$, which is always valid for a nucleus. Since the perturbation V goes as $A^{2/3}/J$ (where $J \sim A^{5/3}/\epsilon_0$), it follows from the condition $\rho'_{\lambda_0\lambda} \ll 1$ that

$$\beta \gg A^{-2/3}, \quad \Delta \ll \varepsilon_0 A^{1/3}\beta^2. \quad (17)$$

For deformed nuclei these conditions are always satisfied ($\beta \sim A^{-1/3}$, $\Delta \sim \epsilon_0 A^{-2/3}$) and, therefore, perturbation theory is applicable.

The calculation of the difference of the moments of inertia of even and odd nuclei was carried out using the formula

$$\delta J = \sum_\lambda \frac{4\Delta |M^x_{\lambda_0\lambda}|^2}{\varepsilon^2_\lambda} - \frac{2(g_1+g_2)\nu_2^2 g_2}{\Delta(\nu_1^2 g_1 + \nu_2^2 g_2)^2}\left[n_z(N-n_z) + \frac{N+n_z}{2}\right] + J_e(\varkappa_o) - J_e(\varkappa_e). \quad (18)$$

The main term in expression (18) is the first term. It gives the principal dependence of δJ on the state of the odd nucleon λ_0. In deriving it no specific form was used for the nuclear potential. It was therefore calculated using the wave functions for the Nilsson potential,[6] which gives a good description of the state of the odd particle in a deformed nucleus. The second term in Eq. (18) was obtained by utilizing the oscillator potential. It contributes to δJ less than the first term ($\sim 25\%$) and, furthermore, depends rather smoothly on the state λ_0. Therefore the evaluation of the second term utilizing asymptotic wave functions of a deformed oscillator in the representation $(N, n_z, \Lambda, \Omega)$ introduces no significant errors.

Migdal[3] obtained an expression for $J_e(\kappa)$, where $\kappa = \hbar\omega_0\beta/2\Delta$ and $\hbar\omega_0 = 41\ A^{-1/3}$ Mev.[6] In order to calculate the difference $J_e(\kappa_o) - J_e(\kappa_e)$ it is necessary to know Δ_o, Δ_e, β_o, and β_e for neighboring nuclei. The quantity Δ may be obtained from data on nuclear masses and binding energies with an accuracy not better than 10 or 15%.[7] In many cases, particularly for odd proton nuclei, the necessary data for Δ do not exist. Therefore in those cases Δ was found by interpolation. In those cases when one of the neighboring Δ_e and Δ_o could not be determined directly we made use of the relation

$$\Delta_e = \Delta_o + 1/\rho_0,$$

where ρ_0 is the density of single-particle levels near the Fermi surface for particles of the given type. The values of β were taken from references 2, 8, and 9. Values of β corresponding to the neighboring nuclei were used in those cases when β was not known. The results of a comparison of theory and experiment are shown in Tables I and II. It is seen that the theory gives a satisfactory description of the dependence of the moments of inertia of odd nuclei on the state occupied by the odd particle. In several cases such a comparison may be useful for a classification of the state occupied by the odd nucleon in the Nilsson scheme. The tables also make it possible to estimate the quantity Δ in those cases when it is not known from other data.

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APPENDIX

Let us find the solution of Eq. (7) for $f'(\mathbf{r})$, under the assumption that the nonvanishing matrix elements $f'_{\lambda\lambda'}$, similarly to $f^0_{\lambda\lambda'}$, are quasi-classically equal for a given λ. Then utilizing the quasi-classical independence of $\varphi_\lambda\varphi^*_{\lambda'}$, $\dot{M}_{\lambda\lambda'}$, $f^0_{\lambda\lambda'}$, and $f'_{\lambda\lambda'}$ on λ' for a fixed λ and employing the well known method[3] for evaluating the sums we reduce Eq. (7) to the form

$$\frac{1}{2}(\nu_1^2 g_1 + \nu_2^2 g_2) f'(\mathbf{r})\rho(\varepsilon_0, \mathbf{r})$$

$$= \frac{1}{4\Delta^2}\left(\frac{1}{\nu_1^2} + \frac{1}{\nu_2^2}\right)\dot{M}^x(\mathbf{r})\,|\varphi_{\lambda_0}|^2 + \frac{f^0(\mathbf{r})\,|\varphi_{\lambda_0}|^2}{\Delta}, \qquad \text{(A.1)}$$

where $\rho(\epsilon_0, \mathbf{r})$ is the density of particles with energy ϵ_0. Introducing into Eq. (A.1) the expression

TABLE I. Relative change of the moments of inertia for nuclei with an odd number of neutrons

Nucleus	Classification of the state occupied by the odd particle $(N, n_z, \Lambda, \Omega)$	β_o	β_e	$\Delta_o^{(n)}$ Mev	$\Delta_e^{(n)}$ Mev	$\delta J/J_s$, % Experiment	$\delta J/J_s$, % Theory
$_{64}Gd^{155}$	$521^3/_2$	0.31	0.3	1	1.15	26	22
$_{64}Gd^{157}$	$521^3/_2$	0.31	0.41	0.8	1	17	16
$_{66}Dy^{161}$	$642^5/_2$	0.3	0.35	0.9	1.1	62	47
$_{66}Dy^{161}$	$521^3/_2$	0.3	0.35	0.9	1.1	15	14
$_{66}Dy^{161}$	$523^5/_2$	0.3	0.35	0.9	1.1	13	17
$_{66}Dy^{163}$	$523^5/_2$	0.3	0.36	0.7	0.9	14	14
$_{68}Er^{167}$	$633^7/_2$	0.29	0.33	0.7	0.9	27	23
$_{70}Yb^{173}$	$512^5/_2$	0.28	0.31	0.6	0.8	7	9
$_{72}Hf^{177}$	$514^7/_2$	0.27	0.29	0.6	0.75	7	9
$_{72}Hf^{179}$	$624^9/_2$	0.26	0.31	0.6	0.75	16	12
$_{90}Th^{229}$	$633^5/_2$	0.22	0.22	0.65	0.85	23	21
$_{90}Th^{231}$	$633^5/_2$	0.23	0.23	0.67	0.87	21	20
$_{92}U^{233}$	$633^5/_2$	0.24	0.24	0.43	0.67	17	16
$_{92}U^{233}$	$631^3/_2$	0.24	0.24	0.43	0.67	17	15
$_{92}U^{235}$	$743^7/_2$	0.24	0.24	0.62	0.81	22	22
$_{94}Pu^{239}$	$743^7/_2$	0.26	0.26	0.44	0.71	28	20
$_{94}Pu^{241}$	$622^5/_2$	0.27	0.27	0.35	0.44	7	5
$_{96}Cm^{245}$	$624^7/_2$	0.26	0.26	0.5	0.7	8	5
		0.20	0.20	0.5	0.7	8	10
$_{96}Cm^{245}$	$734^9/_2$	0.26	0.26	0.5	0.7	19	10
		0.20	0.20	0.5	0.7	19	20

TABLE II. Relative change of the moments of inertia for nuclei with an odd number of protons

Nucleus	N, n_z, Λ, Ω	β_o	β_e	$\Delta_o^{(p)}$ Mev	$\Delta_e^{(p)}$ Mev	$\delta J/J_s$, % Experiment	$\delta J/J_s$, % Theory
$_{67}Ho^{165}$	$523^7/_2$	0.32	0.39	0.8	0.9	8	15
$_{69}Tm^{169}$	$523^7/_2$	0.35	0.34	0.78	0.9	13	21
$_{75}Re^{183}$	$514^9/_2$	0.21	0.23	0.75	0.85	3	4
$_{75}Re^{187}$	$402^5/_2$	0.19	0.24	0.75	0.85	2	3
$_{91}Pa^{233}$	$642^5/_2$	0.24	0.24	0.5	0.6	112	35
$_{93}Np^{237}$	$642^5/_2$	0.25	0.25	0.5	0.6	29	27
$_{93}Np^{239}$	$642^5/_2$	0.26	0.26	0.5	0.65	33	29
$_{93}Np^{237}$	$523^5/_2$	0.25	0.25	0.5	0.6	11	11
$_{93}Np^{239}$	$523^5/_2$	0.26	0.26	0.5	0.65	10	12
$_{95}Am^{241}$	$523^5/_2$	0.27	0.27	0.45	0.55	10	9
$_{95}Am^{243}$	$523^5/_2$	0.27	0.27	0.45	0.55	11	9

for $f^0(\mathbf{r})$ from Eq. (12) we obtain

$$f' = \frac{\dot{M}^x(\mathbf{r})\,|\varphi_{\lambda_0}|^2}{2\Delta^2\rho(\varepsilon_0,\mathbf{r})}\,\frac{(\nu_2^2-\nu_1^2)(g_1\nu_2^2-g_2\nu_1^2)}{\nu_1^2\nu_2^2(g_1\nu_1^2+g_2\nu_2^2)}\,.$$

Since $\rho(\epsilon_0, \mathbf{r})$ depends on $\mathbf{r}$ only weakly we have $\rho(\epsilon_0, \mathbf{r})V = \rho_0$, where ρ_0 is the level density at the Fermi surface, and V is the volume of the system. Since $|\varphi_{\lambda_0}|^2$ is a rapidly oscillating function it may be replaced by 1/2V. Thus the matrix $f'_{\lambda\lambda'}$ has the properties assumed in the derivation of $f'(\mathbf{r})$. Estimating f' for $\nu_1 \sim 1$, $\nu_2 \sim 10$ we find that

$$f'/f^0 \approx 1/2\rho_0\Delta \sim A^{-1/3}.$$

Consequently the function f' is quasiclassically small in comparison with f^0.

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