ON THE SCATTERING OF TRANSVERSELY POLARIZED FERMIONS

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The elastic scattering of transversely polarized fermions is considered. It is shown that in the ultrarelativistic case scattering of the longitudinally polarized fermions is characterized by the four dimensional intrinsic angular momentum tensor. In the latter case the scattered fermions remain transversely polarized in interactions proportional to the matrices ρ_1 and ρ_2; on the contrary, in interactions proportional to the matrices ρ_3 and ρ_4, the longitudinal component may appear after scattering (if the particle has a non-zero rest mass).

1. INTRODUCTION

In the motion of a free fermion, described by the Dirac equation

$$\psi = L^{-3/2}\sum_s C_s b_s \exp\{-i\varepsilon cKt + i\mathbf{kr}\}, \qquad (1)$$

where $K = \sqrt{k^2 + k_0^2}$ is the positive ($\epsilon = 1$) energy of the particle, $\mathbf{k}$ is its momentum, k_0 is the rest mass; we shall characterize the polarization property by the unit three dimensional vector $\mathbf{s}^0$.

For the longitudinal (s_3^0 parallel to the vector $\mathbf{k}$) and transverse ($s_{1,2}^0$ perpendicular to the vector $\mathbf{k}$) components we have

$$s_1^0 = (C_1^+C_{-1} + C_{-1}^+C_1)\,s_0^{-1}, \qquad s_2^0 = i\,(C_{-1}^+C_1 - C_1^+C_{-1})\,s_0^{-1},$$
$$s_3^0 = (C_1^+C_1 - C_{-1}^+C_{-1})\,s_0^{-1}, \qquad (2)$$

respectively, where

$$s_0 = C_1^+C_1 + C_{-1}^+C_{-1}. \qquad (3)$$

As is well known, either the four dimensional spin pseudovector ($\hat{\xi}_\mu = \boldsymbol{\sigma},\ i\rho_1$):

$$\zeta_3 = Ks_3^0, \quad \zeta_{1,2} = k_0 s_{1,2}^0 \quad \zeta_t = -i\zeta_4 = ks_3^0, \qquad (5)$$

or the intrinsic angular momentum tensor of the electron ($\xi_{23} = \rho_3\sigma_1$; $-i\hat{\xi}_{14} = \rho_2\sigma_1$, etc)

$$\mu_3 = \zeta_{12} = k_0 s_3^0, \quad \mu_{1,2} = Ks_{1,2}^0, \quad \varepsilon_1 = -i\zeta_{14} = ks_2^0,$$
$$\varepsilon_2 = -ks_1^0,\ \varepsilon_3 = 0 \qquad (5a)$$

can be formed from the unit three dimensional vector $\mathbf{s}^0$ by averaging of the type

$$\zeta = \int \psi^+ K\hat{\zeta}\psi d^3x \qquad (4)$$

with corresponding matrices $\hat{\xi}$.

Equation (4) can be generalized to the case of the presence of states of both positive and negative energies by using the relation

$$K\hat{\zeta} \to \tfrac{1}{2}(\boldsymbol{\alpha}\mathbf{k} + \rho_3 k_0)\hat{\zeta} + \tfrac{1}{2}\hat{\zeta}(\boldsymbol{\alpha}\mathbf{k} + \rho_3 k_0). \qquad (6)$$

In the case of the free motion, the generalized matrices will, in similar fashion, commute with the Hamiltonian, and will lead to the result (4) in averaging for states with positive energy.

For matrices of the spin pseudovector (see references 1 — 3) and for the intrinsic momentum tensor, we shall have here

$$K\boldsymbol{\sigma} \to k_0\rho_3\boldsymbol{\sigma} + \rho_1\mathbf{k}, \qquad K\rho_1 \to \boldsymbol{\sigma}\mathbf{k},$$
$$K\rho_3\boldsymbol{\sigma} \to k_0\boldsymbol{\sigma} - \rho_2\,[\mathbf{k}\times\boldsymbol{\sigma}], \qquad -iK\rho_2\boldsymbol{\sigma} \to \rho_3\,[\boldsymbol{\sigma}\times\mathbf{k}] \qquad (6a)$$

respectively. From the latter matrices we can form the matrix

$$\boldsymbol{\sigma}^0 = \mathbf{k}^0(\boldsymbol{\sigma}\mathbf{k}^0) + \rho_3(\boldsymbol{\sigma} - \mathbf{k}^0(\boldsymbol{\sigma}\mathbf{k}^0)) \qquad (\mathbf{k}^0 = \mathbf{k}/k), \quad (6b)$$

which also commutes with the Hamiltonian. The eigenvalues of the components of this matrix will be equal to the components of the unit three dimensional spin vector $\mathbf{s}^0$. The transformation laws in the Lorentz rotations for the three dimensional $\mathbf{s}^0$ can be found from the corresponding transformation rules for the four dimensional ξ_μ or $\xi_{\mu\nu}$ (see references 4 and 5). They have a somewhat peculiar form:

$$s_3'^0 = s_3^0\cos\gamma + s_1^0\sin\gamma, \qquad s_1'^0 = s_1^0\cos\gamma - s_3^0\sin\gamma,$$
$$s_2'^0 = s_2^0, \qquad (7)$$

where

$$\cos\gamma = (\beta_1 - \beta\cos\theta)\,[(\beta_1 - \beta\cos\theta)^2 + \beta^2(1-\beta_1^2)\sin^2\theta]^{-1/2},$$
$$\sin\gamma = (\beta\sqrt{1-\beta_1^2}\sin\theta)\,[(\beta_1 - \beta\cos\theta)^2 + \beta^2(1-\beta_1^2)\sin^2\theta]^{-1/2},$$

β is the velocity of motion of the primed system of coordinates relative to the unprimed, which we direct along the z axis. The x axis is chosen in such a fashion that the velocity of motion of the particle $\beta_1 = k/K$ relative to the unprimed system lies in the zx plane.

It follows from these equations that interchanges of the longitudinal and transverse components of the polarization will take place for particles with nonzero rest mass ($k_0 \neq 0$) in the transition from one inertial frame to another. Only in the case $\theta = 0$, i.e., when the direction of the velocity of motion β_1 of the particle coincides with the velocity of motion β of the primed system will the longitudinal components remain longitudinal and the transverse components transverse. Conversely, for $\beta = \beta_1/\cos\theta$, the longitudinal components as a whole transform into the transverse and vice versa, as a result of the Lorentz transformations, if $\mathbf{s}_2^0 = 0$.

For $k_0 \neq 0$, one can characterize the polarization both by a four dimensional spin pseudovector ζ_μ and by the intrinsic momentum tensor $\zeta_{\mu\nu}$, or by the unit three dimensional vector $\mathbf{s}^0$. All these quantities will be mutually connected by the relations (4) and (5). If the mass of the fermion is equal to zero ($k_0 = 0$) then $\beta_1 = 1$ in all systems of coordinates, and the angle $\gamma = 0$. Hence both the longitudinal and the transverse components of the polarization preserve their invariance separately ($s_3'^0 = s_3^0$; $s_{1,2}'^0 = s_{1,2}^0$).

For $k_0 = 0$, the longitudinally polarized fermions (for example, the neutrino) must be described by a spin pseudovector ($\zeta_3 = ks_3^0$, $\zeta_{1,2} = 0$), while the transversely polarized, by an intrinsic magnetic moment tensor ($\mu_{1,2} = ks_{1,2}^0$, $\mu_3 = 0$).

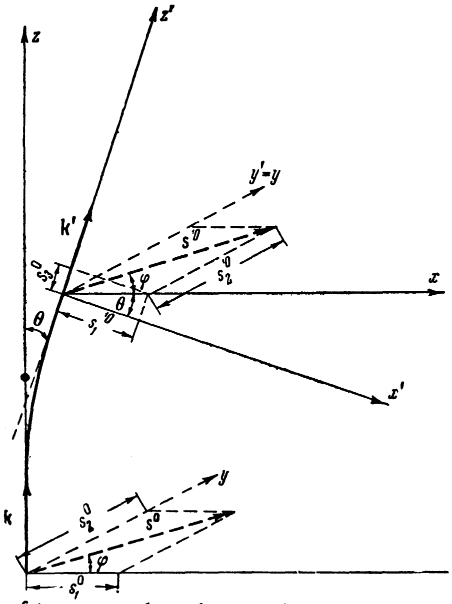
2. FUNDAMENTAL FORMULAS FOR SPIN REVERSAL IN ELASTIC SCATTERING

In the investigation of the behavior of the polarization of a fermion in elastic scattering ($K' = K$, and the primes denote quantities referring to the scattered state), it is first necessary to compute the values of the coefficients $C'_{s'}$ which will be connected with the initial coefficients C_s by the relation

$$C'_{s'} = \frac{1}{2}\sum_s \bar{\rho}_\mu (s', s)\, \bar{\sigma}_\nu (s', s)\, C_s, \tag{8}$$

where the values for $\bar{\rho}_\mu(s', s)$, $\bar{\sigma}_\nu(s', s)$ are given by Eqs. (12) — (14) of reference 6, in which the scattering of longitudinally polarized fermions ($C_1 = 1$, $C_{-1} = 0$) was considered. Here it is best to characterize the polarization by a spin pseudovector.

In the present work, we want to investigate the scattering of transversely polarized fermions; we shall therefore describe the polarization properties by an intrinsic magnetic-moment tensor.



Scattering of transversely polarized fermions by a center of force.

Let the fermion move along the z axis before scattering, and let its spin be characterized by the vector $\boldsymbol{\mu}$ directed perpendicularly to the motion, making angle φ with the x axis. In this case, we must set $C_1 = 1/\sqrt{2}$, $C_{-1} = e^{i\varphi}/\sqrt{2}$ inasmuch as we have here

$$s_0 = 1, \qquad s_1^0 = \cos\varphi, \qquad s_2^0 = \sin\varphi, \qquad s_3^0 = 0,$$

as is seen from Eqs (2) and (3). Without affecting the generality of the discussion, we can choose the zx plane so that the momentum of the scattered fermion lies in this plane (see drawing).

We shall find general formulas that characterize the polarization after scattering as a function of the character of the interaction.

a) First we consider the case of interactions proportional to the matrices ρ_1 and ρ_2 (see reference 6), i.e., $A^t = \rho_1\sigma_4$ (temporal component of the pseudovector interaction), $P = \rho_2\sigma_4$ (pseudoscalar interaction), $V^s = \rho_1 \boldsymbol{\sigma}\cdot\mathbf{n}$ (spatial component of the vector interaction) and $T^t = \rho_2 \boldsymbol{\sigma}\cdot\mathbf{n}$ (space-time component of the tensor interaction). The three dimensional vector $\mathbf{n}$ defines some direction which is connected with the scattering center and which is specified by the directions of the magnetic moment and the momenta of the motion.

In all these cases it is easy to prove that the longitudinal component of the polarization (i.e., the projection of the spin of the scattered fermion on the z′ axis, which we direct along the momentum $\mathbf{k}$) vanishes ($s_3'^0 = 0$). In other words, in these interactions, the longitudinally polarized fermion remains longitudinal, as was shown in reference 6, while the transversely polarized fermion remains transverse.

To determine the rotation of the vector of the transverse polarization of the scattered fermion, we have the following expressions from (2) and (8):

$$s_1^{\prime 0} = R_1 \cos\varphi + Q_1 \sin\varphi, \qquad s_2^{\prime 0} = R_2 \sin\varphi + Q_2 \cos\varphi, \quad (9)$$

$$Q_{1,2}^{A,P} = 0, \qquad R_1^{A,P} = \mp 1, \qquad R_2^{A,P} = -1. \quad (10)$$

In Eq. (10), the minus sign refers to R_1^A and the plus to R_1^P, and so forth. The corresponding value for the quantity s_0', which characterizes the total (for finite spin s') effective cross section, will be equal to

$$s_0^{\prime A,P} = (k^2/2K^2)(1 \pm \cos\theta). \quad (11)$$

In a similar way we find

$$s_0^{\prime V} = \frac{k^2}{K^2}(a^2+b^2), \qquad R_{1,2}^V = \frac{a^2-b^2}{a^2+b^2}, \qquad Q_{1,2}^V = \pm\frac{2ab}{a^2+b^2},$$

$$s_0^{\prime T} = \frac{k^2}{K^2}(a'^2+b'^2), \qquad R_{1,2}^T = \mp\frac{a'^2-b'^2}{a'^2+b'^2},$$

$$Q_{1,2}^T = \frac{2a'b'}{a'^2+b'^2}; \quad (12)$$

$$\left.\begin{matrix} a \\ a' \end{matrix}\right\} = n_1\sqrt{\frac{1\mp\cos\theta}{2}} \pm n_3\sqrt{\frac{1\pm\cos\theta}{2}},$$

$$\left.\begin{matrix} b \\ b' \end{matrix}\right\} = \pm n_2\sqrt{\frac{1\mp\cos\theta}{2}}, \quad (13)$$

where θ is the scattering angle.

b) In the case of interactions proportional to the matrices ρ_4 and ρ_3:

$$V^{\mathrm{sp}} = \rho_4\sigma_4, \qquad S = \rho_3\sigma_4, \qquad T^{\mathrm{sp}} = \rho_3(\sigma\mathbf{n}), \qquad A^{\mathrm{sp}} = \rho_4(\sigma\mathbf{n}) \quad (14)$$

we find that as a result of the scattering, in addition to the rotation of the transverse component of the polarization vector, an additional longitudinal component must appear (if the mass of the fermion $k_0 \neq 0$). In this case we have

$$s_0^{\prime V,S} = \tfrac{1}{2}(1+k_0^2/K^2) \pm \tfrac{1}{2}(1-k_0^2/K^2)\cos\theta,$$

$$Q_{1,2}^{V,S} = 0, \quad R^{V,S} = 1, \quad R_1^{V,S} = \frac{(K^2+k_0^2)\cos\theta \pm (K^2-k_0^2)}{(K^2+k_0^2)\pm(K^2-k_0^2)\cos\theta}, \quad (15)$$

$$s_0^{\prime A,T} = \bar{a}'^2 + \bar{b}'^2 + \bar{a}^2 + \bar{b}^2,$$

$$R_{1,2}^{A,T} = \frac{(\bar{b}^2-\bar{a}^2)\mp(\bar{b}'^2-\bar{a}'^2)}{\bar{a}^2+\bar{b}^2+\bar{b}'^2+\bar{a}'^2}, \quad Q_{1,2}^{A,T} = -\frac{2\bar{a}'\bar{b}'\mp 2\bar{a}\bar{b}}{\bar{a}^2+\bar{b}^2+\bar{a}'^2+\bar{b}'^2}. \quad (15a)^*$$

In Eqs. (15) and (15a), one must set $\bar{a}' = k_0a'/K$, $\bar{b}' = k_0b'/K$, $\bar{a} = a$, $\bar{b} = b$ in the case of the A interaction, and $\bar{a}' = a$, $\bar{b}' = b$, $\bar{a} = k_0a/K$, $\bar{b} = k_0b/K$ for the T interaction.

*The different signs on the right side of Eq. (15a) refer to the indices 1 and 2, respectively.

As we have already noted for the interactions (14), longitudinal polarization can appear as a result of the scattering; this polarization should be characterized by the components

$$s_3^{\prime 0} = R_3\cos\varphi + Q_3\sin\varphi; \quad (16)$$

$$R_3^{V,S} = \frac{2k_0K\sin\theta}{(K^2+k_0^2)\pm(K^2-k_0^2)\cos\theta},$$

$$R_3^{A,T} = \frac{2(\bar{a}\bar{a}'+\bar{b}\bar{b}')}{\bar{a}'^2+\bar{b}'^2+\bar{a}^2+\bar{b}^2},$$

$$Q_3^{V,S} = 0, \qquad Q_3^{A,T} = \frac{2(\bar{a}'\bar{b}-\bar{a}\bar{b}')}{\bar{a}'^2+\bar{b}'^2+\bar{a}^2+\bar{b}^2}; \quad (17)$$

the values of the coefficients $\bar{a}$, $\bar{b}$, etc are the same as in Eqs. (15) and (15a).

It is seen from (16) and (17) that the longitudinal polarization for fermions with zero rest mass ($k_0 = 0$) should not appear in the scattering of transversely polarized fermions.

3. POLARIZATION EFFECTS IN SCALAR AND VECTOR INTERACTIONS

We shall investigate in more detail the polarization properties in certain special cases. First we note that these polarization properties in elastic scattering (in the first Born approximation) depend only on the initial polarization, the scattering angle and the character of the interaction, inasmuch as the quantities s_0' and s'^0 do not depend on the matrix element of the energy of interaction

$$u_{\mathbf{k}'\mathbf{k}} = \int e^{i\varkappa\mathbf{r}} u(r)\, d^3x, \quad (18)$$

(see reference 6), where $\varkappa = \mathbf{k} - \mathbf{k}'$, and $u(r)$ is the spatial part of the interaction energy.

For example, as is seen from Eq. (15), the component of the unit vector $\mathbf{s}'^0$ along the axis $y' = y$ for V^t or S interactions remains unchanged ($s_2'^0 = s_2^0$), while the spin vector in the ($x'z'$) plane, located in a system of coordinates associated with the motion of the fermion,* is turned through some angle. As a result, a longitudinal component appears, as is seen from Eq. (17).

In the nonrelativistic case ($k_0 = K$), it follows from Eqs. (15) and (17) that, both for V^t and for S interactions, the same result holds:

$$s_3^{\prime 0} = \sin\theta\cos\varphi,$$

$$s_1^{\prime 0} = \cos\theta\cos\varphi, \qquad s_2^{\prime 0} = \sin\varphi, \quad (19)$$

*We shall denote by $x'y'z'$ the auxiliary set of coordinates connected with the motion of the fermion. Before scattering of the fermion, the $x'y'z'$ system coincides with the xyz system. After scattering, only the y' axis coincides with the y axis. The x', z' axes are rotated, relative to the x, z axes, respectively, through the scattering angle θ (see drawing).

i.e., just as in the scattering of longitudinally polarized particles (see reference 6), the spin vector of the scattered fermion maintains the direction of the original spin: $\mathbf{s}'^0 = \mathbf{s}^0$. This specific case is also shown in the drawing.

On the other hand, in the ultrarelativistic case ($k_0 \ll K$), the longitudinal component of the polarization will gradually vanish with increase in energy, and the fermion will again become transversely polarized. In this case the spin in V^t interaction maintains its orientation relative to the x′ axis ($s_1'^0 = s_1^0 = \cos\varphi$). In the case of S interaction, it changes this orientation to the opposite ($s_1'^0 = -s_1^0 = -\cos\varphi$).

We note that in the general case, in connection with the appearance of longitudinal polarization as the result of the scattering, the intrinsic magnetic moment is rotated in the z′x′ plane through an angle

$$\tan\alpha' = \mu_3'/\mu_1' = 2k_0^2\sin\theta \,/\, [(K^2+k_0^2)\cos\theta \pm (K^2-k_0^2)]. \qquad (20)$$

The plus sign here refers to V^t interaction, and the minus to S interaction.

It is interesting to note that for these same interactions an analogous formula was obtained in reference 6 for the angle of rotation in the system connected with the motion of the particle. This was done for the case of longitudinally polarized fermions. However, this rotation did not refer to the intrinsic magnetic moment, directed in our case perpendicular to the momentum, but to the spin pseudovector of the fermion, which was directed parallel to the momentum.

As another example, we consider the scattering of a charged fermion by a fixed magnetic moment μ'. In this case, we can represent the interaction energy in the form

$$U = -ie\rho_1(\sigma\mathbf{A}),$$

where $\mathbf{A}$ is the vector potential created by the magnetic moment. The matrix element will be proportional to the following expression (see reference 6):

$$V^t = -ie\rho_1\sigma\mathbf{n}, \quad \text{and} \quad \mathbf{n} = [\varkappa\cdot\mu'], \quad \varkappa = \mathbf{k}-\mathbf{k}'. \qquad (21)$$

As a result of scattering, the transversely polarized fermion should remain transversely polarized (inasmuch as this interaction is proportional to the matrix ρ_1). However, in the plane perpendicular to the direction of the momentum of the scattered fermion (i.e., in the x′y′ plane), the intrinsic magnetic moment ought to rotate through some angle which depends on the direction of the magnetic moment μ'.

If the magnetic moment μ' is directed along the momentum $\mathbf{k}$ of the incident fermion ($\mu_y' = \mu_x' = 0$), we shall have, in the system connected with the motion of the fermion:

$$s_1'^0 = -\cos\varphi, \qquad s_2'^0 = -\sin\varphi, \qquad s_3'^0 = 0. \qquad (22)$$

If the magnetic moment μ' is perpendicular to the direction $\mathbf{k}$ ($\mu_z' = 0$), we get

$$s_{1,2}'^0 = \frac{\mu_y'^2 - \mu_x'^2\sin^4(\theta/2)}{\mu_y'^2 + \mu_x'^2\sin^4(\theta/2)}\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix} + \frac{2\mu_x'\mu_y'\sin^2(\theta/2)}{\mu_y'^2+\mu_x'^2\sin^4(\theta/2)}\begin{pmatrix}-\sin\varphi\\ \cos\varphi\end{pmatrix}, \qquad (23)$$

i.e., if $\mu_y' = 0$ and $\mu_x' \neq 0$, we again obtain the expressions (22) for the unit vector, and for $\mu_x' = 0$ and $\mu_y' \neq 0$ we have

$$s_1'^0 = \cos\varphi, \qquad s_2'^0 = \sin\varphi, \qquad s_3'^0 = 0. \qquad (24)$$

Hence it is seen that if the magnetic moment μ' is perpendicular to the momentum of the incident and scattered mesons (i.e., perpendicular to the plane $\mathbf{k}'\cdot\mathbf{k}$), then $\mu_x' = \mu_z' = 0$ and, therefore, in accord with (24), the polarization vector maintains its direction in the system of coordinates connected with the motion of the particle. Now, if μ' lies in the plane $\mathbf{k}'\cdot\mathbf{k}$, then the polarization vector changes its direction to the opposite. We recall that before scattering the components of the spin were equal to $s_1^0 = \cos\varphi$, $s_2^0 = \sin\varphi$, $s_3^0 = 0$.

4. AZIMUTHAL ASYMMETRY IN THE SCATTERING OF POLARIZED FERMIONS

In our previous work,[6] we considered the scattering of longitudinally polarized fermions, and found that the differential effective cross section was the same as in the scattering of unpolarized fermions. This result is completely understandable, since no asymmetry should be noted for longitudinal fermions relative to the azimuthal scattering angle φ, and therefore, neither in the case of a linear combination of different interactions, nor in the case of a more exact solution of this problem (for example, by the damping method) should an azimuthal asymmetry connected with longitudinal polarization be obtained.

It should be emphasized that in the scattering of longitudinal fermions by the magnetic moment μ' directed perpendicularly to the momentum $\mathbf{k}$ ($\mu_x' \neq 0$, $\mu_y' \neq 0$), azimuthal asymmetry can be observed. However, this asymmetry characterizes only the azimuthal asymmetry connected with the presence of a transverse component of the scattering magnetic moment μ', and is completely independent of the initial degree of longitudinal polarization of the scattered fermions.

In the scattering of transversely polarized fermions, we can expect the phenomenon of azimuthal asymmetry. It is true that in carrying out calculations in the Born approximation only for the presence of one form of interaction, we found that the quantity s'_0 does not depend on the angle φ (see Sec. 2). However, this dependence can appear a) in the case of scattering in the presence of a linear combination of separate interactions, or b) in the presence of a single form of interaction but for a more exact solution of the problem, for example, by the method of damping.

We shall consider each of these cases in more detail.

a) Combination of separate interactions can take place, for example, in the investigation of the scattering of a particle possessing an electric charge e and a magnetic moment $\boldsymbol{\mu} = \mu\boldsymbol{\sigma}$ by a fixed center which has an electric charge e′ and a magnetic moment μ'.

In this case the matrix element characterizing the spin part of the interaction takes the form

$$C'_{s'} = \frac{1}{2}\sum_{s=-1,1}(ee'\bar{\rho}_4(s',s)\bar{\sigma}_4(s',s) - ie'\mu\bar{\rho}_2(s',s)\bar{\boldsymbol{\sigma}}(s',s)\boldsymbol{\varkappa}$$
$$+ ie\bar{\rho}_1(s',s)\bar{\boldsymbol{\sigma}}(s',s)[\boldsymbol{\varkappa}\times\boldsymbol{\mu}'] + \mu\bar{\rho}_3(s',s)\bar{\boldsymbol{\sigma}}(s',s)[\boldsymbol{\varkappa}\times[\boldsymbol{\varkappa}\times\boldsymbol{\mu}']])C_s,$$

where $\boldsymbol{\kappa} = \mathbf{k} - \mathbf{k}'$.

In order to avoid the effects of azimuthal asymmetry connected with the presence of a transverse component of the magnetic moment μ' of the scattering center, we direct this moment along the z axis ($\mu'_z = \mu'$, $\mu'_x = \mu'_y = 0$). Here we find for the coefficients $C'_{s'}$ [by means of Eq. (8)]:

$$C'_{s'} = \sum_s\left\{\frac{1}{2}(1+ss')(p-sq) - \frac{1}{2}s(1-ss')m\right\}C_s; \quad (25)$$

$$p = ee'\cos\frac{\theta}{2}, \qquad q = 2\mu'(e+2\mu k_0)\frac{k^2}{K^2}\sin^2\frac{\theta}{2}\cos\frac{\theta}{2},$$
$$m = e'(ek_0 - 2\mu k^2)\frac{1}{K}\sin\frac{\theta}{2}. \quad (26)$$

If we assume for transversely polarized fermions $C_1 = 1/\sqrt{2}$, $C_{-1} = e^{i\varphi}/\sqrt{2}$, we obtain the following expression for the quantity s'_0 which characterizes the effective cross section:*

$$s'_0 = (p^2+q^2+m^2)(1-\delta\cos\varphi), \quad (27)$$

where the asymmetry coefficient δ is equal to

$$\delta = 2qm/(p^2+q^2+m^2). \quad (28)$$

*We recall that the connection of s'_0 with the differential effective cross section is determined by Eq. (15) of reference 6.

It is evident from the latter formula that the azimuthal asymmetry must hold in the case in which the scattering center also possesses an electric charge ($e' \neq 0$) and a magnetic moment ($\mu' \neq 0$) (see also reference 7, where the azimuthal asymmetry was investigated for double scattering).

b) Exactly the same azimuthal asymmetry can be obtained in the scattering of transverse fermions in the presence of one form of interaction but with allowance for damping. For example, in the simplest case of the scattering of a fermion of charge e by a fixed charge e′ (V^t interaction) with account of damping, we find (see references 8 and 9):

$$s''_0 = \frac{1}{2}K^{-2}[(K^2+k_0^2) + \cos\theta\,(K^2-k_0^2)](1+\delta\sin\varphi), \quad (29)$$

where the asymmetry coefficient is

$$\delta = 2\alpha'\sin\theta\,Kk_0/[(K^2+k_0^2)+\cos\theta'(K^2-k_0^2)],$$
$$\alpha' = \frac{k^3}{16\pi^2 c\hbar KU_{\mathbf{k}'\mathbf{k}}}\oint d\Omega'' u_{\mathbf{k}'\mathbf{k}''}u_{\mathbf{k}''\mathbf{k}}\frac{(\mathbf{k}-\mathbf{k}'')(\mathbf{k}+\mathbf{k}')}{\mathbf{k}(\mathbf{k}+\mathbf{k}')}.$$

Here $d\Omega''$ is the solid angle of the vector $\mathbf{k}''$.

In a similar way one can easily determine the azimuthal asymmetry in the presence of other interactions, solving the problem by the method of damping.

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