ALPHA-DEUTERON MODEL OF THE Li^6 NUCLEUS

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On the assumption that the nucleons in the Li^6 nucleus are predominantly grouped in subsystems in the form of an alpha particle and a deuteron, the energy of the relative motion of these subsystems is calculated and it is shown that it has a minimum in the region of negative values. The energy of the O^+, $T = 1$ excited state of the Li^6 nucleus is also determined.

1. It is known that each nucleon in a nucleus can be regarded in first approximation as moving in a certain average field of the other nucleons. Recently it has become clear, however, that for a better description of the properties of nuclear matter one should take account more carefully of the interaction of the nucleons. In the uniform theory of the nucleus which was developed as a result of this consideration, this interaction is taken into account by considering the collective motions of the whole nucleus, while in the many-body theory which was developed for the case of the nucleus system by Brueckner and Bethe, the approximation which gives the average field consists in the introduction of the so-called correlated pairs. However, for the validity of either theory it is necessary that the number of nucleons in the nucleus be sufficiently large.

In the case of the lightest nuclei, containing only a few nucleons, the collective description is not applicable, and it is not excluded that the effect of the interaction may with good probability result in the formation within the nucleus of individual internally-bound subsystems. In other words, in the case of the lightest nuclei one of the possible ways for taking account of the interaction of the nucleons, more rigorously than is done by introducing an overall average field with a single center of symmetry, may consist in the assumption that the nucleons within such a nucleus are bound into separate groups with a center of symmetry in each of these groups. In addition, it is known that temporary agglomerations of nucleons occur preferentially in the surface layer of the nucleus. Since, as we go toward light nuclei, the relative importance of surface effects increases, it should be expected that in them these agglomerations of nucleons forming substructures will have relatively higher stability.

Recently many investigators[1-6] have called attention to the idea of the existence of substructures within atomic nuclei in the form, for example, of deuterons, tritons, and alpha particles. Of course, the original assumption that there exist in the nucleus alpha particles in the form of stable and unchangeable structural units is not applicable. However, it is entirely possible that the nucleons in light nuclei preferentially group themselves into alpha particles where the composition of each of these particles, because of the exchange of nucleons, is subjected to a continual change.

On the basis of this assumption, Biel[7] made a calculation of the binding energy of Be^8 and C^{12} and found satisfactory agreement with experiment for an appropriate choice of the mixture of Serber and symmetric forces. It should, however, be noted that all the work in this direction refers to light nuclei of the type 4n, containing 2, 3, or 4 alpha particles.

But, one asks, what can one say about a nucleus in which in addition to alpha particles there are two or three nucleons? Can one, for example, treat the Li^6 nucleus in some approximation as a system of an alpha particle and a deuteron, assuming consequently that the two extra nucleons beyond the alpha particle form a bound system in the form of a deuteron?

Before attempting to answer this question, we point out that from the non-existence of the nuclei He^5 and Li^5, it follows that one nucleon is not bound with the first closed shell consisting of two protons and two neutrons constituting an alpha particle. On the other hand, there exists the stable nucleus Li^6 containing two nucleons outside a closed shell.

In order to make these two results consistent one can adopt two assumptions: 1) the simultaneous joining of a proton and a neutron to the alpha particle results in the break-up of the latter and in the formation of a complicated system of six nucleons with new couplings; 2) in the formation of the Li^6 nucleus, the alpha particle is not broken up, but is effectively bound, not with each of the nucleons individually, but with the system of proton and neutron together.

The adoption of the second assumption enables one to make the natural conclusion that the proton and neutron outside the closed shell in the Li^6 nucleus should form a bound system. In fact, if these nucleons were independent of one another, then from the fact that each of them is not bound to the closed shell, it would follow that both nucleons simultaneously are not bound to this shell, which would imply the impossibility of formation of the Li^6 nucleus. The existence of the Li^6 nucleus means consequently that the proton and neutron in this nucleus are not independent of one another, but are in some bound state.

Later it will be shown that when there is a coupling between the two nucleons which are outside the closed shell in the Li^6 nucleus, the Schrödinger equation for the whole nucleus actually has a solution corresponding to a negative total energy. In connection with this, we investigate in the present paper the conditions of stability of a system of three protons and three neutrons forming subsystems with four and two nucleons respectively; in other words, we solve the problem of whether the energy of the relative motion of these subsystems has a minimum in the region of negative values of the energy and, if it does have such a minimum, whether the energy at the minimum corresponds to the binding energy of the two subsystems to one another as determined from data on the mass defect of the Li^6 nucleus.

2. Following Biel, we assume that both the forces between the two nucleons and their wave functions have a Gaussian shape, and we try to find the conditions under which the energy of the system of six nucleons in the Li^6 nucleus, which is divided into two subsystems which are individually bound in the form of an alpha particle and a deuteron and are continually interchanging nucleons, is a minimum. We also determine the parameters which characterize the Li^6 nucleus in its ground and excited states.

Each state of the nucleon will be expressed as a vector $\mathbf{i}(i_1, i_2, i_3)$, where i_1 denotes the spin of the nucleon and can take on two values: 1 or 2; i_2 is the isotopic spin of the nucleon also taking on values 1 and 2, and i_3 is equal to 1 or 2 indicating to which of the subsystems, alpha particle or deuteron, the given nucleon belongs.

It is obvious that there are altogether eight states: (111), (121), (211), (221), (112), (122), (212) and (222). We denote them respectively by the numbers 1, 2, 3 . . . 8. The six nucleons are to be distributed over these states. Without loss of generality we can assume that, for example, the first four nucleons fill the states 1, 2, 3, and 4 forming an alpha particle. The remaining two nucleons are distributed over the other four states: 5, 6, 7, and 8. The number of such distributions is obviously equal to six. These six distributions correspond to different states of the deuteron in the nucleus. These states will also correspond to states of the Li^6 nucleus as a whole since, by our assumption, for not too large excitations the alpha particle in the nucleus will always be in its ground state with a total spin and isospin equal to 0.

If we denote the total spin and isospin of the Li^6 nucleus by S and T and their projections, respectively, by M_S and M_T, these six distributions will correspond to states with $S = 1$, $T = 0$, $M_S = 0, \pm 1$ and $S = 0$, $T = 1$, $M_T = 0, \pm 1$. The first three of these states correspond to the normal state of the deuteron and determine the ground level of the Li^6 nucleus, while the other three correspond to a singlet state of the pair of nucleons. We assume that the singlet state of the nucleon pair forming the deuteron ($S = 0$, $T = 1$, $M_T = 0$), determines one of the excited states of the Li^6 nucleus. As for the remaining two states ($S = 0$, $T = 1$, $M_T = \pm 1$), they characterize the energies of the isobaric nuclei Be^6 and He^6. Ascribing the total angular momentum of the Li^6 nucleus to the spin of the deuteron, we assume that the orbital angular momentum of the relative motion of the deuteron and alpha particle in both the ground state and in the excited state which we are considering are equal to zero.

We write the antisymmetrized wave functions $\Psi = \Psi^{ST}_{M_S M_T}$, corresponding to given values of S, T, M_S and M_T in the form

$$\Psi^{10}_{10} = \sum_p \pm p \alpha^1_{11} \alpha^2_{12} \alpha^3_{21} \alpha^4_{22} \alpha^5_{11} \alpha^6_{12} \psi(1234; 56), \quad (1)$$

$$\Psi^{01}_{01} = \sum_p \pm p \alpha^1_{11} \alpha^2_{12} \alpha^3_{21} \alpha^4_{22} \alpha^5_{11} \alpha^6_{21} \psi(1234; 56), \quad (2)$$

where $\alpha^f_{i_1 i_2}$ are products of the spin and isospin wave functions of the nucleon f with given values of i_1 and i_2; $\psi(1234; 56)$ is the spatial part of the wave function of the Li^6 nucleus in which the first four indices correspond to nucleons in the alpha particle and the last two to the nucleons forming the deuteron; the summation extends over all permutations p of the superscripts, where the sign is determined by the parity of the corresponding permutation. We note that we have not introduced expressions for the other four functions $\Psi^{ST}_{M_S M_T}$ since they are not used in calculating the energy of the system.

Neglecting the Coulomb interaction of the protons, we have for the total Hamiltonian of the system

$$\hat{H} = -\frac{\hbar^2}{4}\left(\frac{1}{M_n} + \frac{1}{M_p}\right)\sum_i \nabla_i^2 + \sum_{i>j} V(i, j), \qquad (3)$$

where M_n and M_p are the masses of the neutron and proton,

$$V(i, j) = (W + BP^{\sigma}_{ij} - \mathcal{H}P^{\tau}_{ij} - MP^{\sigma}_{ij}P^{\tau}_{ij})\, F(i, j), \qquad (4)$$

where P^{σ}_{ij} and P^{τ}_{ij} are the Bartlett and Heisenberg exchange operators; W, B, $\mathcal{H}$ and M give the fractions corresponding to interactions of the Wigner, Bartlett, Heisenberg, and Majorana type; F(i, j) is the radial part of the nucleon-nucleon interaction.

Our problem consists in calculating the mean value of the energy

$$E = \int \Psi^* \hat{H} \Psi d\tau \Big/ \int \Psi^* \Psi d\tau, \qquad (5)$$

and minimizing it with respect to the parameter appearing in Ψ. To obtain the energy in the ground state we must set $\Psi = \Psi^{01}_{01}$, while $\Psi = \Psi^{10}_{10}$ will determine the energy of one of the excited states of the Li^6 nucleus with angular momentum 0^+, close in energy to the isobaric nuclei He^6 and Be^6.

As mentioned earlier, we choose both the potential energy of the two nucleons and the spatial wave function of the system to have a Gaussian shape, i.e.,

$$\psi\,(1234; 56) = \exp\left\{-\tfrac{1}{2}\lambda s_{12}^2 - \tfrac{1}{2}\mu \sum_{i>j=1}^{4} r_{ij}^2 - \tfrac{1}{2}\nu r_{56}^2\right\}, \qquad (6)$$

$$F(i, j) = V_0 \exp(-\beta r_{ij}^2), \qquad (7)$$

where $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{s}_{12}$ is the radius-vector between the centers of the alpha particle and the deuteron. V_0 and β are taken from data on the scattering of nucleons by alpha particles and are equal[8] to $V_0 = -45$ Mev, $\beta = 0.266 \times 10^{26}\ \mathrm{cm}^{-2}$; μ is determined from the binding energy of the alpha particle and is equal to $0.316 \times 10^{26}\ \mathrm{cm}^{-2}$; λ is a variational parameter determining the equilibrium distance between the centers of the alpha particle and the deuteron. The coefficient ν which is a parameter in the trial function for the system of the two nucleons 5 and 6, forming a deuteron in the triplet state, is determined from the condition that the energy of this system for the given values of V_0 and β have a minimum. From this condition we find that $\nu = 0.150 \times 10^{26}\ \mathrm{cm}^{-2}$.

We mention that with this value of ν the binding energy of the system consisting of the two nucleons 5 and 6 turns out to be negative and numerically equal to 0.41 Mev. Thus, the values of the parameters μ, ν, V_0 and β chosen by us lead to bound stable states of He^4 and H^2 with a binding energy of 27 Mev for the alpha particle and 0.41 Mev for the deuteron, instead of the experimental values of 28.1 and 2.2 Mev for the free alpha particle and deuteron.

One might get better agreement with experiment with regard to the binding energy of the deuteron and the nucleus at the expense of a slight change in the binding energy of the alpha particle, but this would require us to choose a somewhat different value for μ, different from that used by us and taken from reference 8. However, since we are interested not in the total energy of the Li^6 nucleus, but only in the difference $\epsilon(Li^6) = \epsilon(\alpha) - \epsilon(d)$, which gives the relative energy of alpha particle and deuteron, the use of an exact value of $\epsilon(d)$ is not significant. Besides, it can be shown that changing the binding energy of the deuteron within the limits considered has no essential effect upon the position of the minimum in the energy of the relative motion of the alpha particle and deuteron.

3. It is not possible to find the energy minimum analytically because of the complicated nature of the expressions. We therefore went over to a numerical method. In Fig. 1 we show the energy curves for the ground state of Li^6. Curve 1 corresponds to a Serber force and curve 2 to a symmetric force, where the ordinate is the difference

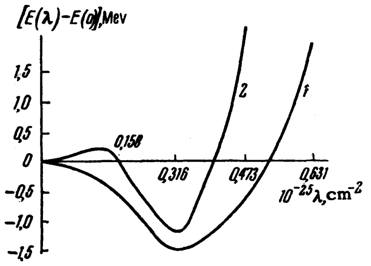

FIG. 1

$E(\lambda) - E(0)$, where $E(0)$ is the energy of the system when the alpha particle and the deuteron are at infinity. We see that the curves actually do have a minimum, which shows that the nucleon configuration in the Li^6 nucleus which we are considering is stable. Also, in both cases the minimum in the energy occurs at the same value, $\lambda = 0.0316 \times 10^{26}\ \mathrm{cm}^{-2}$. For the corresponding values of the energy in the case of the Serber forces we have -1.58 Mev, and for the case of the symmetric forces -1.42 Mev.

In treating the problem of Be^8 and C^{12}, Biel[7] obtained better agreement with experiment by assuming that the total nucleon-nucleon interaction consists of a mixture of Serber and symmetric forces with relative weights of 0.7 and 0.3. If we choose this same mixture for our case, the relation between the energy and the parameter λ has the form shown in Fig. 2. In this case the value

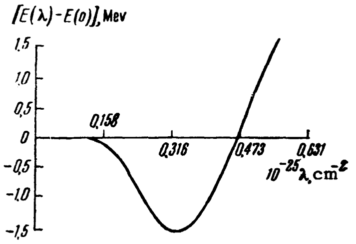

FIG. 2

obtained for the energy at the minimum is -1.52 Mev, which is in good agreement with the experimental value of the binding energy of the deuteron to the alpha particle in the Li^6 nucleus, as found from data on the mass defect and equal to -1.48 Mev. The value $\lambda = 0.0316 \times 10^{26}$ cm^{-2} corresponds to an equilibrium distance between the centers of the alpha particle and deuteron equal to 2.52×10^{-13} cm. Thus we see that the centers of gravity of the subsystems in the Li^6 nucleus are moved with respect to one another, while the distance between them exceeds their individual radii.

Now let us consider the excited state of the Li^6 nucleus. Such a state, according to our model, can arise as a result of excitation in general of the alpha particle and of the deuteron and from their relative motion. But we know that the alpha particle has no bound states with excitation energy below 20 Mev, so that the observed levels of the Li^6 nucleus lying below 20 Mev cannot be the result of excitation of the alpha particle.

The wave function $\Psi^{01}_{0M_T}$ corresponds to a state of the Li^6 nucleus with $S = 0$ and $T = 1$. In the spectrum of the Li^6 nucleus the level with $T = 1$ which is a component of the isotopic triplet ($M_T = 0$) has an energy of 3.57 Mev.[9] The remaining components of the isotopic triplet ($M_T = \pm 1$) correspond to the ground states of the He^6 and Be^6 nuclei. Since this level corresponds to spin 0^+, in our model the transition from the ground state with angular momentum 1^+ to this excited state can occur only via a transition of the deuteron in the nucleus from a triplet state to a singlet state. In such a transition we must also consider the possibility of a change in the average energy of relative motion. Therefore the total change in energy will consist of two parts, one of which is caused by a change in the binding energy of the deuteron and the other by a change in the energy of relative motion.

In Fig. 3 we show the dependence of the difference in energy of relative motion $E^*(\lambda) - E^*(0)$ on the parameter λ for the case of a mixture of Serber and symmetric forces. This curve also has a minimum, where the energy at the minimum is 0.66 Mev. Using the fact that the energy of relative motion at the minimum in the ground state is equal to -1.52 Mev, we have 0.86 Mev for the change in this energy in the transition considered.

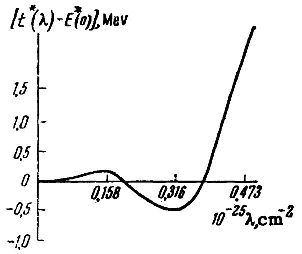

FIG. 3

As for the change in energy occurring as a result of a change in the spin state of the deuteron, our calculation shows that it is equal to 3.91 Mev. Thus, we obtain a value of 4.77 Mev for the total excitation energy of the Li^6 nucleus for the transition to the state $S = 0$, $T = 1$. This value differs somewhat from the experimental value of the energy of the corresponding level, which is 3.57 Mev.

We point out that if we chose for the change in energy of the deuteron when only the spin state changes the experimental value of 2.23 Mev, the total excitation energy would be 3.1 Mev, which would allow us to conclude that there is more or less satisfactory agreement with experiment. Consequently, the actual disagreement with experiment, though not insignificant if we consider the approximate nature of our calculations, leads to a discrepancy with experiment for the value of the change in internal energy of the deuteron in a transition to the singlet state.

This discrepancy is apparently caused by the fact that in both the ground and excited states we have used the same value of β, changing only the depth of the nucleon-nucleon interaction for the singlet state by 0.6. Actually the values of β for the singlet and triplet state differ markedly from one another. We were forced to proceed in this way because the variation of β in the transition from the triplet state to the singlet state could not be accounted for consistently in our calculations because of the presence of exchange integrals in the expressions for the potential energy. We therefore assigned an average value to the coefficient β throughout our calculations, this value being obtained from experimental data on the scattering of nucleons by alpha particles. As we see from this paper, this value of β together with $V_0 = -45$ Mev assures the stability of the He^4 nucleus and gives the correct value for its binding energy.

The average value of the coefficient β which we used differs little from its correct value in the triplet state of the deuteron, so that the shift to the average value should not essentially affect our calculations for the ground state of the Li^6 nucleus. For the excited state such a change may lead to a fundamental error since the value of the coefficient β in the singlet state is considerably different from its value in the triplet state. This error in our opinion also would lead to the result that we obtain 4.77 Mev for the excitation energy instead of the experimentally observed 3.57 Mev. It is possible also that the slight rise in the curves of Figs. 1 — 3 near the origin is due to an error in the choice of the value of the coefficient β.

In connection with the results obtained above, we should mention the following. In references 10 and 11 it is stated that the wave function of a nucleus of type 4n built up on the alpha particle model is identical, after antisymmetrization in all the nucleons, to the wave function in the single particle approximation. However, it is easy to see that such an identity is obtained only if, in the individual wave functions referring both to the internal state of the alpha particle and to the relative motion, the corresponding parameters of the oscillator functions coincide and are equal to the parameter appearing in the wave function of the single particle approximation. In the cited paper of Biel, the parameter in the wave function describing the relative motion of the alpha particles in the Be^8 nucleus is treated as a variational parameter, and by applying the condition for minimum energy it is shown that it is not equal to the parameter characterizing the internal state of the alpha particles. In the same way, as we see from (6), for the case of the Li^6 nucleus, the parameters appearing in the wave functions for the internal states of the alpha particle and deuteron and the wave function of their relative motion differ from one another. One can therefore conclude that our treatment preserves a definite individuality of the alpha particle and deuteron in the Li^6 nucleus and is not identical to a description of this nucleus in the single particle approximation.

In conclusion it is our duty to express our gratitude to the computing center of the Academy of Sciences of the Georgian S.S.R. in the persons of D. A. Kveselav and E. N. Dekanosidze, on the one hand, and to the computing center of the Academy of Sciences of the Armenian S.S.R. in the persons of of R. A. Aleksandryan and F. M. Ter-Mikaélyan on the other, for carrying out our requests for tabulation of functions and computation of a large number of fifth-order determinants.

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340