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NONRESONANCE ABSORPTION OF OSCILLATING MAGNETIC FIELD ENERGY BY A FERROMAGNETIC DIELECTRIC, II

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Spin-wave theory is used to calculate the imaginary part of the transverse magnetic susceptibility of a ferromagnetic dielectric.

We have previously studied the nonresonance absorption of alternating magnetic field energy by a ferromagnetic dielectric, with the field in the direction of easy magnetization.[1] The present paper discusses the analogous problem for a field perpendicular to the direction of easy magnetization. Unlike the case of the "longitudinal" field (in the direction of easy magnetization), the energy of the "transverse" field can be absorbed even when dissipative processes are absent if the field frequency coincides with that of ferromagnetic resonance. We shall not consider ferromagnetic resonance and the associated questions such as the shape of the absorption line. Our results therefore pertain basically to frequencies far from resonance.

The dissipative processes associated with energy absorption result from the interaction between the magnetic field and spin waves. As previously,[1] we shall assume saturation magnetization of the ferromagnetic dielectric at a given temperature ($T \ll \Theta_C$). The sample is thus a single domain and is also assumed to be of sufficient purity for the disregard of impurities.

1. The Hamiltonian of the ferromagnet in an alternating magnetic field **h** is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}, \qquad \mathcal{H}_{int} = -\int \hat{\mathbf{M}}\mathbf{h}\, dv, \tag{1}$$

where $\hat{\mathcal{H}}_0$ is the part of the Hamiltonian which does not contain the alternating magnetic field.

Let the field **h** be monochromatic and polarized circularly in a plane perpendicular to the easy magnetization axis, which is the direction of the static field $H_0(0, 0, H_0)$:

$$h_x = h_0 \cos(\omega t - \mathbf{kr}),$$
$$h_y = h_0 \sin(\omega t - \mathbf{kr}), \qquad k = \omega / c. \tag{2}$$

Introducing the operators a and a^* through the formulas[2,3]

$$\hat{M}^+ \equiv \hat{M}_x + i\hat{M}_y = (2\mu M_0)^{1/2} a^* (1 - \mu a^* a / 2M_0)^{1/2},$$
$$\hat{M}^- \equiv \hat{M}_x - i\hat{M}_y = (2\mu M_0)^{1/2} (1 - \mu a^* a / 2M_0)^{1/2} a \tag{3}$$

and expanding the radicals in powers of a^*a, we obtain, up to third-order terms,

$$\hat{\mathcal{H}}_{int} = -\tfrac{1}{2}(2\mu M_0)^{1/2}\int (a^* h^- + a h^+)\, dv$$
$$+ \frac{\mu}{8M_0}(2\mu M_0)^{1/2}\int (a^* a^* a h^- + a^* a a h^+)\, dv, \tag{4}$$

where

$$h^{\pm} = h_x \pm i h_y.$$

The first integral in the interaction Hamiltonian (4) describes resonance absorption of the magnetic field since it represents the excitation of a spin wave with crystal quasimomentum $\hbar\mathbf{k}$ and energy $\hbar\omega$. As already mentioned, this process will not be considered here. We confine our attention to the second integral in (4).

When the temperature T is higher than the characteristic temperature of magnetic interaction $2\pi\mu M_0 \sim 1°K$ we can neglect the magnetic interaction energy in the unperturbed part $\hat{\mathcal{H}}_0$ of the Hamiltonian compared with the exchange energy.[2,3] We know that in this case

$$a = \frac{1}{V^{1/2}}\sum_\lambda a_\lambda e^{i\mathbf{k}_\lambda \mathbf{r}}, \qquad a^* = \frac{1}{V^{1/2}}\sum_\lambda a_\lambda^* e^{-i\mathbf{k}_\lambda \mathbf{r}}, \tag{5}$$

where a_λ^* and a_λ are the creation and annihilation operators of the spin wave with quasimomentum $\hbar\mathbf{k}_\lambda$.

Substituting (5) in (4), we obtain

$$\hat{\mathcal{H}}_{int} = \frac{\mu h_0}{8M_0}\left(\frac{2\mu M_0}{V}\right)^{1/2} e^{i\omega t}\sum_{\lambda\mu\nu} a_\lambda a_\mu a_\nu^* + \text{c.c.}\ (\mathbf{k}_\lambda + \mathbf{k}_\mu = \mathbf{k}_\nu + \mathbf{k}). \tag{6}$$

It thus appears that the nonresonance absorption of magnetic field energy is associated with the production of two spin waves through the "collision" of a photon with a spin wave. Photon absorption is also possible in the following perturbation approximations through processes in which a larger number of spin waves participate. The most important of these processes is of the fifth order in the operators a_λ and a_λ^*, and occurs in the second perturbation approximation because of exchange-interaction terms in $\mathcal{H}_0$ which are of the fourth order in a_λ and a_λ^* and because of the resonance term in (4). Although the photon absorption probability due to this process contains as a factor the large constant of the exchange interaction, its importance is reduced by the high power of the temperature (the number of spin waves being proportional to $T^{3/2}$). Therefore the process under consideration plays the principal part at some not too low temperature. In accordance with (6) the probability for this process is

$$W^{n_\lambda n_\mu n_\nu 1_\omega}_{n_\lambda+1\, n_\mu+1\, n_\nu-1\, 0_\omega} = \frac{2\pi}{\hbar} \left| (\mathcal{H}_{int})^i_f \right|^2 \delta(E_i - E_f)$$
$$= \frac{\pi}{4} \frac{\mu^3 h_0^2}{V\hbar M_0} (n_\lambda + 1)(n_\mu + 1) n_\nu \delta(\varepsilon_\lambda + \varepsilon_\mu - \varepsilon_\nu - \hbar\omega)$$
$$\times \Delta(\mathbf{k}_\lambda + \mathbf{k}_\mu - \mathbf{k}_\nu - \mathbf{k}), \tag{7}$$

where n_λ is a spin-wave occupation number and

$$\Delta(\mathbf{k}) = \begin{cases} 1 & \text{for} \quad \mathbf{k} = 0, \\ 0 & \text{for} \quad \mathbf{k} \neq 0; \end{cases}$$

ϵ_λ is the energy of the spin wave with the wave vector k_λ. For $\Theta_c \gg T \gg 2\pi\mu M_0$ we have

$$\varepsilon_\lambda = \Theta_c (a k_\lambda)^2 + \mu H_e, \qquad H_e = H_0 + \beta M_0, \tag{8}$$

β being the anisotropy constant. (8) is also valid for $H_e \gg 2\pi M_0$ at all low temperatures.

The magnetic energy absorption coefficient is

$$\Gamma = Q \Big/ \frac{h_0^2}{8\pi} V,$$

where Q is the amount of energy absorbed per second in the entire volume:

$$Q = \sum_{\lambda\mu\nu} \hbar\omega \left\{ W^{n_\lambda n_\mu n_\nu 1_\omega}_{n_\lambda+1\, n_\mu+1\, n_\nu-1\, 0_\omega} - W^{n_\lambda \quad n_\mu \quad n_\nu \quad 0_\omega}_{n_\lambda-1\, n_\mu-1\, n_\nu+1\, 1_\omega} \right\}. \tag{9}$$

From (7) and (9) we obtain

$$\Gamma = \frac{\pi^2}{4} \frac{\mu^3 \omega}{M_0} \sum_{\lambda\mu\nu} (n_\lambda n_\mu - n_\lambda n_\nu - n_\mu n_\nu - n_\nu)\, \delta(\varepsilon_\lambda + \varepsilon_\mu - \varepsilon_\nu - \hbar\omega)$$
$$\times \Delta(\mathbf{k}_\lambda + \mathbf{k}_\mu - \mathbf{k}_\nu - \mathbf{k}). \tag{10}$$

When the frequency of the field $\mathbf{h}$ is large compared with the reciprocal of the spin-wave relaxation time the n_λ in (10) may reasonably be regarded as equilibrium Bose functions:

$$n_\lambda = (e^{\varepsilon_\lambda / T} - 1)^{-1}. \tag{11}$$

The frequency limitation does not apply to the case under consideration since the field $\mathbf{h}$, which is perpendicular to the equilibrium magnetic moment of the ferromagnet, appears quadratically in the expression for the spin-wave energy.* Therefore the distribution function of n_λ ($\dot{n}_\lambda \sim \dot{\epsilon}_\lambda$) is not affected in the approximation which is linear in $\mathbf{h}$.

*For $\omega\tau \ll 1$, where τ is the relaxation time, we can consider quasi-static energy levels of the spin waves.[1]

(10) and (11) thus determine the absorption coefficient Γ at all frequencies.

Our study is subject to a high-frequency condition ($\hbar\omega \ll \Theta_c$), since we are using the dispersion law (8), which is valid only at low energies ($\epsilon_\lambda \ll \Theta_c$), and it follows from energy conservation that $\hbar\omega = \epsilon_\lambda + \epsilon_\mu - \epsilon_\nu$.

The wave vector $\mathbf{k}$ of the magnetic field is practically always small compared with the wave vector of the spin wave, and can be neglected. When the summation in (10) is changed to integration over all angles we obtain

$$\Gamma = \frac{1}{32\pi^2} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \omega (1 - e^{-\hbar\omega/T}) I(\eta, \nu), \tag{12}$$

where

$$w = \mu^2 / a^3, \qquad \eta = \mu H_e / T, \qquad \nu = \hbar\omega / T,$$

and $I(\eta, \nu)$ is defined by

$$I(\eta, \nu) = \iint \frac{e^{x+y}\, dx\, dy}{(e^x - 1)(e^y - 1)(e^{x+y-\nu} - 1)}, \tag{13}$$

with the region of integration given by the inequalities

$$(\nu - \eta)^2 \leqslant 4(x - \eta)(y - \eta), \qquad x > \eta, \qquad y > \eta. \tag{14}$$

Performing a single integration in (13) in accordance with (14), we can represent $I(\eta, \nu)$ by

$$I(\eta, \nu) = \int_\eta^\infty \frac{e^y}{(e^y - 1)(1 - e^{y-\nu})} \ln \frac{\exp\{\eta + (\eta - \nu)^2 / 4(y - \eta)\} - 1}{\exp\{\eta + (\eta - \nu)^2 / 4(y - \eta)\} - e^{\nu - y}} dy. \tag{13'}$$

2. We now compute (13) (or (13′)) in the different limiting cases.

<u>Low frequencies ($\nu \ll 1, \eta$).</u> In (13′) ν can now be set equal to zero. Introducing the new variable $z = y/\eta$, we obtain

$$I(\eta, 0) = \eta \int_1^\infty \frac{e^{\eta z}}{(e^{\eta z} - 1)^2} \ln \frac{\exp\{\eta[1 + 1/4(z - 1)]\} - e^{-\eta z}}{\exp\{\eta[1 + 1/4(z - 1)]\} - 1} dz. \tag{15}$$

(15) will now be integrated for large and small values of H_e ($\eta \ll 1$, $\eta \gg 1$).

a) $\eta \ll 1$. Expanding all exponentials in powers of η, we obtain after integration

$$I(\eta, 0) \approx \frac{4}{\eta\sqrt{3}} \left\{ \ln(2 + \sqrt{3}) - \frac{2}{\sqrt{3}} \ln 2 \right\} \approx \frac{2.1}{\eta\sqrt{3}}. \tag{16}$$

b) $\eta \gg 1$. The argument of the logarithm is now expanded about unity, from which it differs very little, yielding the following approximate expression for (15):

$$I(\eta, 0) \approx \eta e^{-\eta} \int_0^\infty \exp\left\{ -\eta \left[y + \frac{1}{4(y - 1)} \right] \right\} dy.$$

The method of steepest descents easily leads to the result

$$I(\eta, 0) \approx \sqrt{\pi\eta/2}\, e^{-3\eta}. \quad (17)$$

Resonance frequency ($\nu = \eta$). At $\hbar\omega = \mu H_e$ the absorption is mainly of resonance character. Calculations are shown here only for the purpose of estimating the contribution of nonresonance absorption near resonance.

At $\nu = \eta$ we easily obtain from (13′):

$$I(\eta, \eta) = \int_\eta^\infty \frac{e^y}{(e^y - 1)(1 - e^{y-\eta})} \ln \frac{1 - e^{-\eta}}{1 - e^{-y}}\, dy.$$

a) $\eta = 1$. When, as previously, we make the substitution $y = \eta z$ and expand the exponentials in powers of η, we obtain

$$I(\eta, \eta) \approx \frac{1}{\eta} \int_0^\infty \frac{z dz}{e^z - 1} = \frac{\pi^2}{6\eta}. \quad (18)$$

b) $\eta \gg 1$. After expansion of the logarithm about unity we obtain

$$I(\eta, \eta) \approx e^{-\eta}. \quad (19)$$

High frequencies. High frequencies are understood to be those which are high compared with at least one of the frequencies $T/\hbar$ or $\mu H_e/\hbar$, but small compared with $\Theta_c/\hbar$.

a) $1 \gg \nu \gg \eta$. We may substitute $\eta = 0$ in (13′). Substituting the new variable $z = y/\nu$ and expanding the exponentials in powers of ν, we obtain

$$I(0, \nu) \approx \frac{2}{\nu} \int_0^\infty \frac{\ln|2z - 1|\, dz}{z(z-1)} = \frac{3\pi^2}{20\nu}. \quad (20)$$

b) $\nu \gg 1 \gg \eta$. It is now more convenient to use (13) for $I(\eta, \nu)$. Substituting $\eta = 0$, we find that for $\nu \gg 1$ most of the integral comes from the region of integration about the point with the coordinates $x = y = \nu/2$. We can therefore neglect unity compared with e^x and e^y. We thus have

$$I(0, \nu) \approx \iint \frac{dx\, dy}{e^{x+y-\nu} - 1} \qquad (4xy \geqslant \nu^2;\ x > 0,\ y > 0).$$

After integrating once we have

$$I(0, \nu) \approx -\int_0^\infty \ln\left[1 - \exp\left\{-\frac{(\nu - 2x)^2}{4x}\right\}\right] dx.$$

Taking $\nu \gg 1$ into account, we obtain

$$I(0, \nu) \approx \sqrt{2\nu} \int_0^\infty \frac{z^{1/2} dz}{e^z - 1} = \zeta(3/2) \sqrt{\frac{\pi\nu}{2}}. \quad (21)$$

c) $\eta \gg \nu \gg 1$. The asymptotic value (13′) now does not differ from its value for $\eta \gg 1$ at low frequencies [see Eq. (17)].

d) $\nu \gg \eta \gg 1$. We again use (13) for $I(\eta, \nu)$. Since $\eta \gg 1$ while $x, y > \eta$, we have

$$I(\eta, \nu) \approx \iint \frac{dx\, dy}{e^{x+y-\nu} - 1}, \qquad (\nu - \eta)^2 < 4(x - \eta)(y - \eta),\quad x > \eta,\quad y > \eta.$$

After a single integration and expansion of the logarithm about unity, the method of steepest descents yields

$$I(\eta, \nu) \approx \sqrt{\pi\nu/2}. \quad (22)$$

3. Using the asymptotic values of $I(\eta, \nu)$, we derive the following values of the absorption coefficient. For $\mu H_e \ll T$:

$$\Gamma \approx \frac{0{,}53\omega}{8\sqrt{3}\,\pi^2} \frac{w^2 T^2}{\mu M_0 \Theta_c^2} \frac{\hbar\omega}{\mu H_e} \qquad (\hbar\omega \ll \mu H_e \ll T),$$

$$\Gamma \approx \frac{\omega}{192} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \qquad (\hbar\omega = \mu H_e \ll T),$$

$$\Gamma \approx \frac{3\omega}{640} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \qquad (T \gg \hbar\omega \gg \mu H_e),$$

$$\Gamma \approx \frac{\zeta(3/2)\,\omega}{32\sqrt{2}\,\pi^{3/2}} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \sqrt{\frac{\hbar\omega}{T}} \qquad (\hbar\omega \gg T \gg \mu H_e).$$

For $\mu H_e \gg T$:

$$\Gamma \approx \frac{\omega}{32\sqrt{2}\,\pi^{3/2}} \frac{w^2 \hbar\omega}{\mu M_0 \Theta_c^2} \sqrt{\frac{\mu H_e T}{\Theta_c^2}} \exp\left\{-\frac{3\mu H_e}{T}\right\} \quad (\hbar\omega \ll T \ll \mu H_e),$$

$$\Gamma \approx \frac{\omega}{32\pi^2} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \exp\left\{-\frac{\hbar\omega}{T}\right\} \qquad (\hbar\omega = \mu H_e \gg T),$$

$$\Gamma \approx \frac{\omega}{32\sqrt{2}\,\pi^{3/2}} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \sqrt{\frac{\mu H_e}{T}} \exp\left\{-\frac{3\mu H_e}{T}\right\} \quad (\mu H_e \gg \hbar\omega \gg T),$$

$$\Gamma \approx \frac{\omega}{32\sqrt{2}\,\pi^{3/2}} \frac{w^2 T^2}{\mu M_0 \Theta_c^3} \sqrt{\frac{\hbar\omega}{T}} \qquad (\hbar\omega \gg \mu H_e \gg T).$$

With the absorption coefficient Γ known, the dispersion equation for a circularly polarized electromagnetic wave can be used to determine the imaginary part of the transverse magnetic susceptibility

$$\mu''_\perp = \mu'_\perp \Gamma/\omega,$$

where $\mu'_\perp$ is the real part of the magnetic susceptibility. At low frequencies ($\omega \ll gB_e$) we have $\mu'_\perp = B_e/H_e$, where $B_e = H_e + 4\pi M_0$; at high frequencies ($\omega \gg gB_e$) the result is $\mu'_\perp = 1$.

An estimate of $\mu''_\perp$ near resonance shows that nonresonance absorption is entirely negligible in this region, while it evidently plays an important part far from resonance.

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