APPLICATION OF THE DISPERSION METHOD FOR INVESTIGATION OF THE SIMPLEST GREEN'S FUNCTIONS IN QUANTUM ELECTRODYNAMICS

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The dispersion relation technique is used to study the electron propagation function $G(\hat{p})$ and the current matrix element $\langle 0|\eta(0)|p', k\rangle$ as functions of $p^2 = (p' + k)^2$. It is shown that in the two-particle approximation (no more than two particles in the intermediate state) the analytical properties of the matrix element under study (with respect to the variable p^2) lead to a Hilbert boundary problem, which is then solved. Explicit expressions for the electron spectral propagation functions are derived in the given approximation, and the asymptotic behavior of $G(\hat{p})$ and $\Gamma_\mu(p, p'; k)$ is determined for $-p^2 \to \infty$, $p'^2 = -m^2$ and $k^2 = 0$.

1. INTRODUCTION

AT the present time it is possible to develop a new approach to the study of the problems of quantum field theory. This method is connected with the use of the dispersion relations for different Green's functions (or matrix elements) of the interacting fields. In such an approach, an infinite set of interlocking integral equations is generated; these connect all possible matrix elements on the energy surface, and definite boundary conditions are necessary for the solution of this set.

In quantum electrodynamics, and especially in quantum field theory, the dispersion technique has already furnished a number of interesting results; in these cases, correspondence with perturbation theory is chosen as a reasonable boundary condition. Thus, for example, in the work of Faĭnberg,[1] the behavior of the photon propagation function $D_{\mu\nu}(k)$ and the matrix element of the current $\langle 0|j_\mu(0)|p, p'\rangle$ were studied as functions of the square of the photon momentum $k^2 = (p + p')^2$ for $p^2 = p'^2 = -m^2$. In addition, a number of other problems was also considered. In this connection, the study of similar quantities as functions of the square of the electron momentum p^2 is of interest. This is the object of the present research.*

By definition,

$$G_{\alpha\beta}(x-y) = \langle 0|T\psi_\alpha(x)\overline{\psi}_\beta(y)|0\rangle = \frac{1}{(2\pi)^4}\int G_{\alpha\beta}(p)\, e^{ip(x-y)}\, dp, \tag{1.1}$$

where $\psi(x)$ is the renormalized Heisenberg operator which satisfies the equation

$$(\hat{\nabla}_x + m)\psi(x) = \eta(x), \qquad \hat{\nabla}_x = \gamma_\mu \partial/\partial x_\mu,$$
$$\gamma_\mu = \gamma_\mu^+, \qquad xy = \mathbf{xy} + x_4 y_4, \qquad x_4 = ix_0. \tag{1.2}$$

It is known that the Fourier transform of the electron propagation function has the spectral representation

$$G(\hat{p}) = \frac{1}{i}\left\{\frac{-i\hat{p}+m}{p^2+m^2-i\varepsilon} + \int\limits_{-\infty}^{-m^2} \frac{-i\hat{p}\rho_1(p'^2)+\rho_2(p'^2)}{p^2-p'^2-i\varepsilon}\, dp'^2\right\}; \tag{1.3}$$

$$\rho_1(p^2) = \frac{(2\pi)^3}{4(p^2+m^2)^2}\,\mathrm{Sp}\left\{\left(2m + \frac{p^2-m^2}{i\hat{p}}\right) \times \sum_n \langle 0|\eta(0)|n\rangle\langle n|\overline{\eta}(0)|0\rangle\,\delta(p-p_n)\right\},$$

$$\rho_2(p^2) = \frac{(2\pi)^3}{4(p^2+m^2)^2}\,\mathrm{Sp}\left\{\left(m^2 - p^2 + \frac{2mp^2}{i\hat{p}}\right) \times \sum_n \langle 0|\eta(0)|n\rangle\langle n|\overline{\eta}(0)|0\rangle\,\delta(p-p_n)\right\}. \tag{1.4}$$

Thus the spectral functions ρ_1 and ρ_2 are expressed bilinearly by the current matrix elements $\langle 0|\eta(0)|n\rangle$. In the present research, the behavior of the first non-vanishing matrix element* $\langle 0|\eta(0)|p', r;\ k, \lambda\rangle$ is studied by the dispersion relation technique as a function of $p^2 = (p' + k)^2$.

2. ANALYTICAL PROPERTIES OF THE FORM FACTOR

We consider the electron form factor

$$\tilde{e}_\mu^\lambda (2\omega)^{-1/2} F_\mu^{in}(p)\, u_-(p') = i\langle 0|\eta(0)|p',\ r;\ k,\ \lambda;\ in\rangle, \tag{2.1}$$

*A similar investigation into the application to mesodynamics has been made in the research of Malakhov, Rashevskaya, and Faĭnberg.[2]

*For brevity, we call the matrix element $i\langle 0|\eta(0)|p', k\rangle$ the form factor of the electron.

where $p + p' + k = 0$; $\tilde{e}^{\lambda}_{\mu} = e^{\lambda}_{\mu} - k_{\mu}k^{-2}(e^{\lambda}k)(1-\sqrt{d_l})$ are the four photon polarization vectors in arbitrary scale; $u_-(p')$ satisfies the equation $(i\hat{p}' + m)u_-(p') = 0$.

It is easy to prove that for $p'^2 = -m^2$ and $k^2 = 0$,

$$F^{in}_{\mu}(p) = ie(-i\hat{p} + m)\,G(-\hat{p})\,\Gamma_{\mu}(-p,\ p';\ k) \qquad (2.2)$$

The requirements of relativistic and gauge invariance lead to a general expression for the form factor:

$$F^{in}_{\mu}(p) = \left(\gamma_{\mu} - \frac{p'_{\mu}\hat{k}}{p'k}\right)F_1(p^2) + i\sigma_{\mu\nu}k_{\nu}F_2(p^2) + ik_{\mu}F_3(p^2) + k_{\mu}\hat{k}F_4(p^2) + e\frac{p'_{\mu}\hat{k}}{p'k}, \qquad (2.3)$$

where $\sigma_{\mu\nu} = \frac{1}{2}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})$. In accord with (2.2),*

$$k_{\mu}F^{in}_{\mu}(p)\,u_-(p') = e\hat{k}u_-(p'). \qquad (2.4)$$

We now investigate the analytical properties of $\langle 0|\eta(0)|p', k\rangle$ in the variable p^2. Making use of the reducing formula of reference 4, we transform to the relation

$$\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F^{in}_{\mu}(p)\,u_-(p') = \int dx\,e^{-ipx}f^{in}_{\lambda}(x, k)\,u_-(p'), \qquad (2.5)$$

where

$$f^{in}_{\lambda}(x, k) = -e^{-ikx}\{\theta(-x_0) \times\langle 0|[\eta(0),\ \bar{\psi}(x)]_+|k,\ \lambda\rangle\}(-\hat{\nabla}_x + m) = 0$$

for $x_0 > 0$, $x^2 > 0$.

By virtue of the fact that $k^2 = 0$ in the system of coordinates $\mathbf{p}' = 0$, we have $|\mathbf{p}| = -p_0 - m$. Therefore, for $p_0 \to p_0 + i\Gamma$, the factor $\exp\{\Gamma|x_0 + \mathbf{e}\cdot\mathbf{x}|\}$ appears under the integral in (2.5), thus guaranteeing the regularity of the desired function in the lower half plane of the complex variable p_0 or, what is the same thing, the regularity of p^2. (The indeterminacy at the cone $x^2 = 0$ is removed as usual by the addition of the factor $\exp\{-\epsilon\mathbf{x}^2\}$ $\epsilon \to +0$).

Similar to the above, the quantity

$$\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F^{out}_{\mu}(p)\,u_-(p') = i\langle 0|\eta(0)|p',\ r;\ k,\ \lambda;\ out\rangle$$
$$= \tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}\left\{\left(\gamma_{\mu} - \frac{p'_{\mu}\hat{k}}{p'k}\right)F_1^*(p^2) + i\sigma_{\mu\nu}k_{\nu}F_2^*(p^2) + ik_{\mu}F_3^*(p^2) + k_{\mu}\hat{k}F_4^*(p^2) + e\frac{p'_{\mu}\hat{k}}{p'k}\right\}u_-(p') \qquad (2.6)$$

can be put in the form

$$\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F^{out}_{\mu}(p)\,u_-(p') = \int dx\,e^{-ipx}f^{out}_{\lambda}(x, k)\,u_-(p'),$$

$$f^{out}_{\lambda}(x, k) = e^{-ikx}\{\theta(x_0)\,\langle 0|[\eta(0),\ \bar{\psi}(x)]_+|k,\ \lambda\rangle\}(-\hat{\nabla}_x + m) = 0 \qquad (2.5')$$

*Here we have used the relation:[3]

$$G^{-1}(-\hat{p}) - G^{-1}(\hat{p}') = -k_{\mu}\Gamma_{\mu}(-p, p'; k).$$

for $x_0 < 0$, $x^2 > 0$, and is therefore regular in the upper half plane of p^2.

In addition,

$$\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}[F^{in}_{\mu}(p) - F^{out}_{\mu}(p)]\,u_-(p') = -(2\pi)^4\sum_n\langle 0|\eta(0)|n\rangle\langle n|\bar{\eta}(0)|k,\ \lambda\rangle\,\delta(p + p_n) = 0 \qquad (2.7)$$

for $p^2 > -m^2$.

Consequently, there exists a single analytical function $\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F_{\mu}(z)u_-(p')$ which is regular through the complex z plane, with the exception of the cut along the real axis from the point $-m^2$ to $-\infty$, so that

$$\lim_{z\to p^2-i\varepsilon}\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F_{\mu}(z)\,u_-(p') = \tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F^{in}_{\mu}(p)\,u_-(p'),$$
$$\lim_{z\to p^2+i\varepsilon}\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F_{\mu}(z)\,u_-(p') = \tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}F^{out}_{\mu}(p)\,u_-(p'). \qquad (2.8)$$

3. THE HILBERT PROBLEM FOR THE FORM FACTOR

We have thus established the analytical properties of the form factor as a function of p^2. For the determination of this function, however, it is necessary to have some sort of condition on the line of the cut. Such a condition is usually obtained in the expansion of $\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}[F^{in}_{\mu}(p) - F^{out}_{\mu}(p)]\,u_-(p')$ over the complete set of states $|n\rangle$, if we break off the resultant infinite series at some term. We note that for such a cut off it is necessary to sum over the intermediate states of $|n\rangle$ in the following fashion in order to preserve the Hermitian character of $i[F_k(p^2) - F_k^*(p^2)]$:

$$\sum_n|n\rangle\langle n| = \tfrac{1}{2}\sum_n\{|n,\ in\rangle\langle n,\ in| + |n,\ out\rangle\langle n,\ out|\}.$$

Taking into account the vanishing of the current matrix elements over the vacuum and the single-particle state, and limiting ourselves to the first term in the sum over $|n\rangle$ (the two-particle approximation), we obtain

$$\tilde{e}^{\lambda}_{\mu}(2\omega)^{-1/2}[F^{in}_{\mu}(p) - F^{out}_{\mu}(p)]\,u_-(p') = \frac{1}{2(2\pi)^6} \times\int d\mathbf{p}'_1\,d\mathbf{k}_1\sum_{r_1,\lambda_1}\tilde{e}^{\lambda_1}_{\mu_1}\frac{1}{\sqrt{2\omega_1}}[F^{out}_{\mu_1}(p_1)\,u_-(p'_1)\,f_{p'_1,k_1,p',k} - F^{in}_{\mu_1}(p_1)\,u_-(p'_1)\,f^*_{p'_1,k_1,p',k}], \qquad (3.1)$$

where $f_{p'_1,k_1,p',k}$ is the Compton effect amplitude. This relation makes it possible to express the desired form factor in terms of the phase of the Compton scattering, but we shall limit ourselves, for simplicity, to the solution of the problem for the case in which the amplitude $f_{p'_1,k_1,p',k}$ is computed by perturbation theory in the Born approximation:

$$f_{p_1', k_1, p', k} = -ie^2(2\pi)^4\delta(p_1' + k_1 + p' + k)(4\omega\omega_1)^{-1/2}$$

$$\times \bar{u}_-(p_1')\Big[\hat{e}^{\lambda_1}\frac{-i(\hat{p}'+\hat{k})+m}{(p'+k)^2+m^2}\hat{e}^{\lambda}$$

$$+\hat{e}^{\lambda}\frac{-i(\hat{p}'-\hat{k}_1)+m}{(p'-k_1)^2+m^2}\hat{e}^{\lambda_1}\Big]u_-(p'). \quad (3.2)$$

Then, after a number of calculations, we obtain

$$F_l(p^2) - F_l^*(p^2) = \sum_{k=1}^{2} ig_{lk}(p^2)[F_k(p^2) + F_k^*(p^2)] + ig_{l0}(p^2)e;$$

$$g_{11}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)\Big[1 + \frac{m^2(p^4 - 4p^2m^2 - m^4)}{2p^4(p^2+m^2)}$$

$$+\frac{m^2(p^2+3m^2)}{(p^2+m^2)^2}\ln\frac{-p^2}{m^2}\Big],$$

$$g_{12}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)m^3\Big[-\frac{3p^2+m^2}{2p^4}$$

$$+\frac{1}{p^2+m^2}\ln\frac{-p^2}{m^2}\Big],$$

$$g_{10}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)$$

$$\times\frac{2m^2}{p^2+m^2}\Big[\frac{-p^2+m^2}{p^2} - \frac{2m^2}{p^2+m^2}\ln\frac{-p^2}{m^2}\Big],$$

$$g_{21}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)$$

$$\times\frac{m^3}{p^2+m^2}\Big[\frac{3p^2+m^2}{2p^4} - \frac{1}{p^2+m^2}\ln\frac{-p^2}{m^2}\Big],$$

$$g_{22}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)$$

$$\times\Big[1 + \frac{m^2(p^2+m^2)}{2p^4} + \frac{m^2}{p^2+m^2}\ln\frac{-p^2}{m^2}\Big],$$

$$g_{20}(p^2) = -\frac{e^2}{8\pi}\theta(-p^2-m^2)$$

$$\times\frac{m}{p^2+m^2}\Big[\frac{p^2-m^2}{p^2} + \frac{2m^2}{p^2+m^2}\ln\frac{-p^2}{m^2}\Big]. \quad (3.3)$$

The g_{lk} for $l = 3, 4$ are certain functions (for brevity, we shall not write them out) having the following asymptotic values for $-p^2 \to \infty$:

$$g_{31}(p^2) \to \frac{e^2 m}{8\pi p^2}, \quad g_{32}(p^2) \to \frac{e^2}{8\pi}, \quad g_{30}(p^2) \to -\frac{e^2 m}{2\pi p^2}\ln\frac{-p^2}{m^2},$$

$$g_{41}(p^2) \to -\frac{3e^2m^4}{4\pi p^6}\ln\frac{-p^2}{m^2}, \quad g_{42}(p^2) \to -\frac{e^2 m}{8\pi p^2},$$

$$g_{40}(p^2) \to -\frac{e^2}{4\pi p^2}.$$

We note that, since $g_{lk}(-m^2) = 0$, $l = 1, \ldots, 4$, we have $F_l(-m^2) - F_l^*(-m^2) = 0$ and, consequently, in contrast with the work of Faĭnberg,[1] no difficulties arise here connected with the infrared divergence.

The relations (3.3) represent the inhomogeneous Hilbert boundary problem for two unknown functions $F_1(p^2)$ and $F_2(p^2)$, which are analytic in the region described above. An exact solution of this problem meets with considerable difficulty and we shall therefore solve it by the iteration method. Assuming that $F_2(p^2)$ falls off at infinity not more slowly than a finite power of p^2, and neglecting the asymptotically small term $ig_{12}(F_2 + F_2^*)$ in (3.3), we get

$$F_1(p^2) - F_1^*(p^2) = ig_{11}(p^2)[F_1(p^2) + F_1^*(p^2)] + ig_{10}(p^2)e,$$

$$F_2(p^2) - F_2^*(p^2) = ig_{21}(p^2)[F_1(p^2) + F_1^*(p^2)]$$

$$+ ig_{22}(p^2)[F_2(p^2) + F_2^*(p^2)] + ig_{20}(p^2)e. \quad (3.3')$$

The general solution of the Hilbert problem with the boundary condition (3.3′) has the following form:[5]

$$F_l(z) = \exp[\Gamma_l(z)]\Big\{\mathcal{P}_l(z) - \frac{1}{2\pi}\int_{-\infty}^{-m^2}\frac{g_l(t)\,dt}{(t-z)\exp[\Gamma_l^+(t)]}\Big\}, \quad (3.4)$$

where the notation

$$\Gamma_l(z) = -\frac{z+m^2}{\pi}\int_{-\infty}^{-m^2}\frac{\operatorname{arc\,tg} g_{ll}(t)\,dt}{(t+m^2)(t-z)}, \quad g_1 = \frac{eg_{10}}{1+ig_{11}},$$

$$g_2 = \frac{g_{21}(F_1+F_1^*)+g_{20}e}{1+ig_{22}}$$

is used.

The solution (3.4) of the Hilbert problem was determined with accuracy up to an arbitrary polynomial $\mathcal{P}_l(z)$. Here as always in the dispersion approach, there is the problem of the boundary conditions that set off the solution uniquely. As such a condition, we require that the expansion of the imaginary part of the resultant solution in a series in the coupling constant coincide with the corresponding series of perturbation theory, which in this case does not contain any divergences. Furthermore, we get still another condition from (2.2), $F_1(-m^2) = e$.

The solution satisfying this boundary condition is

$$F_1(p^2) = e\exp[\Gamma_1^-(p^2)]\Big\{1 - \frac{p^2+m^2}{2\pi}$$

$$\times\int_{-\infty}^{-m^2}\frac{g_{10}(t)\,dt}{(t+m^2)[1+ig_{11}(t)]\exp[\Gamma_1^+(t)](t-p^2+i\varepsilon)}\Big\},$$

$$F_2(p^2) = -\frac{\exp[\Gamma_2^-(p^2)]}{2\pi}$$

$$\times\int_{-\infty}^{-m^2}\frac{g_{21}(t)[F_1(t)+F_1^*(t)] + g_{20}(t)e}{[1+ig_{22}(t)]\exp[\Gamma_2^+(t)](t-p^2+i\varepsilon)}\,dt,$$

$$\Gamma_l^{\mp}(p^2) = \lim_{z\to p^2\mp i\varepsilon}\Gamma_l(z). \quad (3.5)$$

As $-p^2 \to \infty$,

$$F_1(p^2) \to \text{const}\cdot\Big(\frac{-p^2}{m^2}\Big)^{e^2/8\pi^2}, \quad F_2(p^2) \to \frac{\text{const}}{m}\Big(\frac{-p^2}{m^2}\Big)^{-1}. \quad (3.6)$$

It is not difficult to note that further iteration does not change the asymptotic value of $F_1(p^2)$, and of $F_2(p^2)$ either. Thus, for $-p^2 \to \infty$, we get

$$F_{\mu}^{in,\,out}(p) \to \text{const} \cdot \gamma_{\mu}\left(\frac{-p^2}{m^2}\right)^{e^2/8\pi^2},$$

which differs significantly from the known result $-p = p'$, when $(i\hat{p} + m)G(\hat{p})\Gamma_{\mu}(p, p; 0) \xrightarrow[-p^2 \to \infty]{}$ const $\cdot\, \gamma_{\mu}$ (Ward's theorem).

The solution (3.5) found for the Hilbert problem is the asymptotic solution of the set of dispersion equations

$$F_1(p^2) = e + \frac{p^2 + m^2}{2\pi i}\int\limits_{-\infty}^{-m^2} \frac{F_1(t) - F_1^*(t)}{(t + m^2)(p^2 - t - i\varepsilon)}dt,$$

$$F_2(p^2) = \frac{1}{2\pi i}\int\limits_{-\infty}^{-m^2} \frac{F_2(t) - F_2^*(t)}{p^2 - t - i\varepsilon}dt \qquad (3.7)$$

with $F_l - F_l^*$ determined from (3.3).

4. THE ELECTRON PROPAGATION FUNCTION IN THE TWO-PARTICLE APPROXIMATION

The results we have obtained make it possible to compute the spectral functions ρ_1 and ρ_2 in the two-particle approximation in the following fashion:

$$\rho_1(p^2) = \frac{1}{16\pi^2}\Big\{-\frac{p^2 + m^2}{p^4}|F_1|^2 - m\frac{p^2 + m^2}{p^4}(F_1F_2^* + F_1^*F_2)$$
$$+ \frac{p^4 - m^4}{p^4}|F_2|^2 - \frac{m^2}{p^4}e(F_1 + F_1^*)$$
$$- \frac{m(p^2 + m^2)}{2p^4}e(F_2 + F_2^*) - \frac{m(3p^2 - m^2)}{2p^4}ed_l(F_3 + F_3^*)$$
$$+ \frac{p^4 - m^4}{2p^4}ed_l(F_4 + F_4^*) + \frac{e^2}{p^2 + m^2}\Big[\frac{2m^2(m^2 - p^2)}{p^4}$$
$$+ (1 - d_l)\frac{3p^4 - p^2m^2 + 2m^4}{2p^4}\Big]\Big\},$$

$$\rho_2(p^2) = \frac{1}{16\pi^2}\Big\{\frac{p^2 + m^2}{p^2}(F_1F_2^* + F_1^*F_2) + 2m\frac{p^2 + m^2}{p^2}|F_2|^2$$
$$+ \frac{m}{p^2}e(F_1 + F_1^*) + \frac{p^2 + m^2}{2p^2}e(F_2 + F_2^*)$$
$$+ \frac{p^2 - 3m^2}{2p^2}ed_l(F_3 + F_3^*) + m\frac{p^2 + m^2}{p^2}ed_l(F_4 + F_4^*)$$
$$+ \frac{me^2}{p^2 + m^2}\Big[-\frac{4m^2}{p^2} + (1 - d_l)\frac{2p^2 - m^2}{p^2}\Big]\Big\}. \qquad (4.1)$$

It is natural that the functions $F_3(p^2)$ and $F_4(p^2)$, which are connected in elementary fashion with $F_1(p^2)$ and $F_2(p^2)$, drop out of the expressions for ρ_1 and ρ_2 in the case $d_l = 0$, since, in such a case, as is seen from (2.1) and (2.3), they do not enter into the current matrix element. In calibration with $d_l = 1$, in the e^2 approximation, the relations (4.1) coincide with the results of Gell-Mann and Low.[6]

It is seen from (4.1) that $\rho_1(p^2)$ and $\rho_2(p^2)$ have a simple pole at the point $p^2 = -m^2$, as was to be expected. This corresponds to the infrared divergence in $G(p)$, but it is not important for the asymptotic case of interest to us. Asymptotically, we have for $-p^2 \to \infty$:

$$\rho_1(p^2) \sim \left(\frac{-p^2}{m^2}\right)^{-1+e^2/4\pi^2}, \qquad \rho_2(p^2) \sim m\left(\frac{-p^2}{m^2}\right)^{-1+e^2/8\pi^2}, \quad (4.2)$$

whence we find that the renormalization constant

$$Z_2^{-1} = 1 + \int\limits_{-\infty}^{-m^2} \rho_1(p^2)\,dp^2 \qquad (4.3)$$

is infinite. Nevertheless, it is not possible to confirm that this result is preserved upon consideration of higher approximations, because it can be shown* that the exact spectral functions vanish more rapidly than in the two-particle approximation.

In the given approximation, we get from (2.2), (3.6), and (4.2),

$$G(\hat{p}) \sim \frac{\hat{p}}{p^2}\left(\frac{-p^2}{m^2}\right)^{e^2/4\pi^2}, \qquad \Gamma_{\mu}(p, p') \sim \gamma_{\mu}\left(\frac{-p^2}{m^2}\right)^{-e^2/8\pi^2}. \quad (4.4)$$

Thus, in the two-particle approximation, in spite of the infinity of the renormalization constant Z_2^{-1}, a false pole does not arise in $G(\hat{p})$, by virtue of the vanishing of $\Gamma_{\mu}(p, p')$ as $-p^2 \to \infty$.[7] However, complete investigation of this problem in the dispersion method, as also of a number of other similar problems, requires consideration of higher approximations, which entail great difficulties.

In conclusion, the authors wish to express their sincere thanks to V. Ya. Faĭnberg for his constant attention to the research and for numerous discussions.

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*The complete set of states in quantum electrodynamics includes states with a negative norm. Furthermore, it must be remembered that the result (4.2) coincides with the Born approximation for $f_{p_1', k_1, p', k}$.