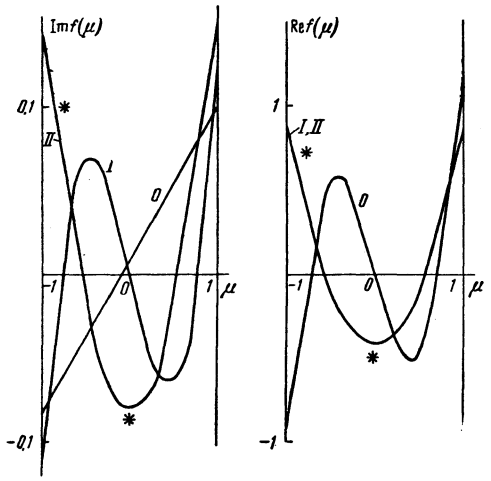
When $R_{n+1}(\mu)$ passes through zero the sign of $R_{n+1}(\mu)$ is fixed by the requirement of continuity of the derivative.

The integrals in Eq. (4) lose their meaning if $R_n(\mu')$ has zeroes μ_s in the interval $(-1, 1)$. To regularize the integrals we can use their principal values:

$$\Phi(R(\mu)) = \int_{-1}^{1} \frac{\varphi(\mu)}{R(\mu)} d\mu = \int_{-1}^{1} \left[\frac{\varphi(\mu)}{R(\mu)} - \sum_s \frac{\varphi(\mu_s)}{(\mu - \mu_s) R'(\mu_s)} \right] d\mu + \sum_s \frac{\varphi(\mu_s)}{R'(\mu_s)} \ln \frac{1-\mu_s}{1+\mu_s}. \quad (5)$$

It was found that such applications of the theory of generalized functions assure convergence even in cases in which the numbers of zeroes of the solution and of the zeroth approximation are not the same (see diagram). Equation (4) was solved with an electronic computer by reducing it to an algebraic system of linear equations. The number of equations is then equal to the number of points used in the numerical-integration formula (that of Gauss) plus the number of zeroes of the real part of the scattering amplitude [cf. the factor $\varphi(\mu_S)$ in Eq. (5)]. In the examples solved the number of zeroes does not exceed three, and the number of points for the integration formula was six. As a rule the functions chosen as zeroth approximations were extremely far removed from the actual solutions of the system.

In spite of this, rapid convergence of the iteration process was found in seven out of the ten examples solved. The correction to the zeroth approximation, which was often larger in magnitude than $I_0(\mu)$ and $R_0(\mu)$ themselves, at once gave almost the correct value of $|R(\mu)|$. The next approximation led to the correct value of $I(\mu)$. Three or four approximations were usually enough to get three-figure accuracy (see diagram). The convergence is slower in the three examples in which either R or R_0 has no zeroes, i.e., in which the regularization by Eq. (5) either is unnecessary ($R_0 > 0$) or leads to a $\Phi[R_0(\mu)]$ that differs sharply from $\Phi[R(\mu)]$ ($R > 0$, R_0 changes sign). In all ten examples the scattering amplitude in forward directions ($\mu > 0$) was reconstructed with particularly high accuracy; the reconstruction for R was better than that for I.



Convergence of the iteration process. Curves 0, I, II are respectively the zeroth, first, and second approximations, and * is the exact solution.

The numerical experiment which we have made allows us to hope that also in the more complicated case of scattering of particles with spin direct solution of the system of nonlinear equations can give the desired scattering matrix, bypassing phase-shift analysis.[1]

For a detailed exposition of the results of the present work, see reference 3.

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ELECTRONIC PARAMAGNETIC RESONANCE OF THE Ti^{3+} *ION IN CORUNDUM*

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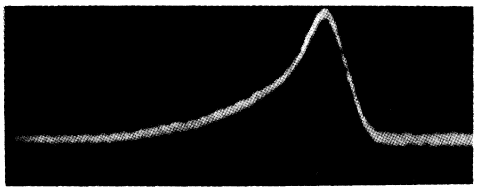
UP to recently, electronic paramagnetic resonance (e.p.r.) of the Ti^{3+} ion was observed only in cesium-titanium alum.[1,2] The authors of these

papers mentioned certain difficulties in the interpretation of the results obtained.

We observed e.p.r. of Ti^{3+} ions, introduced isomorphically in the crystal lattice of corundum (Al_2O_3), at temperatures of liquid helium. The average concentration of the titanium ions in each of the three investigated samples was several hundredths atomic percent. The investigations were made at wavelengths near 3 cm.



The e.p.r. line of the Ti^{3+} ion in corundum for $\theta = 0°$ (the magnetic field increases to the right).

We observed one e.p.r. line, the shape of which had an unusual asymmetry with a steeper descent on the side of increasing magnetic field (see photograph). An investigation of the behavior of the line with varying angle θ between the directions of the permanent external magnetic field **H** and the trigonal axis of the electric field of the crystal has shown that the theoretical interpretation can be carried out with the aid of the spin Hamiltonian[3]

$$\hat{H} = g_{\parallel}\beta H_z \hat{S}'_z + g_{\perp}\beta (H_x \hat{S}'_x + H_y \hat{S}'_y),$$

where $g_{||}$ and $g_{\perp}$ are the g factors in directions parallel and perpendicular to the trigonal axis of the crystal field, β is the Bohr magneton, and $\hat{S}'$ is the effective-spin operator, with value $S' = \frac{1}{2}$.

The g factors, determined by the points of maximum line intensity, were found to be

$$g_{\parallel} = 1.067 \pm 0.001; \quad g_{\perp} \leqslant 0.1.$$

The estimate of $g_{\perp}$ was made by studying the dependence of the line position on the angle θ as the latter was varied from 0 to 67 deg. The e.p.r. line was studied here at magnetic field values from 6,000 to 16,000 oe, and the upper estimate of $g_{\perp}$ was chosen to obtain good agreement with the theoretical relationship $H = h\nu/g\beta$, where $g = (g_{||}^2 \cos^2\theta + g_{\perp}^2 \sin^2\theta)^{1/2}$. This estimate of $g_{\perp}$ agrees with the extremely low density of the observed e.p.r. line of the Ti^{3+} ion, the peak value of which at $\theta = 0$ deg is approximately equal to the peak intensity of the e.p.r. lines observed in the same specimens and produced by Cr^{3+} and Fe^{3+} ions at a concentration 10^3 or 10^4 times smaller than the concentration of the Ti^{3+} ions. This is explained by the fact that when the high-frequency magnetic field is perpendicular to the external magnetic field the intensity of the absorption line is proportional to the square of the quantity

$$g' = g_{\parallel} g_{\perp} (g_{\parallel}^2 \cos^2\theta + g_{\perp}^2 \sin^2\theta)^{-1/2}.$$

The width of the e.p.r. line of the Ti^{3+} ion in corundum at $\theta = 0°$, measured between points of maximum slope of the line at 4.2°K, was found to be 50 oe. The shape and the width of the line did not change as the temperature was reduced to 1.55°K. A study of the dependence of the line width (in oersteds) on the angle shows the presence of a considerable deviation from the law $\Delta H = h\Delta\nu/g\beta$, which in this case, assuming $\Delta\nu$ to be constant, can be represented with sufficient accuracy by $\Delta H = \text{const}/\cos\theta$. Actually, the line broadening was much faster, particularly at small values of θ. A satisfactory explanation for this fact can be proposed by assuming for the individual ions a dispersion of order 0.5 to 1° in the directions of the trigonal axis of the crystal electric field, about a certain average direction. At all values of θ, the line retained the characteristic asymmetric form.

When the specimen was heated, the line began to broaden noticeably with increasing temperature, starting with ~ 9°K. This enabled us to estimate the spin-lattice relaxation time τ_1, for the indicated temperature, at approximately 5×10^{-8} seconds. We also measure τ_1 by the continuous-saturation method. If it is assumed that $g_{\perp} = 0.1$, the values of τ_1 will be approximately 10^{-4} sec at 4.2°K and approximately 10^{-1} sec. at 1.55°K. As in the case of the Co^{2+} in corundum,[4] τ_1 has a rather strong temperature dependence at these temperatures. This has not yet been satisfactorily explained.

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