SOME CONSEQUENCES OF THE SYMMETRY OF THE UNIVERSAL FERMI WEAK INTERACTION

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It is shown that the invariance of the universal V — A Fermi weak interaction with respect to the Fierz transformation forbids the capture of a μ meson by a proton in the triplet S state. The capture of a μ meson by a proton in the singlet S state with the emission of one photon is also forbidden. It is shown further that the state of two fermions, or that of two antifermions, produced in any weak-interaction process, can be described by one wave function if we neglect the effects of other interactions of these particles; functions describing the correlation of the polarizations are given. The asymmetry of the angular distribution of the neutron emitted by a nucleus after capture of a μ meson is discussed.

1. Zel'dovich and Gershteĭn,[1] and also Chou Kuang-Chao and Maevskiĭ,[2] have shown that the capture of a μ meson by a proton in the triplet S state is forbidden within the framework of the universal Fermi weak interaction proposed by Feynman and Gell-Mann[3] and by Sudarshan and Marshak.[4] Later Dye and others[5] remarked that radiative capture of a μ meson by a proton is also forbidden in the singlet S state with emission of one photon. It is interesting to see whether the vanishing of the matrix elements that describe these processes is due to the existence of some symmetry law. Since the consequences that come from symmetry properties are usually of quite general validity, their study can be useful in testing the universality of the V—A Fermi weak interaction. On the other hand, since the presence of strong interactions usually leads to the disappearance of certain symmetry properties of weak interactions, such a study could also be useful in finding out the effects of the renormalization of the strong interactions on the weak interaction.

In the following section it is shown that the state of two fermions or of two antifermions produced in any weak interaction can be described by one wave function, if we neglect the effects of other interactions of these two particles. In their center-of-mass system these two particles have zero total angular momentum. We shall give a rigorous proof of the results of Zel'dovich and Gershteĭn[1] and of Dye and others,[2] which were previously obtained by the use of the nonrelativistic approximation and by taking into account only the first nonvanishing approximation of perturbation theory.

In the third section we present a function that describes the correlation of the polarizations of two fermions or two antifermions produced in any process described by the universal Fermi weak interaction, when other interactions are neglected; we consider the various possible forms of the Fermi interaction proposed by Gell-Mann and Feynman.[3] Measurements of the correlation of the polarizations can be used for testing the universality of the V—A Fermi weak interaction, and can also give information about the effects of the renormalization of the strong interaction. In connection with the two-component spinor theory[3] we also consider the polarization of a photon emitted in a weak-interaction process.

In the last section it is pointed out that the presence of relativistic effects can cause an asymmetry in the angular distribution of the neutrons emitted by nuclei after the capture of μ mesons, even if we neglect the influence of renormalization and the effect of interaction in the final state.

2. It is well known that the universal Fermi weak interaction

$$H_i = (A, B)(C, D) \equiv (G/\sqrt{2})\bar{\psi}_A\gamma_\alpha(1+\gamma_5)\psi_B\bar{\psi}_C\gamma_\alpha(1+\gamma_5)\psi_D, \quad (1)$$

where ψ_A, ψ_B, ψ_C, and ψ_D are the fields that describe four types of particles A, B, C, and D with half-integral spins, has a number of interesting symmetry properties. For example, under the transformation studied by Fierz:[6]

$$\psi_B \rightleftarrows \psi_D, \quad (2)$$

we have

$$(A, B)(C, D) \rightleftarrows (A, D)(C, B). \quad (3)$$

For convenience in our further arguments it is helpful to subject the operator (1) to the transformation of charge conjugation:

$$H_i = -(G/\sqrt{2})\bar{\psi}_{\bar{B}}\gamma_\mu(1-\gamma_5)\psi_{\bar{A}}\bar{\psi}_C\gamma_\mu(1+\gamma_5)\psi_D. \quad (4)$$

Here $\bar{A}$ and $\bar{B}$ denote the respective antiparticles; thus $\psi_{\bar{A}}$ is the operator that annihilates the antiparticle $\bar{A}$ or produces the particle A. After the Fierz transformation the operator (4) takes the form

$$H_i = \sqrt{2}G\bar{\psi}_C(1-\gamma_5)\psi_{\bar{A}}\psi_{\bar{B}}(1+\gamma_5)\psi_D. \quad (5)$$

Let us study a process in which two fermions A and C are emitted as a result of the interaction (5). If we neglect other interactions of the particles A and C, the Feynman diagram can be represented schematically by Fig. 1. The corresponding element of the S matrix in the momentum representation is of the form

$$\langle \mathbf{p}_A, s_A; \mathbf{p}_C, s_C; r|S|i\rangle = \bar{u}(\mathbf{p}_C, s_C)(1-\gamma_5)v(-\mathbf{p}_A, -s_A)R, \quad (6)$$

where $\mathbf{p}_A$, s_A, and $\mathbf{p}_C$, s_C are respectively the momentum and spin of particle A and the momentum and spin of particle C; r denotes all other quantum numbers of the final state; $|i\rangle$ is the wave function of the initial state; the factors u, v, and $(1-\gamma_5)$ represent respectively the external lines and part of the vertex shown in Fig. 1, u being the wave function of a state with positive energy and v that of a state with negative energy; and, finally, R represents all the other parts of the diagram, which are represented in the drawing by a block. The final state of the system can be represented in the form

$$|f\rangle = |\mathbf{p}_A, s_A; \mathbf{p}_C, s_C; r\rangle\langle \mathbf{p}_A, s_A; \mathbf{p}_C, s_C; r|S|i\rangle. \quad (7)$$

In particular, when the energy and momentum $\mathbf{p}_A + \mathbf{p}_C$ imparted to the system consisting of particles A and C are prescribed, the wave function W of this system is

$$W(\mathbf{p}_A, s_A; \mathbf{p}_C, s_C) = \{\bar{u}(\mathbf{p}_C, s_C)(1-\gamma_5)v(-\mathbf{p}_A, -s_A)\} \times u(\mathbf{p}_A, s_A)u(\mathbf{p}_C, s_C) \quad (8)$$

and does not depend on R. Thus the wave function of the partial system consisting of particles A and C does not depend on what has happened in the intermediate state, if the energy and momentum imparted to this system are prescribed and if we neglect the other interactions of the particles A and C.

It is easy to calculate the wave function W. In the center-of-mass system of particles A and C their total internal angular momentum is zero. Thus W is a superposition of $^1S_0$ and $^3P_0$ states. If one of the particles is a neutrino, the $^1S_0$ and $^3P_0$ states have equal weights. If, however, the velocities of A and C are small, then the contribution of the $^3P_0$ state is small – of the order of v (v is the speed of particle A or particle C, with the speed of light taken as unity). This result is easy to understand directly from the expression (5).

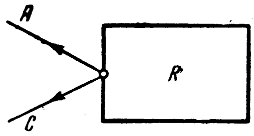

FIG. 1

It is convenient to regard particles A and C as a single system. If we denote $\bar{\psi}_C(1-\gamma_5)\psi_{\bar{A}}$ by Φ, then Eq. (5) takes the form

$$H_i = \sqrt{2}G\Phi\psi_{\bar{B}}(1+\gamma_5)\psi_D. \quad (9)$$

Φ behaves like a scalar under proper Lorentz transformations. Thus the intrinsic angular momentum of the system consisting of the particles A and C emitted as a result of the interaction (5) must be zero.

The results obtained earlier[1,2,5] are now easily understood. Capture of a μ meson by a proton is rigorously forbidden in the ^3S state because of the law of conservation of angular momentum. Since a radiative transition from a state with J = 0 to another state with J = 0 with the emission of one photon is impossible, radiative capture of a μ meson by a proton in the ^1S state with emission of one photon is also rigorously forbidden.

3. Since the wave function W does not depend on what happens in the intermediate state, it is very convenient for applications. As an illustration let us consider the correlation of the polarizations of particles A and C. The table shows the relative probabilities of the various combinations of polarization states; these probabilities are easily obtained by the use of the function W. (In this table v_A and v_C are the speeds of particles A and C, and θ_{AC} is the angle between $\mathbf{p}_A$ and $\mathbf{p}_C$). The functions in the third column of the table give the amounts of correlation of the polari-

s_A, $\mathbf{p}_A$	s_C, $\mathbf{p}_C$	Relative probability
parallel	parallel	$\frac{1}{2}(1-v_A)(1-v_C)(1-\cos\theta_{AC})$
parallel	antiparallel	$\frac{1}{2}(1-v_A)(1+v_C)(1+\cos\theta_{AC})$
antiparallel	parallel	$\frac{1}{2}(1+v_A)(1-v_C)(1+\cos\theta_{AC})$
antiparallel	antiparallel	$\frac{1}{2}(1+v_A)(1+v_C)(1-\cos\theta_{AC})$

zations of fermions emitted in any weak interaction, under the condition that we can neglect the other interactions of these fermions. Let us investigate, for example, the correlation of the polarizations of the neutrino and neutron in the radiative capture of a μ meson by a proton. Let A denote the neutrino and C the neutron; then it can be seen from the table that the neutrino must be a left-hand screw particle, and the degree of longitudinal polarization of the neutrino is

$$(\cos\theta - v)/(1 - v\cos\theta), \tag{10}$$

where v is the speed of the neutron and θ is the angle between the momenta of the neutrino and the neutron. The table can also be applied to the decay of the μ meson, to β decays of nucleons and hyperons, and so on.

Since the theory is invariant with respect to time reversal, the correlation functions given in the table can be used to describe the dependence of the transition probability on the polarization states of the fermions in the initial state, and in particular the dependence of the rate of capture of μ mesons on the hyperfine structure of the μ-mesic atom. In this last case we can assume as a first approximation that only the $^1S_0$ state of the μ meson and proton contributes to the capture. For a more accurate treatment, one must take into account the contribution of the $^3P_0$ state.

Since the V — A Fermi interaction is invariant with respect to combined inversion, the functions that describe the correlation of the polarizations of antifermions can be obtained from the table by the replacements

$$A \to \bar{A}, \qquad C \to \bar{C} \tag{11}$$

together with change of sign of the spins of particles $\bar{A}$ and $\bar{C}$ (the operation "parallel $\rightleftarrows$ antiparallel"). Lack of agreement between the conclusions obtained by using these functions and experimental results would either cast doubt on the universality of the V — A theory of the Fermi weak interaction or indicate an influence of other interactions, for example, a renormalization effect.

As Feynman and Gell-Mann have remarked,[3] one could equally suitably take as the Hamiltonian of the universal weak interaction either the expression

$$H_i = (G/\sqrt{2})\,\bar{\psi}_A\gamma_\alpha(1-\gamma_5)\psi_B\bar{\psi}_C\gamma_\alpha(1-\gamma_5)\psi_D, \tag{12}$$

or the expression

$$H_i = (G/\sqrt{2})\,\bar{\psi}_A\gamma_\alpha(1-\gamma_5)\psi_B\bar{\psi}_C\gamma_\alpha(1+\gamma_5)\psi_D. \tag{13}$$

To settle which of the Hamiltonians (1), (12), or (13) is the correct one, it is necessary to find out whether they lead to different physical consequences. Therefore it is useful to note that Eqs. (1), (12), and (13) give different correlations of the polarizations.

The expression (12) can be obtained from (1) by space inversion, and consequently all formulas obtained from Eq. (1) are converted by space inversion into formulas that follow from Eq. (12). In particular, scalar quantities will remain unchanged, whereas pseudoscalar quantities will be changed in sign; thus to distinguish between Eqs. (1) and (12) one must measure pseudoscalar quantities. The correlation of the polarizations of the fermions in the process described by the Hamiltonian (12) can be obtained from the table by the interchange "parallel $\rightleftarrows$ antiparallel" mentioned earlier. The correlation for antifermions can be obtained in a similar way.

After a charge-conjugation transformation the expression (13) can be written in the form

$$-(G/\sqrt{2})\,\bar{\psi}_{\bar{B}}\gamma_\alpha(1+\gamma_5)\psi_{\bar{A}}\bar{\psi}_C\gamma_\alpha(1+\gamma_5)\psi_D. \tag{14}$$

Thus the correlation of the polarizations given by the Hamiltonian (13) can be obtained from the table by the replacement

$$A \to \bar{B}. \tag{15}$$

Accordingly Eq. (13) leads to a universal correlation of polarizations between fermion and antifermion.

The expression for the interaction Hamiltonian composed of two-component spinors, as proposed by Feynman and Gell-Mann,[3] leads to interesting consequences regarding the polarization state of a photon emitted by a fermion in a weak-interaction process, if the fermion is initially at rest. Let us take as an example the process described by the diagram of Fig. 2. The vertex represents the

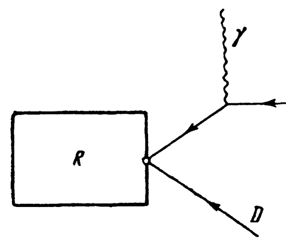

FIG. 2

V — A Fermi interaction, and the photon is emitted by particle B. The corresponding element of the S matrix is of the form

$$R\gamma_\alpha(1+\gamma_5)\frac{i(\hat{p}_B-\hat{k})-m_B}{(p_B-k)^2+m_B^2}\,\frac{\hat{e}^*}{\sqrt{2\omega}}\,u(s_B, p_B). \tag{16}$$

R represents the rest of the diagram, excluding the part that begins with the external line B and ends at the weak-interaction vertex. The rest of the notation is obvious. We have (particle B is initially at rest)

$$\{i(\hat{p}_B - \hat{k}) - m_B\}\hat{e}^* = \hat{e}^*\{i\hat{k} - i\hat{p}_B - m_B\}. \quad (17)$$

Since

$$\{i\hat{p}_B + m_B\}u(\mathbf{s}_B, \mathbf{p}_B) = 0, \quad (18)$$

the expression (16) takes the form

$$iR\gamma_\alpha(1+\gamma_5)\hat{e}^*\hat{k}u(s_B, p_B)/\{(p_B - k)^2 + m_B^2\}\sqrt{2\omega}. \quad (19)$$

We note that for a left-circularly polarized photon

$$\hat{e}^*\hat{k} = -2^{-1/2}(1-\gamma_5)\mathbf{e}^*\sigma\omega. \quad (20)$$

If follows from Eqs. (19) and (20) that the emission of a left-circularly polarized photon by particle B is forbidden — a result first pointed out by Dye and others[5] and by Manacher and Wolfenstein.[7] It is easy to see that if the weak interaction is described by the expression (12) the emission of a right-circularly polarized photon by a particle at rest is forbidden. Therefore measurement of the polarization of a photon emitted by a particle at rest offers a possible way to settle the question as to whether the interaction Hamiltonian is of the form (1) or (12).

4. As an example of the application of the results of the preceding sections let us consider the angular distribution of neutrons emitted by nuclei that have captured μ mesons. Dolinskiĭ and Blokhintsev[8] have shown that both the neutrons emitted by polarized nuclei that have captured unpolarized mesons and the neutrons emitted by unpolarized nuclei that have captured polarized mesons have isotropic distributions if the weak interaction that leads to the capture is of the V — A type and if there are no effects of renormalization and of interaction in the final state. The nonrelativistic approximation was used in their calculations. In particular, they assumed that the speed of the proton is very small and dropped the small components of the proton wave function. The results of Dolinskiĭ and Blokhintsev can now be understood without any calculations. In the center-of-mass system of the meson and proton a μ meson can be captured by a proton only if these two particles are in a state with J = 0. Therefore it is obvious that the angular distribution of the emitted neutron must be isotropic, regardless of whether the nucleus and μ meson are polarized or unpolarized, provided that the μ meson and the proton in the nucleus are regarded as being at rest before the capture.

The results presented above can be affected by two things, namely: 1) the presence of strong interaction, in particular, renormalization effects and interactions in the final state, and 2) relativistic effects associated with the motion of the proton. Dolinskiĭ and Blokhintsev[8] have shown that if one takes into account effects of strong interaction, a sizable anisotropy would be observed. The main terms in their expression for the asymmetry parameter are proportional to:

$$E_\nu / m_N \approx m_\mu / m_N, \quad (21)$$

where E_ν is the energy of the emitted neutrino, and m_μ and m_N are the masses of the μ meson and the nucleon. Because of interference between the weak-magnetism term and the pseudoscalar term the numerical coefficient of the factor (21) has the large value 4.

By using the table one can show easily that relativistic effects can also lead to an asymmetry in the angular distribution, which is proportional to m_π/m_N (m_π is the mass of the π meson). This expression is of the order of magnitude of the momentum of a nucleon in a nucleus. As has already been established above, the neutron is emitted isotropically in the system in which the center of mass of the μ meson and the proton is at rest. In a system in which the center of mass is in motion the neutron is emitted mainly in the forward direction. Let us consider the case in which the proton that captures the μ meson is in an $s_{1/2}$ state. We assume that the μ meson is at rest and is completely polarized in the direction of the z axis. The velocity of the proton, and therefore also that of the center of mass of the proton and the meson, is distributed isotropically in all directions. Since, as can be seen from the table, the capture probability depends on the spin and the velocity, relativistic effects easily lead to an asymmetry in the angular distribution of the neutron.

As illustrations let us consider two cases in detail: 1) the proton is moving in the direction of the z axis; 2) the proton is moving in the opposite direction. The particles A and C in the table must now be taken to be the proton and the μ meson, respectively. If the spin of the proton is parallel to that of the μ meson, the capture probability must be zero, according to the table. If the spin of the proton is antiparallel to that of the μ meson, the capture probability is proportional to 1 + v in case 1, whereas in case 2 the capture probability is proportional to 1 - v (v is the speed of the proton). The proton has a large probability for capturing the μ meson if its velocity is parallel to the spin of the μ meson, and thus more neutrons are emitted along the direction of polarization of the μ mesons. The asymmetry is obviously proportional to $v \approx m_\pi/m_N$. This result is confirmed by calculations, if as the proton wave function we take the wave function for a particle in a spherical region, and for the neutron wave function we take a plane wave.

The argument that has been given shows that here, as is indeed usual, the influence of renormalization is weakened by the relativistic effects. In an accidental way the effects of renormalization and the relativistic effect work against each other in this case (the effect of renormalization causes the emission of a larger number of neutrons in the direction opposite to that of the spin of the μ meson[8]). Inclusion of interactions in the final state, such as the spin-orbit interaction, complicates the situation still more. It is interesting to note that experiments by Astbury and others[9] with the isotope S^{32} have given a negative asymmetry parameter, whereas experiments of Baker and Rubbia[10] with magnesium have given a positive parameter. It would be useful to make experiments with a nucleus of simple structure, such as Ne^{20}. Since in this case the nucleons of the outer shell are in the 2s state, there is no difficulty with inclusion of the spin-orbit interaction in the final state.

It has been shown experimentally[11] that the β-decay interaction is mainly of the V − A type. It is interesting to see whether the weak interaction that leads to μ capture is also an interaction of the V − A type. Telegdi[12] has shown that the μ-capture interaction cannot be of the form (13). The experiment he uses does not, however, show that the μ-capture interaction must necessarily be of the V − A type, namely, of the form (1). Since the quantity measured in the experiment is a scalar, the experiment can also be explained by an interaction of the type (12). In order to assign the μ-capture interaction to the type V − A it is necessary to measure a pseudoscalar quantity, such as the longitudinal polarization of the neutron emitted in the capture.

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