THE FORM FACTOR AND THE μ CAPTURE BY LIGHT NUCLEI WITH SPIN ½

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Various processes of μ capture by light nuclei with spin ½ without emission of neutrons or protons are investigated. The calculations show that the form factor gives a significant correction to the effects accompanying these processes.

1. INTRODUCTION

THERE has been a great deal of interest recently in studying the effect of the form factor on weak interaction processes.[1] The role of the form factor in the μ capture by light nuclei without emission of neutrons or protons has been investigated by several authors.[2] Strictly speaking, however, the formulas given by these authors are useful only for the description of a nucleus in the ground state with zero spin, since the rate of capture and the state of polarization of the nucleus in the final state depend to a great extent on the hyperfine structure of the μ-mesic atoms, This circumstance was noted by Bernstein et al. and also by Shmushkevich.[3] It is therefore of interest to investigate the effect of the form factor on the process of μ capture without emission of neutrons with account of the hyperfine structure.

2. FORMULATION OF THE PROBLEM

The weak-interaction Hamiltonian including the form factor can be written in the form

$$H_W = \frac{1}{\sqrt{2}} G\overline{\Psi}_p (V_\alpha + A_\alpha)\Psi_n\overline{\Psi}_\nu\gamma_\alpha(1+\gamma_5)\Psi_\mu,$$

$$V_\alpha = \gamma_\alpha + (\mu'/2m_p)\sigma_{\alpha\beta}K_\beta, \qquad A_\alpha = \lambda\gamma_\alpha\gamma_5 + (if'/m_\mu)\gamma_5 K_\alpha, \tag{1}$$

where

$$K = p_p - p_n; \qquad \sigma_{\alpha\beta} = -\tfrac{1}{2} i(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha);$$

λ is the ratio of the Gamow-Teller and Fermi coupling constants and is equal to 1.2 in the case of β decay; the term involving μ' is the "weak magnetism" term [μ' is the sum of the anomalous gyromagnetic ratios, $\mu' = \mu_p - \mu_n - 1 = 3.7$ (reference 4)], and the term involving f' represents an induced pseudoscalar (in order of magnitude f' is equal to 8λ in the case of μ capture by a proton[5]).

If the μ meson is captured by a nucleus, the matrix element can be expressed in the form

$$\frac{G}{\sqrt{2}}\int d\tau \langle\nu|\gamma_\alpha(1+\gamma_5)|\mu\rangle\Big\langle f\Big|\sum_{i=1}^{A}\tau_i^{(-)}(V_\alpha + A_\alpha)\Big|i\Big\rangle\delta(\mathbf{r}-\mathbf{r}_i), \tag{2}$$

where ν, μ, f, and i are the wave functions of the neutrino, the μ meson, and of the final and the initial states of the nucleus, respectively; $\tau_i^{(-)}$ is an operator which converts a proton into a neutron, and $d\tau$ is the element of the nuclear volume.

Before the capture the μ meson is almost at rest, so that we may use the non-relativistic wave function of the μ meson. We also neglect the momentum of the protons in the initial state. As was shown by Primakoff and Ioffe,[2] this leads to an error which does not exceed a few percent. With these approximations the matrix element takes the form

$$\tfrac{1}{2} G\Big[\langle v_\nu|1-\boldsymbol{\sigma}\mathbf{n}|v_\mu\rangle\langle f|\sum_{i=1}^{A}\tau_i^{(-)}(V - P\boldsymbol{\sigma}\mathbf{n})e^{-i\mathbf{q}\mathbf{r}_i}\varphi_\mu|i\rangle$$
$$-\langle v_\nu|(1-\boldsymbol{\sigma}\mathbf{n})\boldsymbol{\sigma}|v_\mu\rangle\langle f|\sum_{i=1}^{A}\tau_i^{(-)}A\boldsymbol{\sigma}e^{-i\mathbf{q}\mathbf{r}_i}\varphi_\mu|i\rangle\Big], \tag{3}$$

where $\mathbf{q}$ is the momentum of the neutrino, $\mathbf{n}$ is a unit vector in the direction of the momentum of the neutrino, $\varphi_\mu = \pi^{-1/2}(Z/a_\mu)^{3/2}\exp(-Zr/a_\mu)$ is the Coulomb wave function of the μ meson, $a_\mu = \hbar^2/m_\mu e^2$ is the Bohr radius of the μ-mesic hydrogen atom, v_ν and v_μ are the spin wave functions of the neutrino and the μ meson, and

$$V = 1 + \tfrac{1}{2}\beta, \quad A = \lambda + \tfrac{1}{2}\mu\beta, \quad P = \tfrac{1}{2}\beta(f+\mu),$$
$$\beta = q/m_p, \qquad \mu = \mu' + 1, \qquad f = f' - \lambda. \tag{4}$$

Here we restrict ourselves to the calculation of the μ capture process by a nucleus with spin ½. After the capture the nucleus goes into a state with spin ½ or 3/2 without emitting a neutrino. The μ meson is depolarized as it slows down and transfers part of its polarization to the nucleus during the formation of a definite state of the hyperfine structure. As it is well known that the hyperfine splitting of the ground state level in mesic hydrogen is appreciably larger than $\hbar/\tau$ (τ is the half-life of the μ meson), the states of the hyperfine structure are incoherent.

It was shown by Shmushkevich[6] that the corresponding density matrices for the initial states are

$$\rho_+ = \frac{1}{4}\left(1 + \varkappa \mathbf{j}(\sigma_p + \sigma_\mu) + \frac{1}{3}\sigma_p\sigma_\mu\right) \text{ (triplet state)},$$
$$\rho_- = \frac{1}{4}(1 - \sigma_p\sigma_\mu) \text{ (singlet state)}. \quad (5)$$

It is seen from (5) that in the triplet state the polarization vectors of the proton and the μ meson are identical and equal to $\kappa\mathbf{j}$. This implies that half of the polarization of the μ meson is transferred to the nucleus.

The theory proposed by Shmushkevich is, in a sense, confirmed by the latest experiments performed by Ignatenko et al.[7] They measured the polarization of a μ meson captured in the K shell of a phosphorus nucleus with spin $1/2$ and obtained the value 8.5%. This is exactly half the value of the polarization ($\sim$ 17%) of the μ meson in the capture by a nucleus with spin zero. Clearly, half of the polarization is in this case transferred to the nucleus.

Let us assume that the state of the mesic atom is a mixture of two possible states of the hyperfine structure. Then the density matrix before the capture is given by the formula (b is the statistical weight of the singlet state)

$$\rho = (1-b)\rho_+ + b\rho_- = 1 + \xi_p\sigma_p + \xi_\mu\sigma_\mu + \varepsilon\sigma_p\sigma_\mu,$$

$$\varepsilon = \frac{1}{3}(1-4b), \quad \xi_p = \xi_\mu = (1-b)\varkappa\mathbf{j}. \quad (6)$$

The value of ξ_μ can be easily determined by measuring the asymmetry coefficient for the decay of the μ meson before capture. The quantity b is equal to $1/4$ if we assume that the μ meson goes into the ground state and emits γ rays. However, if there exists some mechanism which involves transitions between two states of the hyperfine structure, the value of b is less than $1/4$. Its value can be estimated from the measured value of the asymmetry parameter of the μ meson, if we assume that the μ meson has the polarization predicted by the theory before this mechanism comes into effect, i.e., 8.5%, and then compare the theoretical value with the experimental one, since the transition from the triplet state to the singlet state involves an additional depolarization.

3. RESULTS OF THE CALCULATIONS

After the capture of the μ meson the nucleus goes over into a state with spin $1/2$ or $3/2$. Treating the transitions into either of these states separately and using the usual method of calculation, we can derive formulas for the transition probability, the angular distribution, the polarization of the nucleus in the direction of flight of the μ meson and of the recoil nucleus, and also for the angular correlation between $\nu - \beta$ and $\mu - \beta$. We shall give here the relatively simple formulas for the special case $\xi_p = \xi_\mu = \xi$, $a = 1/4$, $\epsilon = 0$; the formulas for the general case are given in the Appendix. The total transition probability is

$$W = (G^2Z^3/2\pi^2a_\mu^3)\,N_0q^2(1 - q/Am_p),$$
$$N_0 = (1+\beta)\,|M_F|^2 + (\lambda^2 + \tfrac{1}{2}\lambda\beta(2\mu - f))\,|M_{GT}|^2,$$
$$|M_F|^2 = \frac{1}{2J_i+1}\left|\langle f|\sum_{i=1}^{A}\tau_i^{(-)}e^{i\mathbf{q}\mathbf{r}_i}\varphi_\mu|i\rangle\right|^2,$$
$$|M_{GT}|^2 = \frac{1}{2J_i+1}\left|\langle f|\sum_{i=1}^{A}\tau_i^{(-)}\sigma e^{i\mathbf{q}\mathbf{r}_i}\varphi_\mu|i\rangle\right|^2. \quad (7)$$

Ioffe has shown that the matrix elements M_F and M_{GT} can be expressed in terms of the matrix element for the β decay in the following way:

$$M_F = M_F^\beta(1 - \tfrac{1}{6}q^2\langle r^2\rangle_e),$$
$$M_{GT} = M_{GT}^\beta(1 - \tfrac{1}{6}q^2\langle r^2\rangle_A), \quad (8)$$

where $\langle r^2\rangle_e$ and $\langle r^2\rangle_A$ are the square radii of the charge corresponding to the vector and pseudovector transitions. The numerical values of M_F^β and M_{GT}^β can be obtained from the experimental data on the β decay, and $\langle r^2\rangle_e$ can be calculated with the help of the experimental data on the inelastic scattering of electrons by nuclei. Since the matrix element $\psi_f^*\sigma\psi_i$ has the same form as the matrix element for the magnetic dipole transition, $\langle r^2\rangle_A$ can be regarded as the square radius in the magnetic radiative transition of the nucleus corresponding to the same isotopic multiplet.

In the above-mentioned formulas we have neglected the contribution of the quadrupole moment, which, according to the calculations of Ioffe, does not exceed a few percent.[2]

The angular distribution of the neutrinos with respect to the direction of flight of the μ meson is given by the formula

$$W(\theta) = 1 - a\xi\mathbf{n};$$
$$a = \frac{1+\beta-(2/\sqrt{3})\,[\lambda-(\beta/2)(f-\lambda)]\,\mathrm{Re}\,\rho + (\lambda^2-\lambda\beta f)\,\rho^2/3}{1+\beta+[\lambda^2+(\lambda\beta/3)(2\mu-f)]\,\rho^2}, \quad J' = 1/2,$$
$$a = -\frac{2\lambda^2+\lambda\beta(3\mu+f)}{3\lambda^2+\lambda\beta(2\mu-f)}, \quad J' = 3/2. \quad (9)$$

Here $\rho = M_{GT}/M_F$, and J' is the total angular momentum of the final state of the nucleus. It is seen from formula (9) that the coefficient of the angular distribution for the pure Gamow-Teller transition is independent of the matrix element, and the form factor gives rise to a large correction to the asymmetry coefficient. This means that we can determine the form factor by such an experiment. In a transition without change of spin of a nucleus with spin $^1/_2$ the coefficient of the angular distribution depends on the ratio of the matrix elements, which can be determined from the experimental data on β decay.

The polarization of the nucleus in the direction of incidence of the μ meson is given by

$$W(m') = 1 + B(m'/j')\,\xi\mathbf{n}_j;$$

$$B = \frac{1 + \beta + (2/\sqrt{3})[\lambda + (\beta/6)(3\lambda + 2\mu - f)]\,\mathrm{Re}\,\rho + [\lambda^2 + (\lambda\beta/3)(2\mu - f)]\rho^2/3}{1 + \beta + [\lambda^2 + (\lambda\beta/3)(2\mu - f)]\rho^2},$$

$$J' = {}^1/_2, \qquad B = 2, \quad J' = {}^3/_2; \tag{10}$$

$\mathbf{n}_j$ is the direction of polarization of the nucleus. In a pure Gamow-Teller transition the coefficient B is independent of the matrix element and of the coupling constants λ, μ, and f. This has the consequence that we cannot obtain any information on the form factor by measuring the polarization in the direction of emission of the μ meson. Strictly speaking, this result is true only for the terms of lowest order in β. If terms involving β^2 are included, the correction amounts to 5 to 10%. This correction is given in the Appendix [formula (21)].

The polarization of the nucleus in the direction of emission of the neutrino is given by

$$W(m') = 1 + C(m'/J')\,\mathbf{n}\mathbf{n}_j + D\left(m'^2 - {}^5/_4\right)\left({}^1/_3 - (\mathbf{n}\mathbf{n}_j)^2\right);$$

$$C = {}^2/_3\,\frac{(\lambda^2 + \lambda\mu\beta)\rho^2 + \sqrt{3}[\lambda + (\beta/2)(\lambda - f)]\,\mathrm{Re}\,\rho}{1 + \beta + [\lambda^2 + (\lambda\beta/3)(2\mu - f)]\rho^2}, \quad J' = {}^1/_2,$$

$$C = \frac{\lambda^2 + \lambda\mu\beta}{\lambda^2 + (\lambda\beta/3)(2\mu - f)}, \quad J' = {}^3/_2,$$

$$D = 0, \quad J' = {}^1/_2,$$

$$D = -\frac{(\beta\lambda/2)(\mu + f)}{\lambda^2 + (\lambda\beta/3)(2\mu - f)}, \quad J' = {}^3/_2. \tag{11}$$

In our case the angular correlation between μ and γ is not of any great interest, since it is always isotropic and independent of whether we do or do not include a form factor. The angular correlation between ν and γ, on the other hand, depends on the form factor very strongly. Moreover, the latter case has the advantage that the strength of the correlation does not depend on the matrix element. Here we shall give the results for a number of special cases and leave the general formulas for the Appendix. Let J_i, J', and J_f be the spins of the initial, intermediate, and final states, respectively. Then the formulas for the angular correlation in the transition $J_i \to J' \to J_f$ are

$$\begin{aligned}
&1 - 3D\cos^2\theta/(3 + D), && 0 \to 1 \to 0;\\
&1 + 3D\cos^2\theta/(6 - D), && 0 \to 1 \to 1;\\
&1 - 3D\cos^2\theta/(30 + D), && 0 \to 1 \to 2;\\
&1 - 3D\cos^2\theta/(6 + 2D), && {}^1/_2 \to {}^3/_2 \to {}^1/_2;\\
&1 + 6D\cos^2\theta/(15 - D), && {}^1/_2 \to {}^3/_2 \to {}^3/_2;\\
&1 + 3D\cos^2\theta/(30 - 5D), && {}^1/_2 \to {}^3/_2 \to {}^5/_2.
\end{aligned} \tag{12}$$

Here θ is the angle between the directions of emission of the neutrino and the γ quantum emitted by the excited nucleus. It is easily seen that the form factor gives a large contribution to the angular correlation. For example, in the transition $0 \to 1 \to 0$ the correlation is isotropic without the form factor, while the presence of the form factor leads to anisotropy in the angular correlations which may be as large as 50%.

The angular correlation between the μ meson and the electron is given by

$$\begin{aligned}
&W(\theta) = 1 + E\xi\mathbf{v}_e;\\
&E = 1 - \beta^2(f + \mu)^2/12\lambda^2, \quad 0 \to 1 \to 0,\\
&E = 2 - \beta^2(f + \mu)^2/12\lambda^2, \quad {}^1/_2 \to {}^3/_2 \to {}^1/_2,
\end{aligned} \tag{13}$$

where $\mathbf{v}_e$ is the velocity of the electron. Generally speaking, this correlation is difficult to observe, except in the case when the life time of the nucleus after capture is very small (for example, in the transitions $C^{12} \rightleftarrows B^{12}$, $C^{13} \rightleftarrows B^{13}$). It may therefore be possible to preserve the polarization of the nucleus after capture in an appropriate magnetic field. It is difficult to determine the form factor in this way, but the transition $^1/_2 \to {}^3/_2 \to {}^1/_2$ as well as the transition $0 \to 1 \to 0$ can be used for the purpose of finding out whether the nucleus is polarized by the meson and of determining the helicity of the μ mesons, since a different helicity of the μ meson leads to different signs in the angular correlations.

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APPENDIX

We give here the general formulas which are valid with an accuracy up to terms of order β^2. Even these formulas are, of course, not sufficiently accurate, since we neglected the momentum of the proton in the initial state. However, under certain circumstances, the terms $\sim\beta^2$ can become comparable with the terms of order β in view of the large coefficient of the anomalous magnetic moment and of the pseudoscalar term. In the derivation of the formulas describing the angular distribution of the neutrino and of the γ quanta for oriented nuclei we used the results of the work of Beretskiĭ et al.[8] and of Falkoff and Ling.[9]

A. Transition $\frac{1}{2} \rightarrow \frac{1}{2}$

1. Total transition probability

$$W = (G^2Z^3/2\pi^2a_\mu^3)\, q^2 (1 - q/Am_p)\, N_0, \qquad N_0 = (1+\tfrac{1}{2}\beta)^2 |M_F|^2 + (\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) + \tfrac{1}{12}\beta^2(2\mu^2 + f^2))\,|M_{GT}|^2$$
$$- 2\varepsilon[\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) - \tfrac{1}{12}\beta^2\mu(2f - \mu)]\,|M_{GT}|^2 - 2\sqrt{3}\varepsilon[\lambda + \tfrac{1}{6}\beta(3\lambda + 2\mu - f) + \tfrac{1}{12}\beta^2(2\mu - f)]\,\mathrm{Re}\, M_F M_{GT}. \tag{14}$$

2. Angular distribution of the recoil nucleus

$$W(\theta) = 1 - \xi_p \mathbf{n} N_1/N_0 - \xi_\mu \mathbf{n} N_2/N_0, \qquad N_1 = \tfrac{2}{3}[(\lambda^2 + \lambda\mu\beta + \tfrac{1}{4}\mu^2\beta^2)\,|M_{GT}|^2 - \sqrt{3}(\lambda + \tfrac{1}{2}\beta(f - \lambda) - \tfrac{1}{4}\beta^2 f^2)\,\mathrm{Re}\, M_F M_{GT}],$$
$$N_2 = (1 + \tfrac{1}{2}\beta)^2|M_F|^2 - \tfrac{1}{3}(\lambda^2 + \lambda\beta(2\mu + f) + \tfrac{1}{4}\beta^2(2\mu^2 - f^2))\,|M_{GT}|^2. \tag{15}$$

3. Polarization of the nucleus in the direction of polarization of the μ meson

$$W(m') = 1 + \xi \mathbf{n}_j 2m' N_5/N_0 + \xi_\mu \mathbf{n}_j 2m' N_6/N_0; \qquad N_5 = (1 + \tfrac{1}{2}\beta)^2|M_F|^2 - \tfrac{1}{3}(\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) + \tfrac{1}{12}\beta^2(2\mu^2 + f^2))\,|M_{GT}|^2,$$
$$N_6 = -(2/\sqrt{3})\,[\lambda + \tfrac{1}{6}\beta(3\lambda + 2\mu - f) + \tfrac{1}{12}\beta^2(2\mu - f)]\,\mathrm{Re}\, M_F M_{GT} + \tfrac{2}{3}[\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) - \tfrac{1}{12}\beta^2\mu(2f - \mu)]\,|M_{GT}|^2 \tag{16}$$

(m' is the magnetic quantum number of the nucleus in the final state).

4. Polarization of the nucleus in the direction of flight of the neutrino

$$W(m') = 1 + \mathbf{n}\mathbf{n}_j\, 2m' N_4/N_0, \qquad N_4 = \tfrac{2}{3}\,[(\lambda^2 + \lambda\mu\beta + \tfrac{1}{4}\mu^2\beta^2)|M_{GT}|^2 + \sqrt{3}(\lambda + \tfrac{1}{2}\beta(\lambda - f) - \tfrac{1}{4}\beta^2 f)\mathrm{Re} M_F M_{GT}]$$
$$- \varepsilon[(1 + \tfrac{1}{2}\beta)^2|M_F|^2 + \tfrac{1}{3}(7\lambda^2 - \beta\lambda(4\mu - 3f) + \tfrac{1}{3}\beta^2(2\mu^2 - 4\mu f + f^2))\,|M_{GT}|^2$$
$$+ (4/\sqrt{3})\,(\lambda + \tfrac{1}{2}\beta(\lambda + \mu) + \tfrac{1}{4}\beta^2\mu)\,\mathrm{Re}\, M_F M_{GT}]. \tag{17}$$

B. Transition $\frac{1}{2} \rightarrow \frac{3}{2}$

1. Total transition probability

$$W = (G^2Z^3/2\pi^2a_\mu^3)\, N_0' q^2 (1 - q/Am_p), \qquad N_0' = [(1 + \varepsilon)\,(\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) - \tfrac{1}{12}\beta^2\mu(2f - \mu) + \tfrac{1}{12}\beta^2(\mu + f)^2]\,|M_{GT}|^2. \tag{18}$$

2. Angular distribution of the recoil nucleus

$$W(\theta) = 1 + \tfrac{1}{3}\xi_p \mathbf{n} N_1'/N_0' + \tfrac{1}{3}\xi_\mu \mathbf{n} N_2'/N_0', \quad N_1' = (\lambda^2 + \lambda\mu\beta + \tfrac{1}{4}\mu^2\beta^2)\,|M_{GT}|^2, \quad N_2' = (\lambda^2 + \lambda\beta(2\mu + f) + \tfrac{1}{4}\beta^2(2\mu^2 - f^2))\,|M_{GT}|^2. \tag{19}$$

3. Polarization of the nucleus in the direction of polarization of the μ meson

$$W(m') = 1 + \tfrac{2}{3}m'\xi_p \mathbf{n}_j N_3'/N_0' + \tfrac{2}{3}m'\xi_\mu \mathbf{n}_j N_4'/N_0', \qquad N_3' = [\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) + \tfrac{1}{12}\beta^2(2\mu^2 + f^2)]\,|M_{GT}|^2,$$
$$N_4' = [\lambda^2 + \tfrac{1}{3}\lambda\beta(2\mu - f) - \tfrac{1}{12}\beta^2\mu(2f - \mu)]\,|M_{GT}|^2. \tag{20}$$

If $\xi_p = \xi_\mu$, then

$$W = 1 + \tfrac{2}{3}[2 - \beta^2(f + \mu)^2/12\,(1 + \varepsilon)\,\lambda^2]\, m'\xi\mathbf{n}_j. \tag{21}$$

4. Polarization of the nucleus in the direction of emission of the neutrino

$$W(m') = 1 + \tfrac{2}{3}m'\mathbf{n}\mathbf{n}_j N_5'/N_0' + (m'^2 - \tfrac{5}{4})[\tfrac{1}{3} - (\mathbf{n}\mathbf{n}_j)^2]\, N_6'/N_0', \quad N_5' = (1 + \varepsilon)\,(\lambda + \tfrac{1}{2}\mu\beta)^2\,|M_{GT}|^2,$$
$$N_6' = -[(1 + \varepsilon)[\tfrac{1}{2}\beta\lambda(\mu + f) + \tfrac{1}{4}\beta^2(\mu^2 + f\mu)] - \tfrac{1}{8}\beta^2(\mu + f)^2]\,|M_{GT}|^2. \tag{22}$$

C. Angular Correlation between the Neutrino and the γ Quanta [$W(m')$ is given by (20)]

$$W(\theta) = \sum_{M_f, m'} W(m')\, P_{M_f, m'}(\theta), \qquad P_{M_f, m'}(\theta) = |\langle J_f, L, M_f, M\,|\,J', m'\rangle|^2\, |E_L|^2\, F_L^M(\theta) + |\langle J_f, L-1, M_f, M\,|\,J', m'\rangle$$
$$\times |^2\, |M_{L-1}|^2\, F_{L-1}^M(\theta) + \langle J_f, L, M_f, M\,|\,J', m'\rangle\langle J_f, L-1, M_f, M\,|\,J'\, m'\rangle (E_L M_{L-1}^* + \text{c.c.})\, F_{L, L-1}^M(\theta);$$
$$F_L^M(\theta) = \frac{4\pi}{L(L+1)}\,[2M^2|Y_L^M|^2 + (L + M)(L - M + 1)\,|Y_L^{M-1}|^2 + (L + M + 1)(L - M)\,|Y_L^{M+1}|^2],$$
$$F_{L, L-1}^M(\theta) = -4\pi[(2L + 1)(L^2 - M^2)/(2L - 1)L^2(L^2 - 1)]^{1/2} \times [2M\,|Y_{L-1}^M|^2 + (L - M - 1)\, Y_L^{M+1}|^2 - (L + M - 1)\,|Y_{L-1}^{M-1}|^2]. \tag{23}$$

D. Angular Distribution of the Electrons in the Direction of Polarization of the μ Meson

$$W(\theta) = \sum_{m'} W(m')(1 + \bar{\beta}_{m'} v \cos\theta),$$

$$\bar{\beta}_{m'} = \{\Delta_{j',m'} + 2m'[j'(j'+1)]^{-1/2}\delta_{j_f,j'}\}[1 + |\rho'|^2\delta_{j_f,j'}]^{-1},$$

$$\Delta_{j',m'} = \begin{cases} m'/j', & j_f = j' - 1 \\ m'/j'(j'+1), & j_f = j' \\ -m'/(j'+1), & j_f = j' + 1 \end{cases} \quad (24)$$

(j_f is the spin of the nucleus in the final state after the β decay, $\rho' = M_F^\beta / M_{GT}^\beta$).

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