TRANSITION RADIATION FOR A CHARGED PARTICLE AT OBLIQUE INCIDENCE

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The transition radiation emitted in the forward direction by a charge in oblique incidence at the boundary between two media is considered. It is shown that the intensity of the radiation is essentially independent of the angle of incidence of the particles so long as this angle is far from 90°.

THE total intensity of the transition radiation emitted by a relativistic particle in the forward direction in passing through a boundary between two media is proportional to the particle energy.[1] It is of interest to investigate the intensity of the transition radiation as a function of the entrance angle of the particle into the medium. The problem of a charge moving into a medium at oblique incidence has been solved earlier in general form[2] (cf. also reference 3). In the present note the energy losses due to the forward radiation of an extreme relativistic particle which moves into a medium at oblique incidence are calculated by the Landau method[4] and by a direct calculation of the energy flux.

Let the plane z = 0 be the boundary separating two media characterized by dielectric constants ϵ_1 and ϵ_2. The particle moves with velocity **v** in the yz plane. As has been shown in reference 2, the Fourier component of the electric vector of the radiation field in the second medium is given by the formula

$$\mathbf{E}_2'(\mathbf{k}) = \boldsymbol{\varkappa} e_{2\varkappa} + \mathbf{j} e_{2y} - \mathbf{n}(\varkappa^2 e_{2\varkappa} + k_y e_{2y})/\lambda_2,$$

$$e_{2\varkappa} = \frac{ei}{2\pi^2}\frac{\lambda_1\lambda_2}{\varepsilon_1\lambda_2 + \varepsilon_2\lambda_1}\left\{\frac{-1/\lambda_1 - v_z/\omega}{k^2 - \omega^2\varepsilon_1/c^2} + \frac{\varepsilon_1/\varepsilon_2\lambda_1 + v_z/\omega}{k^2 - \omega^2\varepsilon_2/c^2} - \frac{k_y v_y (k_z - \lambda_2 + \lambda_1)}{\omega\lambda_1\lambda_2}\left(\frac{1}{k^2 - \omega^2\varepsilon_1/c^2} - \frac{1}{k^2 - \omega^2\varepsilon_2/c^2}\right)\right\}, \quad (1)$$

$$e_{2y} = \frac{ei}{2\pi^2}\frac{v_y}{\lambda_1 + \lambda_2}\frac{\omega(\lambda_1 + k_z)}{c^2}\left(\frac{1}{k^2 - \omega^2\varepsilon_1/c^2} - \frac{1}{k^2 - \omega^2\varepsilon_2/c^2}\right). \quad (2)$$

Here $\omega = \mathbf{k}\cdot\mathbf{v} = k_y v_y + k_z v_z$; **i**, **j** and **n** are unit vectors along the x, y and z axes; κ is the component of the wave vector **k** in the xy plane; $\lambda_{1,2}^2 = \omega^2\epsilon_{1,2}/c^2 - \kappa^2$, where the real and imaginary parts of λ_1 and λ_2 are taken as positive. We also introduce the angle of incidence of the particle at the boundary between the media φ; $v_z = v\cos\varphi$ and $v_y = v\sin\varphi$.

In carrying out the calculation it is convenient to rotate the coordinate axes about the x direction through an angle φ such that the new z′ axis coincides with the particle trajectory. Then, denoting all quantities in the new coordinate system by primes we have

$$E_{2z'}'(\mathbf{k}') = [k_y\sin\varphi - (\varkappa^2/\lambda_2)\cos\varphi]\, e_{2\varkappa} + [\sin\varphi - (k_y/\lambda_2)\cos\varphi]\, e_{2y},$$

$$\mathbf{E}_{2t'}'(\mathbf{k}') = \mathbf{i}'k_x' e_{2\varkappa} + \mathbf{j}'\{[k_y\cos\varphi + (\varkappa^2/\lambda_2)\sin\varphi]\, e_{2\varkappa} + [\cos\varphi + (k_y/\lambda_2)\sin\varphi]\, e_{2y}\},$$

$$\mathbf{H}_2'(\mathbf{k}') = c\omega^{-1}[\boldsymbol{\varkappa} + (\mathbf{n}'\cos\varphi - \mathbf{j}'\sin\varphi)\lambda_2] \times \mathbf{E}_2'(\mathbf{k}')] \quad (3)$$

where $\mathbf{E}_{2t'}'(\mathbf{k}')$ is the projection of the Fourier component of the radiation field on the plane perpendicular to the direction of motion of the particle. We must keep in mind the fact that

$$\omega = k_z' v, \qquad k_y = \omega v^{-1}\sin\varphi + k_y'\cos\varphi,$$

$$k_z = \omega v^{-1}\cos\varphi - k_y'\sin\varphi, \quad \boldsymbol{\varkappa} = \mathbf{i}'k_x' + (\mathbf{j}'\cos\varphi + \mathbf{n}'\sin\varphi)k_y.$$

Using Eq. (3) we can compute the force due to the radiation field which acts on the particle (cf. reference 4)

$$F_2(\varphi) = ev\int_0^\infty dt\int E_{2z'}'(\mathbf{k}') \times \exp\left\{ivt\left(\lambda_2 + k_y'\sin\varphi - \frac{\omega}{v}\cos\varphi\right)\cos\varphi\right\} d\mathbf{k}', \quad (4)$$

as well as the Poynting vector in the second medium at a plane z′ = const (cf. reference 1):

$$S_{2z'}(\varphi) = \frac{c}{4\pi}\int_{-\infty}^{+\infty}[\mathbf{E}_2' \times \mathbf{H}_2']_{z'}\, dt dx' dy'. \quad (5)$$

We now sketch the calculation for the second case. Since $k^2 - \omega^2\epsilon_{1,2}/c^2 = k'^2 - \omega^2\epsilon_{1,2}/c^2$, the denominators of the total-radiation expressions contain factors characteristic of normal incidence of the particle on the separation boundary; for an extreme relativistic particle these factors lead to the emission of transition radiation at very small angles to the direction of motion of the particle.

Furthermore, since

$$\lambda_{1,2} = \left[\frac{\omega^2}{v^2}\left(\cos^2\varphi - (1-\beta^2) - \frac{\sigma_{1,2}}{\omega^2}\right) - 2\frac{\omega}{v}k_y'\sin\varphi\cos\varphi - \varkappa'^2 + k_y'^2\sin^2\varphi\right]^{1/2},$$

if

$$\cos\varphi > \sqrt{\sigma_{1,2}/\omega^2}, \qquad \cos\varphi > \sqrt{1-\beta^2} \tag{6}$$

(σ_1 and σ_2 are the plasma frequencies of the media) we find that

$$\mathbf{E}_{2t'}'(\mathbf{k}') = \frac{ei}{2\pi^2}\varkappa'\left(\frac{1}{k'^2 - \varepsilon_2\omega^2/c^2} - \frac{1}{k'^2 - \varepsilon_1\omega^2/c^2}\right). \tag{7}$$

In the present case the magnetic field is

$$\mathbf{H}_2'(\mathbf{k}') = \beta^{-1}[\mathbf{n}'\times\mathbf{E}_{2t'}(\mathbf{k}')],$$

so that the further calculations are very simple and for $\omega > \sqrt{\sigma_{1,2}}$ we obtain formulas which coincide with the formulas for the case of perpendicular incidence.

If the condition in (6) is not satisfied the quantities $\lambda_{1,2}$ become purely imaginary and since the expressions for the radiation fields contain the factor $\exp(i\lambda_2 z)$ these fields fall off exponentially with z.

The condition in (6) can be understood on the basis of the following qualitative considerations. As is well known, transition radiation is formed in some region of space around the trajectory of the particle in the first and second media. For oblique incidence of the particle it is evident that the transition radiation will be generated (just as in the case of perpendicular incidence) if the formation zone satisfies the same conditions. For example, in the second medium the formation zone for radiation of frequency ω is a cone with opening angle (relative to the particle trajectory) given approximately by $\sqrt{\sigma_2}/\omega$ (for frequencies $\omega < \sqrt{\sigma_2}/\sqrt{1-\beta^2}$). Hence it is clear that it is only when $\cos\varphi < \sqrt{\sigma_2}/\omega$ that the formation zone in the second medium is affected, because part of it extends into the first medium.

Thus, as the angle of incidence of the particle increases the softer photons tend to be suppressed: with further increases in φ the suppression extends to harder and harder photons. This effect can be used for generating transition radiation in a narrow frequency range since the long-wave portion of the radiation spectrum can be suppressed by rotation of the medium. The total number of emitted photons remains almost the same because the lower frequency limit appears in the formula logarithmically [cf. reference 1, Eq. (14)].

Results similar to these have been obtained by Korkhmazyan,[5] who used the method of images.

[1] G. M. Garibyan, JETP **37**, 527 (1959), Soviet Phys. JETP **10**, 372 (1960).

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[3] N. A. Korkhmazyan, ibid., **11**, No. 6 (1958).

[4] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред, (Electrodynamics of Continuous Media) Gostekhizdat, 1957.

[5] N. A. Korkhmazyan, Izv. AN Arm. S.S.R., Ser. Fiz.-Mat. Nauk (in press).

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