CONTRIBUTION TO THE THEORY OF ELECTRON GAS CONDUCTIVITY IN A STRONG MAGNETIC FIELD

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The conductivity of an electron gas in perpendicular electric and magnetic fields is investigated for $\omega\tau \gg 1$ (τ is the electron relaxation time, ω is the cyclotron frequency). Elastic scattering of electrons on fixed short-range force centers is considered. Interaction between the electrons and the scatterers is treated without the aid of perturbation theory. In the final result the conductivity is expressed as a function of the magnetic field and the exact amplitude for scattering of a zero energy electron on a single center in the absence of a magnetic field.

In the recent work of Adams and Holstein,[1] galvanomagnetic phenomena were studied in a strong magnetic field. Here, the interaction of the electrons with different scatterers was considered by the authors as a perturbation, which required the assumption of a finite width of the electron levels for the elimination of divergences that appear in the Born approximation.

In the present work, the conductivity of an electron gas is computed for a strong magnetic field in the case $\omega\tau \gg 1$, where τ is the relaxation time of the electrons, $\omega = eH/m$ (the system of units is used for which $\hbar = c = 1$). Only elastic scattering of the electrons on randomly-arranged immovable centers is considered; the radius of action of these centers is assumed to be small in comparison with the wavelength of the electrons and with the mean distance between scatterers. The interaction of the electrons with the scatterers is considered without the aid of perturbation theory, inasmuch as the Born approximation, strictly speaking, is not suitable at low energies. Limiting ourselves to the case in which the directions of the electric and magnetic fields are perpendicular, we begin our calculations with the general expression for the conductivity tensor obtained by Kubo:[2]

$$\sigma_{\mu\nu} = \frac{e^2}{\Omega}\int_0^\infty e^{-\varepsilon t}dt\int_0^\beta d\lambda\,\mathrm{Sp}\,\{\rho v_\nu(-i\lambda)\,v_\mu(t)\},$$

$$\rho = \exp(-\beta\mathcal{H})/\mathrm{Sp}\exp(-\beta\mathcal{H}), \qquad (1)$$

$\mathcal{H}$ is the total Hamiltonian of the system in the absence of an electric field; $v_\mu(t)$ is the Heisenberg operator of the μ component of the velocity of the electron; $\epsilon > 0$; $\beta = 1/kT$; Ω is the normalized volume; $\mu, \nu = x, y, z$. The diagonal elements of the tensor $\sigma_{\mu\nu}$ can be written in the much simpler form

$$\sigma_{\mu\mu} = \frac{\beta e^2}{\Omega}\mathrm{Re}\,\{\mathrm{Sp}\,\rho v_\mu(0)\,X_\mu\}, \qquad (2)$$

$$X_\mu = \int_0^\infty e^{-\varepsilon t}dt e^{i\mathcal{H}t}v_\mu(0)\,e^{-i\mathcal{H}t}. \qquad (3)$$

For calculation of $\sigma_{\mu\nu}$, it is convenient to put the operator X_μ in the form

$$X_\mu = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dE\,\frac{1}{E-\mathcal{H}-i\varepsilon}v_\mu(0)\,\frac{1}{E-\mathcal{H}+i\varepsilon}. \qquad (4)$$

The identity of Eqs. (3) and (4) can easily be seen by taking their matrix elements in the representation of the Hamiltonian $\mathcal{H}$ and carrying out integration over t in (3) and integration over E in (4).

We shall now consider separately the cases of Boltzmann statistics and Fermi statistics.

1. NONDEGENERATE ELECTRON GAS

In this case, it can be assumed that $\mathcal{H}$ is the Hamiltonian of a single electron in the field of the scattering centers, i.e., $\mathcal{H} = \mathcal{H}_0 + V$, where

$$\mathcal{H}_0 = (\mathbf{p} - e\mathbf{A})^2/2m, \qquad V = \sum_i V_i.$$

Here $\mathbf{A}$ is the vector potential of the magnetic field $\mathbf{H}$, V_i is the Hamiltonian of interaction of the electron with the i-th scattering center, located at the point $\mathbf{r}_i$.

We write the vector $\mathbf{A}$ in the form $A_x = A_z = 0$; $A_y = Hx$. The eigenfunctions and the eigen-

values of the operator $\mathcal{H}_0$ have the form

$$\Psi_{nk_yk_z}(\mathbf{r}) = \exp[i(k\ y + k_z z)]\,\varphi_n(x/l - lk_y),$$
$$E_{nk_z} = \omega(n + \tfrac{1}{2}) + k_z^2/2m,$$

where φ_n is the Hermite function of n-th order and $l = (eH)^{-1/2}$. The matrix elements of the operators $v_x(0)$ and $v_y(0)$ are determined by the formula

$$(n'k_y'k_z' \mid v_x(0) + iv_y(0) \mid nk_yk_z)$$
$$= -i(2\omega/m)^{1/2}n'^{1/2}\delta_{n',n-1}\delta_{k_y'k_y}\delta_{k_z'k_z},$$
$$(n'k_y'k_z' \mid v_x(0) - iv\ (0) \mid nk_yk_z)$$
$$= i(2\omega/m)^{1/2}(n+1)^{1/2}\delta_{n',n+1}\delta_{k_y'k_y}\delta_{k_z'k_z}. \qquad (5)$$

In what follows, we shall denote the sets of quantum numbers nk_yk_z by the Greek letters α, β, etc.

Following Luttinger and Kohn,[3] we introduce the "scattering operator" $T^{(\pm)}(E)$, which is determined by the relation

$$(E - \mathcal{H} \pm i\varepsilon)^{-1} = G_E^{(\pm)} + G_E^{(\pm)}T^{(\pm)}(E)\,G_E^{(\pm)},$$
$$G_E^{(\pm)} = (E - \mathcal{H}_0 \pm i\varepsilon)^{-1}. \qquad (6)$$

The operator $T^{(\pm)}(E)$ can be written in the form of a series

$$T^{(\pm)}(E) = \sum_j T_j^{(\pm)}(E) + \sum_{j\neq k} T_j^{(\pm)}(E)\,G_E^{(\pm)}T_k^{(\pm)}(E) + \ldots, \qquad (7)$$

where the "operator of scattering on the j-th center" $T_j^{(\pm)}(E)$ satisfies the integral equation

$$T_j^{(\pm)}(E) = V_j + V_jG_E^{(\pm)}T_j^{(\pm)}(E).$$

The matrix element $(\beta \mid T_j^{(+)}(E) \mid \alpha)$ for $E = E_\alpha$ is the amplitude of the transition from the state α to the state β in the scattering of the electron on the j-th center. In reference 4, the author calculated the amplitude of scattering of an electron of low energy on the short-range potential V in a strong magnetic field [it was assumed that the radius of action of the scattering potential $r_0 \ll (2mE)^{-1/2}$]. According to the results of reference 4,

$$(\beta \mid T_j^{(+)}(E_\alpha) \mid \alpha) = \Psi_\beta^*(\mathbf{r}_j)\,\Psi_\alpha(\mathbf{r}_j)\,\frac{2\pi f/m}{1 + ifK(E_\alpha)}, \qquad (8)$$

where

$$K(E) = l^{-2}\sum_{n=0}^{N}[2m(E - n\omega - \omega/2)]^{-1/2},$$

f is the exact scattering amplitude of a zero energy free electron on the potential V; N in the expression for K is so defined that the last term in the sum over n would be purely imaginary.

By a method completely analogous to that used in reference 4 for the derivation of (8), it can be shown that the matrix elements $T_j^{(+)}(E)$ are determined by an expression similar to (8), i.e., that (8) maintains its force if we replace E by E_α in the argument of $T_j^{(+)}$ and in K. The matrix elements of the operator $T_j^{(-)}(E)$ are connected in simple fashion with the matrix elements of $T_j^{(+)}(E)$:

$$(\alpha \mid T_j^{(-)}(E) \mid \beta) = (\beta \mid T_j^{(+)}(E) \mid \alpha)^*.$$

Expressing X_μ and ρ in terms of the operator $T(E)$, determined by Eq. (7), one can compute the elements of the tensor $\sigma_{\mu\nu}$ for $\mu, \nu = x, y$. It is shown that in the approximation $\omega\tau \gg 1$ and $\beta^{-1}\tau \gg 1$, it is sufficient to consider a single scattering center and multiply the expression obtained in this case for the number of collisions of the electron per unit time by the number of such centers. Therefore, in what follows, in order to make the calculations clearer, we shall understand by $T(E)$ the scattering operator on a single center.

Substituting (4) in (3), we have

$$X_\mu = X_\mu^{(0)} + X_\mu^{(1)} + X_\mu^{(2)}.$$

Here $X_\mu^{(0)}$ does not depend on the operator $T(E)$; $X_\mu^{(1)}$ is proportional to the first power of $T(E)$, and $X_\mu^{(2)}$ is proportional to the second.

$$(\beta \mid X_\mu^{(0)} \mid \alpha) = i(\beta \mid v_\mu(0) \mid \alpha)/\omega_{\beta\alpha}. \qquad (9)$$

$$(\beta \mid X_\mu^{(1)} \mid \alpha) = i\sum_\gamma \{(\beta \mid v_\mu(0) \mid \gamma)\frac{1}{\omega_{\beta\gamma}}(\gamma \mid T^{(+)}(E_\beta) \mid \alpha)\frac{1}{\omega_{\beta\alpha} + i\varepsilon}$$
$$- \frac{1}{\omega_{\alpha\beta} - i\varepsilon}(\beta \mid T^{(-)}(E_\alpha) \mid \gamma)\frac{1}{\omega_{\alpha\gamma}}(\gamma \mid v_\mu(0) \mid \alpha)\}, \qquad (10)$$

where $\omega_{\alpha\beta} = E_\alpha - E_\beta$. In obtaining (10), we took it into account that $T^{(+)}(E)$ has a pole only in the lower half of the complex plane E, while $T^{(-)}(E)$ has a pole only in the upper half-plane. The operator X_μ has a simple physical meaning. $X_\mu^{(0)}$ is the operator of the μ-th coordinate of the electron relative to its center of rotation in the absence of collisions, while $X_\mu - X_\mu^{(0)}$ describes the mean change of X_μ under the action of the scattering of the electron on the potential V. We note that $X_\mu^{(0)}$ makes no contribution to $\sigma_{\mu\mu}$, inasmuch as the term corresponding to it is purely imaginary. One can generally omit $X_\mu^{(2)}$ in the calculation of $\sigma_{\mu\nu}$ in the approximation under consideration, since

$$\sum_{\alpha\alpha'} T^{(-)}(E) \mid \alpha)\frac{1}{E - E_\alpha - i\varepsilon}(\alpha \mid v_\mu(0) \mid \alpha')$$
$$\times \frac{1}{E - E_{\alpha'} + i\varepsilon}(\alpha' \mid T^{(+)}(E)$$

disappears on summation over the x coordinate of the center of rotation of the electron (l^2k_y), because of the orthogonality of Hermite functions of different order. We recall that only $\Psi_\alpha(\mathbf{R})$ and $\Psi^*_{\alpha'}(\mathbf{R})$, which enter into the matrix elements of the operators $T^{(+)}(E)$ and $T^{(-)}(E)$, depend on k_y; here $\mathbf{R}$ is the radius vector of the scattering center.

Let us consider the operator $\rho(\mathcal{H})$. We can represent the quantity $\exp(-\beta\mathcal{H})$ in it in the form

$$\exp(-\beta\mathcal{H}) = S(i\beta)\exp(-\beta\mathcal{H}_0),$$

where $S(i\beta)$ is determined by the equation

$$-\frac{\partial}{\partial\beta}S(i\beta) = S(i\beta)V(i\beta) \qquad (11)$$

and the boundary condition $S(0) = 1$. By direct substitution it can easily be shown that the solution of (11) (with accuracy up to terms of higher order in the operator T') is given by the formula

$$(\alpha|S(i\beta)|\gamma) = \delta_{\alpha\gamma} - \frac{1-\exp(\beta\omega_{\gamma\alpha})}{\omega_{\gamma\alpha}}(\alpha|T'(E_\alpha)|\gamma), \qquad (12)$$

where T' is determined by the integral equation

$$(\alpha|T'(E_\alpha)|\gamma) = (\alpha|V|\gamma) + \sum_\delta (\alpha|T'(E_\alpha)|\delta)\left(\frac{1}{\omega_{\alpha\delta}}\right)_P (\delta|V|\gamma);$$

(here the index P means that the integral over E is taken in the sense of the principal value). The solution of this equation differs from the matrix elements $(\alpha|T^{(\pm)}(E_\alpha)|\gamma)$ only in the fact that it does not contain $\mathrm{Re}\,K(E_\alpha)$.

Substituting the expressions (9), (10), and (12) obtained for X_μ and $\exp(-\beta\mathcal{H})$ in (1) and (2), we obtain, after tedious but elementary transformations,

$$\sigma_{xx} = \beta n_e e^2 \sum_{\alpha\gamma}\rho_0(E_\alpha)|(\alpha|X^{(0)}|\gamma)|^2 2\pi \times\sum_\lambda|(\gamma|T^{(+)}(E_\lambda)|\lambda)|^2\delta(E_\lambda - E_\alpha), \qquad (13)$$

$$\sigma_{yx} = -n_e e^2/m\omega, \qquad (14)$$

where

$$\rho_0(E) = Z^{-1}\exp(-\beta E), \qquad Z = \mathrm{Sp}\exp(-\beta\mathcal{H}_0),$$

n_e is the electron concentration. In the derivation of (13), we made use of the "optical theorem" for the operator $T(E)$:

$$-2\,\mathrm{Im}(\alpha|T^{(+)}(E)|\alpha) = 2\pi\sum|(\alpha|T^{(+)}(E_\beta)|\beta)|^2\delta(E_\beta - E)$$

and also of the fact that

$$\mathrm{Re}(\beta|T^{(+)}(E_\alpha)|\alpha)(\alpha|T'(E_\alpha)|\beta) = |(\beta|T^{(+)}(E_\alpha)|\alpha)|^2.$$

The validity of these expressions follows from the explicit form of the matrix elements of $T^{(+)}$ and T'.

Thus σ_{xx} and σ_{yy} are proportional to the mean square of the Larmor radius of the electron and to the frequency of collisions with the scatterers. Equation (14) shows that $Y^{(1)}$ makes no contribution to σ_{yx} in the given approximation, i.e., the average change of the y coordinate of the electron under action of the collisions is equal to zero if the electric field is directed along the x axis.

We shall compute σ_{xx} in two different limiting cases. Substituting (8) and (9) in (13), we have

$$\sigma_{xx} = \beta\frac{n_e e^2}{m\omega}\sum_\alpha\rho_0(E_\alpha)(n_\alpha + 1/2)\nu(E_\alpha), \qquad (15)$$

$$\nu(E) = n_p\frac{4\pi f^2}{(1+fK'')^2 + (fK')^2}\frac{K'(E)}{m}. \qquad (16)$$

We use here $K = K' - iK''$; n_p is the number of scatterers per unit volume. The quantity $\nu(E)$ is the total number of collisions of the electron with energy E per unit time. In the limiting case $\beta\omega \ll 1$, we have substantially $n_\alpha \approx (\beta\omega)^{-1}$ in (14), and the well-known classical formula

$$\sigma_{xx} \approx n_e e^2/m\omega^2\tau \qquad (17)$$

is obtained for σ_{xx}, where

$$1/\tau = \sum_\alpha\rho_0(E)\nu(E_\alpha).$$

In the other limiting case $\beta\omega \gg 1$, if all the electrons are in states with $n_\alpha = 0$, σ_{xx} has the form

$$\sigma_{xx} \approx \beta\frac{n_e e^2}{m}n_p f^2\left(\frac{2\pi\beta}{m}\right)^{1/2}\mathrm{Ei}\left(\frac{\beta f^2}{4ml^4}\right). \qquad (18)$$

Here,

$$\mathrm{Ei}(u) = \int_0^\infty\frac{dz}{z+u}\exp(-z),$$

that is, in the limiting quantum case, σ_{xx} is almost independent of the value of the magnetic field.

2. DEGENERATE ELECTRON GAS

In the case of Fermi statistics, it is appropriate to calculate in the second-quantization representation. We introduce the operators a^+_α of creation of an electron in the state α and the operators α_β of annihilation of an electron in the state β. The properties of these operators are defined by well-known commutation relations

$$a_\alpha a_\beta + a_\beta a_\alpha = a^+_\alpha a^+_\beta + a^+_\beta a^+_\alpha = 0, \qquad a^+_\alpha a_\beta + a_\beta a^+_\alpha = \delta_{\alpha\beta}.$$

The operators $\mathcal{H}$ and X_μ are single particle operators and have the form

$$\mathcal{H} = \mathcal{H}_0 + V, \qquad \mathcal{H}_0 = \sum_\alpha (E_\alpha - \zeta)\hat{n}_\alpha,$$

$$V = \sum_{\alpha \neq \beta} a_\alpha^+ a_\beta (\alpha | V | \beta), \qquad X_\mu = \sum_{\alpha\beta} a_\alpha^+ a_\beta (\alpha | X_\mu | \beta). \quad (19)$$

Here ζ is the chemical potential and $\hat{n}_\alpha = a_\alpha^+ a_\alpha$ is the operator of the number of electrons in the state α.

The operator $\exp(-\beta\mathcal{H})$ can, as in the non-degenerate case, be written in the form

$$\exp(-\beta\mathcal{H}) = S(i\beta)\exp(-\beta\mathcal{H}_0),$$

where, under the given approximation ($\omega\tau \gg 1$),

$$S(i\beta) = 1 + \sum_{\alpha\gamma} a_\alpha^+ a_\gamma \frac{1 - \exp(\beta\omega_{\gamma\alpha})}{\omega_{\gamma\alpha}} (\alpha | T'(E_\alpha) | \gamma). \quad (20)$$

We now substitute (19) and (20) in (1) and (2), and make use of the relations

$$Z^{-1}\,\mathrm{Sp}\,[e^{-\beta\mathcal{H}_0}\hat{n}_\alpha] = f(E_\alpha),$$

$$Z^{-1}\,\mathrm{Sp}\,[e^{-\beta\mathcal{H}_0}\hat{n}_\alpha\hat{n}_\gamma] = f(E_\alpha) f(E_\gamma) \qquad (\alpha \neq \gamma)$$

etc. where f(E) is the Fermi function.

Carrying out transformations similar to those used in the derivation of (13), and making use of the identity

$$f(E_\alpha) - f(E_\gamma) = [1 - \exp(\beta\omega_{\alpha\gamma})] f(E_\alpha)[1 - f(E_\gamma)],$$

$$\beta f(E)[1 - f(E)] = df(E)/d\zeta,$$

we get

$$\sigma_{xx} = \sigma_{yy} = \frac{e^2}{\Omega} \sum_\alpha \frac{df(E_\alpha)}{d\zeta} l^2 (n_\alpha + 1/2)\nu(E_\alpha), \quad \sigma_{yx} = -n_e e^2/m\omega, \quad (21)$$

where $\nu(E)$ is given by Eq. (16).

In the limiting quantum case $1 \ll \beta(\zeta - \omega/2) < \beta\omega$, all the electrons are found in states with n = 0, and we get

$$\sigma_{xx} = \frac{1}{4\pi} \frac{n_p e^2 f^2 \omega}{\zeta - \omega/2} \quad (22)$$

from Eq. (21) with the aid of (16).

In order to obtain the final expression for the conductivity, it is still necessary to take into account the dependence of the chemical potential ζ on the magnitude of the magnetic field. This dependence can easily be found from the condition

$$n_e(\zeta) = n_e^{(0)}(\zeta_0).$$

Here $n_e^{(0)}$ and ζ_0 are the electron concentration and the chemical potential in the absence of the magnetic field, respectively. In the case under study,

$$\zeta - \omega/2 = 4\zeta_0^3/9\omega^2$$

and the final expression for σ_{xx} has the form

$$\sigma_{xx} = 9 n_p e^2 f^2 \omega^3 / 16\pi\zeta_0^3. \quad (23)$$

In the case $\omega \ll \zeta$, (21) can be reduced to the form

$$\sigma_{xx} = \frac{e^2}{(2\pi)^2}\left(\frac{2m}{\omega}\right)^{1/2} \int_{\omega/2}^{\infty} dE \frac{df(E)}{d\zeta} I(E)\nu(E), \quad (24)$$

where

$$I(E) = \sum_n \frac{n + 1/2}{(E/\omega - n - 1/2)^{1/2}}.$$

The summation is carried out over all n for which the radicand is non-negative.

For computation of K′ and I, we introduce the notation $E = \omega(N + \epsilon + \frac{1}{2})$, where N is an integer, $0 \leq \epsilon < 1$, and make use of the summation formula of Poisson, which is written in the form

$$\sum_{k=-\infty}^{+\infty} \exp(i2\pi kx) = \sum_{n=-\infty}^{+\infty} \delta(x - n). \quad (25)$$

Multiplying (25) by the function $\varphi(x)$, and integrating over x from 0 to $N + \epsilon$, we get

$$\sum_{n=0}^{N} \varphi(n) = \frac{1}{2}\varphi(0) + \sum_{k=-\infty}^{+\infty} \int_0^{N+\epsilon} dx \varphi(x) \exp(i2\pi kx).$$

By means of this formula, it is not difficult to obtain an expression for I and K′ in the case $N \gg 1$:

$$I(E) \approx \frac{4}{3}\left(N + \varepsilon + \frac{1}{2}\right)^{3/2}\left[1 + \frac{3}{2}\left(N + \varepsilon + \frac{1}{2}\right)^{-1/2} \times \sum_{k=1}^{\infty} \frac{\cos(2\pi k\varepsilon - \pi/4)}{(2k)^{1/2}}\right],$$

$$K'(E) \approx (2m\omega)^{1/2}\left(N + \varepsilon + \frac{1}{2}\right)^{1/2}\left[1 + \left(N + \varepsilon + \frac{1}{2}\right)^{-1/2} \times \sum_{k=1}^{\infty} \frac{\cos(2\pi k\varepsilon - \pi/4)}{(2k)^{1/2}}\right]. \quad (26)$$

As ϵ tends to zero, the behavior of the series appearing in Eq. (26) is accurately described by the function $\frac{1}{2}\epsilon^{-1/2}$. This function serves as an integrating factor; therefore in integration over E in (24), we can neglect the departure of the denominator of (16) from unity, for all terms except the term corresponding to the product of the series entering into (26). As a result, the product $I(E)\nu(E)$ can be written in the form

$$\left(\frac{E}{\omega}\right)^2\left[1 + \frac{5}{2}\left(\frac{\omega}{E}\right)^{1/2} \sum_{k=1}^{\infty} \frac{\cos(2\pi k\varepsilon - \pi/4)}{(2k)^{1/2}} + \frac{3\omega}{4E} \frac{1}{2\varepsilon + f^2/l^2}\right]$$

with accuracy to within a common factor.

Expanding the last component in the brackets in a Fourier series and integrating over E in (24) for the case $1 < \beta\omega \ll \beta\zeta$, we obtain

$$\sigma_{xx} \approx \frac{8}{3\pi} e^2 n_p f^2 \left(\frac{\zeta}{\omega}\right)^2 \left\{ 1 + \frac{3}{8}\frac{\omega}{\zeta} \ln \frac{2l^2}{f^2} \right.$$
$$\left. + \sum_{k=1}^{\infty} (-1)^k \left[A_k \cos\left(\frac{2\pi k\zeta}{\omega} - \frac{\pi}{4}\right) + B_k \cos\left(\frac{2\pi k\zeta}{\omega}\right) \right] \right\}, \quad (27)$$

where

$$A_k = \frac{2\pi^2 k}{\beta\omega \sinh(2\pi^2 k / \beta\omega)} \frac{5}{2} \left(\frac{\omega}{\zeta}\right)^{1/2} (2k)^{-1/2},$$

$$B_k = -\frac{2\pi^2 k}{\beta\omega \sinh(2\pi^2 k / \beta\omega)} \frac{3\omega}{4\zeta} \mathrm{Ci}\left(\frac{\pi k f^2}{l^2}\right).$$

The asymptotic expression of the cosine integral has the form

$$\mathrm{Ci}\, z \approx \ln \gamma z \qquad (z \ll 1), \qquad \gamma = 1.7\ldots,$$

for small values of the argument; therefore the term in the conductivity proportional to B_k can play an important role, especially if ω/ζ is not too small.

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[1] E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).

[2] R. Kubo, J. Phys. Soc. Japan **12**, 570 (1957).

[3] J. M. Luttinger and D. W. Kohn, Phys. Rev. **109**, 1892 (1958).

[4] V. G. Skobov, JETP **37**, 1467 (1959), Soviet Phys. JETP **10**, 1039 (1960).

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