A RESONANCE MODEL FOR THE $\pi + N \rightarrow \pi + \pi + N$ REACTION AT MESON ENERGIES OF 300 — 450 Mev

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It is assumed that the energy dependence of the matrix element is only due to a resonance interaction between the nucleon and one of the mesons in the final state ($^3/_2$, $^3/_2$), and that these particles carry away most of the energy of the system. Under these assumptions, expressions containing six parameters are given for the reaction cross sections. It is shown that the results based upon such a model are in agreement with the available experimental data.

It is not possible at present to calculate the cross sections for the $\pi + N \rightarrow \pi + \pi + N$ reaction using the field theory. It is therefore very desirable to describe the process using some general properties of strong interactions or, at least, by means of phenomenological models. At not too high energies of the incident meson (up to 250 Mev) the reaction was studied by Ansel'm and Gribov,[1] who succeeded in showing that the zero-energy amplitudes of the meson-meson scattering can be found from the study of the behavior of the reaction cross section near the threshold. At energies in the 1 — 1.5 Bev range, single π-meson production in meson-nucleon collisions was investigated quite successfully using the isobar model.[2] In the 300 — 600 Mev range, Peierls[3] and Barshay[4] studied the possibility of obtaining information on the meson-meson interaction from the angular correlation of the particles produced in the reaction.

1. BASIC ASSUMPTIONS. RELATIONS BETWEEN THE TOTAL CROSS SECTIONS FOR DIFFERENT REACTIONS.

In the present article we consider the case where the kinetic energy of the particles produced lies within the 100 — 200 Mev range in the c.m.s. We assume that the meson-meson interaction in the energy range under consideration is much smaller than the resonance meson-nucleon interaction. It is further assumed that, in the final state, one of the mesons and the nucleon are produced in the ($^3/_2$, $^3/_2$) state with an energy which is, in the majority of events, close to resonance energy. The kinetic energy of the second meson will, in general, be not higher than 50 Mev. The last assumption can be submitted to an experimental test. The matter will be discussed in detail below.

In addition, it is assumed that the energy dependence of the matrix element is determined by the interaction of the particles in the final state only. In the majority of cases one of the mesons and the nucleon are in the resonant state with high energy, while the second meson has an energy not higher than 50 Mev. The interaction of a 50-Mev meson with a nucleon is considerably weaker than at 100 — 200 Mev. We neglect therefore the interaction between the second meson and the nucleon in the final state. Since it is assumed that the meson-meson interaction at energies below 200 Mev is also weak compared with the resonance interaction, it can be neglected as well.

Thus, only the interaction between one of the mesons and the nucleon in the ($^3/_2$, $^3/_2$) state is considered in the final state. A diagram of the process is shown in Fig. 1a; it is assumed that the shaded circle is constant in the energy range under consideration. The upper part of the diagram represents the scattering of the meson on the nucleon, in which one meson vertex is missing (Fig. 1b). If the product r_0q of the meson-nucleon interaction radius by the meson momentum is small, then the matrix element corresponding to Fig. 1b is given, disregarding angle-dependent factors, by the following expression (neglecting the nucleon recoil):

$$q^{-2}e^{i\delta(q)}\sin\delta(q). \quad (1)$$

where $\delta(q)$ is the phase of meson-nucleon scattering in the ($^3/_2$, $^3/_2$) state for meson momentum q. Since the second meson possesses an energy of the order of 50 Mev, we shall assume that it is in a state with orbital angular momentum L equal to 0

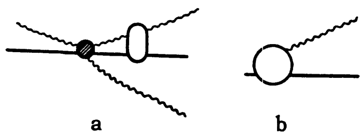

FIG. 1

or 1 with respect to the center of mass of the two remaining particles.

It is necessary to note that when the energy of the incident meson is higher than 450 Mev the kinetic energy of the particles produced will be higher than 200 Mev. The energies of both mesons can then be close to the resonance energy, and it becomes necessary to consider the interaction of both mesons with the nucleon in the final state. The results obtained in the present article are, therefore, not applicable to higher energies.

Similar assumptions were made by Mandelstam[5] in a study of the N + N → π + N + N reaction. His results are in good agreement with experiment, and one can hope, therefore, that a similar approach may be useful in the treatment of the present problem.

Let us now consider the consequences resulting only from the assumption that, in the final state, one of the mesons and the nucleon are produced in the $(^3/_2, {}^3/_2)$ resonance state and carry away most of the energy of the system.

If the meson and the nucleon are characterized by the isotopic spin projections ζ and ν in the initial state, and if in the final state the first and second meson and the nucleon have the isotopic spin projections ξ, η, and μ respectively, then, according to the requirement of isotopic invariance, the transition matrix element is

$$\sum_T (\alpha_T F^T(1,2) + \beta_T F^T(2,1)),$$

$$\alpha_T = C^{T\tau}_{1\zeta\,{}^1/_2\nu} C^{T\tau}_{1\eta\,{}^3/_2\mu+\xi} C^{{}^3/_2\mu+\xi}_{1\xi\,{}^1/_2\mu}, \quad \beta_T = C^{T\tau}_{1\zeta\,{}^1/_2\nu} C^{T\tau}_{1\xi\,{}^3/_2\mu+\eta} C^{{}^3/_2\mu+\eta}_{1\eta\,{}^1/_2\mu}, \quad (2)$$

where T and τ are the total isotopic spin of the system and its projection on the z axis (T can have the values $^1/_2$ and $^3/_2$), 1 and 2 are the energy and momentum of the first and second meson, respectively, and $F^T(1, 2)$ is the transition amplitude, which is dependent on the total isotopic spin and on the energies and momenta of the particles produced. The term containing $F^T(1, 2)$ corresponds to the production of the first π meson in resonance with the nucleon, while the term with $F^T(2, 1)$ corresponds to the production of the second meson in the resonance state. The actual values of α_T and β_T for different processes are given in the Appendix.

The square of the matrix element contains terms with $F^{T'}(1, 2) F^{T*}(1, 2)$ and $F^{T'}(1, 2) F^{T*}(2, 1)$. Moreover, $F^T(1, 2)$ and $F^T(2, 1)$ cannot be simultaneously large: if one meson is produced in resonance with the nucleon, little energy is available for the second meson and the amplitude $F^T(2, 1)$ is small, and vice versa. We can therefore neglect the terms containing $F^{T'}(1, 2) F^{T*}(2, 1)$. The square of the matrix element can then be written in the following form:

$$\sum_{TT'} \{\alpha_T \alpha_{T'} \operatorname{Re}(F^{T*}(1,2) F^{T'}(1,2)) + \beta_T \beta_{T'} \operatorname{Re}(F^{T*}(2,1) F^{T'}(2,1))\}. \quad (3)$$

The contributions of the real parts of Eq. (3) to the total cross section are equal. Using Eq. (3) and Eq. (A1) of the Appendix we thus obtain the following expressions for the total cross sections for the processes under consideration:

$$\sigma(\pi^+ + p \to n + \pi^+ + \pi^+) = {}^4/_{15} A_3, \quad \sigma(\pi^+ + p \to p + \pi^0 + \pi^+) = {}^{13}/_{15} A_3,$$

$$\sigma(\pi^- + p \to n + \pi^- + \pi^+) = {}^{10}/_{27} A_1 + {}^{26}/_{135} A_3 - {}^{14}/_{27} \sqrt{{}^2/_5}\, A_{13},$$

$$\sigma(\pi^- + p \to n + \pi^0 + \pi^0) = {}^8/_{27} A_1 + {}^4/_{135} A_3 + {}^8/_{27} \sqrt{{}^2/_5}\, A_{13},$$

$$\sigma(\pi^- + p \to p + \pi^- + \pi^0) = {}^4/_{27} A_1 + {}^{17}/_{135} A_3 + {}^{10}/_{27} \sqrt{{}^2/_5}\, A_{13}. \quad (4a)$$

A_2, A_3, and A_{13} are functions of the total energy only. A_1 and A_3 are obtained by integrating the squares of the absolute values of the amplitudes, with total isotopic spins equal to $^1/_2$ and $^3/_2$ respectively, over the phase volume, and A_{13} is the result of integrating the interference term. A_1 and A_3 are therefore positive, and A_{13} satisfies the inequality $|A_{13}| \le \sqrt{A_1 A_3}$.

The following two relations between the total cross sections are a consequence of Eq. (4a):

$$\sigma(\pi^+ + p \to n + \pi^+ + \pi^+) = 0.3\, \sigma(\pi^+ + p \to p + \pi^0 + \pi^+),$$

$$1.4\sigma(\pi^+ + p \to n + \pi^+ + \pi^+) + 2\sigma(\pi^- + p \to n + \pi^0 + \pi^0) = 0.6\sigma(\pi^- + p \to n + \pi^- + \pi^+) + 2.5\,\sigma(\pi^- + p \to p + \pi^0 + \pi^-). \quad (4b)$$

Thus, using only the assumption that one of the mesons is produced in the resonance state $(^3/_2, {}^3/_2)$, two relations between the cross sections in the energy range under consideration have been obtained. A comparison of Eqs. (4b) with experimental data will constitute a test of the assumption made above that one of the mesons and the nucleon are produced in the $(^3/_2, {}^3/_2)$ state with an energy which, in general, is close to the resonance energy.

2. TRANSITION-MATRIX ELEMENT

Let us now consider the problem in greater detail. The following transitions are possible under the assumptions made above:

$$D^{-}_{3/2} \to P_{3/2}s_{3/2}, \quad P^{+}_{3/2} \to P_{3/2}\, p_{3/2},$$
$$P^{+}_{1/2} \to P_{3/2}p_{1/2}, \quad F^{+}_{3/2} \to P_{3/2}p_{5/2}. \tag{5}$$

In (5), the expression $P^{+}_{1/2} \to P_{3/2}p_{1/2}$ denotes, e.g., that in the initial state the meson and the nucleon had orbital angular momentum 1, total angular momentum $1/2$, and positive parity, while in the final state on the mesons and the nucleon are in the $P_{3/2}$ resonance state and the second meson is in a p state with respect to the center of mass of the first two particles. We shall neglect the transitions from an initial F state since, at energies on the order of 400 Mev, the contribution of the F state to meson-nucleon scattering is considerably smaller than the contribution of other states.

As will be shown in the following, the assumption that one of the mesons and the nucleon are in the $(3/2, 3/2)$ state with an energy close to the resonance energy in the majority of cases is, for the given model, equivalent to the assumption that the principal transition is

$$D^{-}_{3/2} \to P_{3/2}s_{3/2}.$$

It is possible to write down the matrix element $F^T(1, 2)$ in an approximation in which the nucleon mass is assumed to be infinite. For the case when the total angular momentum of the system and its projection on the z axis are equal to j and M, respectively, $F^T(1, 2)$ is given by

$$a^T_{jL}q_1^{-2}\, e^{i\delta(q_1)} \sin\delta(q_1)\, q_2^L \sum_{ms} C^{jM}_{LM-m\,3/2\,m} C^{3/2\,m}_{1\,m-s\,1/2\,s}$$
$$\times Y_{1\,m-s}(\mathbf{q}_1)\, Y_{LM-m}(\mathbf{q}_2), \tag{6}$$

where $\mathbf{q}_1$ is the momentum of the meson which is in the $(3/2, 3/2)$ state, $\delta(\mathbf{q}_1)$ is the phase shift for the given momentum in the resonance state, $\mathbf{q}_2$ and L are the momentum and angular momentum of the second meson with respect to the center of mass of the first two particles, and a^T_{jL} are certain constants varying with T, j, and L. It follows from Eq. (5) and the assumption about the small contribution of the F state that there are three such constants for a given T.

The matrix element, as given by Eq. (6), is correct to a term of the order of the ratio of the kinetic energy of the particles in the final state to the nucleon mass.

The constants a^T_{jL} should still satisfy certain relations that follow from the fact that the S matrix is a unitary one. As in reference 6, it can be found that

$$a^T_{jL} = b^T_{jL} \exp(i\eta^T_{jL}), \tag{7}$$

where η^T_{jL} is the phase of the meson-nucleon scattering with given T, j, and L corresponding to the incident meson energy, and b^T_{jL} are real (though not necessarily positive) quantities which should be determined by comparing the expressions for the cross sections with experimental data.

3. EXPRESSIONS FOR THE CROSS SECTIONS

In calculating the cross sections for the investigated process we assume the nucleon to be at rest in the final state. The cross sections can be then easily calculated in the usual way. It is only necessary to multiply the amplitudes F^T given in Eq. (6) by a factor $k^{-1/2}C^{jM}_{\lambda 0 \frac{1}{2} M}$, which is due to the expansion of the plane wave in terms of the total angular momentum eigenfunctions of the system (k is the meson momentum in the initial state, and λ is the angular momentum of the meson-nucleon system in the initial state).

The energy distribution of mesons moving within a solid angle element with a given θ (angle between the z axis and the direction of meson emission in the c.m.s.) is given by the expression

$$d\sigma = [a_0 + a_1\cos\theta + \tfrac{1}{2}a_2(3\cos^2\theta - 1)]\, d\omega\, d\Omega/4\pi,$$
$$a_0 = S(\omega)[A_1 + ((\varepsilon-\omega)^2 - 1)B_1] + S(\varepsilon-\omega)[A_2 + (\omega^2 - 1)B_2],$$
$$a_1 = S(\varepsilon-\omega)\sqrt{\omega^2 - 1}\; C,$$
$$a_2 = S(\omega)[D_1 + ((\varepsilon-\omega)^2 - 1)E_1] + S(\varepsilon-\omega)(\omega^2 - 1)E_2,$$
$$S(\omega) = \frac{\sqrt{1+k^2}\sqrt{M^2+k^2}}{k^2(\sqrt{1+k^2} + \sqrt{M^2+k^2}}$$
$$\times(\varepsilon-\omega)\sqrt{(\varepsilon-\omega)^2 - 1}\,\omega\,\frac{\sin^2\delta(\omega)}{(\omega^2-1)^{3/2}}. \tag{8}$$

where M is the nucleon mass, $\omega = \sqrt{1 + q^2}$ is the energy of the meson under consideration (meson mass equal zero), ϵ is the energy of both mesons in the final state (including rest energy) in the c.m.s. (it should be remembered that we neglect the kinetic energy of the nucleon in the final state), $d\Omega = 2\pi \sin\theta\, d\theta$ is the solid-angle element, and $\delta(\omega)$ is the phase shift in the $(3/2, 3/2)$ resonance state. A_1, A_2, ... E_1, E_2 are constants varying with a^T_{jL}, α_T, and β_T (i.e., they are different for different processes). In addition, these constants vary depending on which one of the mesons produced is considered. Explicit expressions for the constants are given in the Appendix.

The total cross section is obtained from Eq. (8) by integrating over all possible values of ω and the angle θ. Using the data of reference 7 to determine $\sin^2\delta(\omega)$, we can calculate the total cross section for the investigated process

$$\sigma = (A_1 + A_2)I_1 + (B_1 + B_2)I_2, \tag{9}$$

where I_1 and I_2 are functions of the total energy only. Numerical values of these functions are given in the table.

Energy of the incident meson in the laboratory system, Mev.	$I_1 \cdot 10^3$	$I_2 \cdot 10^3$	$I_3 \cdot 10^3$
290	0.83	0.42	0.60
320	1.5	1.0	1.2
370	3.2	2.9	3.0
430	6.3	6.6	6.4

The following expression gives the meson angular distribution:

$$4\pi d\sigma/d\Omega = b_0 + b_1 \cos\theta + {}^1/_2\, b_2 (3\cos^2\theta - 1),$$
$$b_0 = (A_1 + A_2) I_1 + (B_1 + B_2) I_2,$$
$$b_1 = C I_3, \quad b_2 = D I_1 + (E_1 + E_2) I_2. \qquad (10)$$

The values of I_3 are given in the table.

According to the assumptions made, the course of the process is such that one of the mesons is produced in a $(^3/_2, {}^3/_2)$ resonance state and, in general, the energy of this meson lies in the resonance range, i.e., is relatively high.

It is essential to check whether the formulas obtained fulfill the requirement that the number of cases in which the meson produced in the $(^3/_2, {}^3/_2)$ state has an energy substantially different from the resonance energy be really small. Integrating Eq. (8) over the angle θ, we obtain the energy distribution for the meson which, to be specific, we shall denote as the "first" meson:

$$d\sigma/d\omega = S(\omega)\,[A_1 + ((\varepsilon - \omega)^2 - 1)\, B_1]$$
$$+ S(\varepsilon - \omega)\,[A_2 + (\omega^2 - 1)\, B_2].$$

The first term is responsible for the production of the "first" meson in the $(^3/_2, {}^3/_2)$ state. It is necessary to check if this term really decreases sufficiently fast with decreasing energy of the "first" meson. The energy dependence of the first term is determined by the quantities $A_1 S(\omega)$ and $B_1(\epsilon - \omega^2 - 1)\, S(\omega)$, of which the first has, in fact, a maximum for $\omega \sim \omega_{res}$ ($\omega_{res} \sim \epsilon - 1$), while the second attains a maximum for smaller ω. This is a result of the additional factor $(\epsilon - \omega)^2 - 1 = q_2^2$, which appears in the matrix element (6) because of the production of the "second" meson with $L = 1$.

If the process goes in such a way that the "second" meson is, in general, produced in a state with $L = 0$, i.e., if $A_1 \gg B_1$ (the quantities $S(\omega)$ and $S(\omega)\,[(\epsilon - \omega)^2 - 1]$ are of the same order of magnitude), then the basic assumptions are satisfied. The same can be said about the second term, responsible for the production of the "second" meson in the $(^3/_2, {}^3/_2)$ state. This term should decrease sufficiently fast with increasing ω if $\epsilon - \omega$ becomes smaller than ω_{res}.

It is thus necessary that $A_1 + A_2 \gg B_1 + B_2$ (for the results it is sufficient that $A_1 + A_2$ be greater than $B_1 + B_2$ by a factor of two or three at least). As will be seen in the following from the analysis of experimental data, the condition $A_1 + A_2 \gg B_1 + B_2$ can, in fact, be satisfied, although B_1 and B_2 cannot be assumed to vanish, for if all mesons produced are in resonance interaction with the nucleon the term $a_1 \cos\theta$, which is essential for the agreement with experiment, disappears from the cross section (8) in the P state.

4. COMPARISON WITH EXPERIMENTAL DATA

The results given above for the cross sections have been compared with the experimental data of Perkins et al.,[8] who measured the cross sections for the $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ reaction at 260, 320, 370, and 430 Mev (Fig. 2). The expression for the total cross section (9) is in good agreement with the experimental results. Attention should be drawn to the fast increase of the cross section between 300 and 450 Mev, which cannot be explained by the increase of the phase volume only. The curve showing the increase of the phase volume with energy is given for comparison in Fig. 2 by the dashed line. It should also be noted that, essentially, the total cross sections (9) depend only on one parameter $A_1 + A_2$, since $B_1 + B_2$ is small under the assumptions made above.

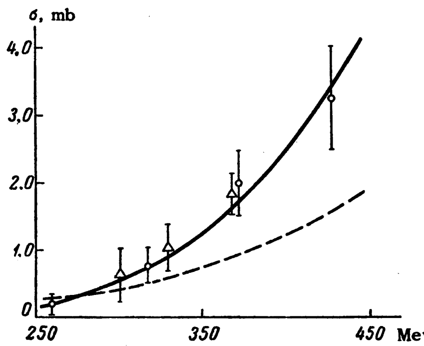

FIG. 2. Total cross section for the $\pi^- + p \rightarrow n + \pi^+ + \pi^-$ reaction, calculated according to Eq. (9) for $A_1 + A_2 = 53$ mb, $B_1 + B_2 = 3$ mb (solid line). Dashed line represents the increase in the phase volume. o – values of the total cross section $\sigma(\pi^- + p \rightarrow n + \pi^+ + \pi^-)$,[8] Δ – values of $\sigma(\pi^- + p \rightarrow n + \pi^+ + \pi^-) + 0.35\sigma(\pi^- + p \rightarrow \pi^- + \pi^0 + p)$.[9] The x axis represents the incident meson energy in the laboratory system.

It can be seen from Figs. 3 and 4 that the data on the angular distribution of positively charged mesons at 320 and 430 Mev also are in agreement with the theoretical curves.

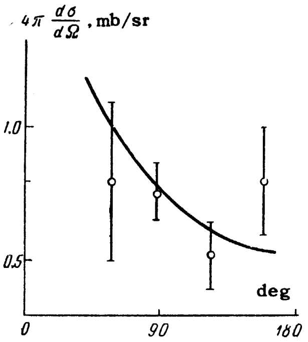

FIG. 3. Angular distribution in the c.m.s. of the mesons produced in the $\pi^- + p \to n + \pi^+ + \pi^-$ reaction at 320 Mev incident meson energy.[8] Solid curve represents the angular distribution calculated according to Eq. (10) for C = 30 mb, $D_1 = 2.5$ mb, and $E_1 + E_2 = 2.5$ mb.

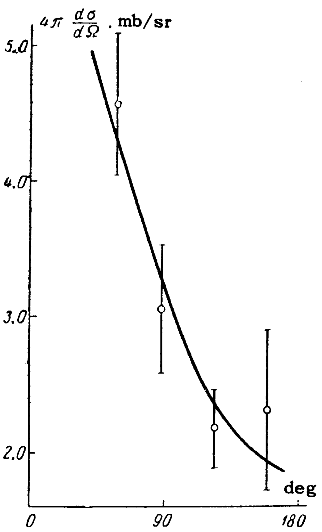

FIG. 4. Angular distribution in c.m.s. of π^+ mesons produced in the $\pi^- + p \to n + \pi^+ + \pi^-$ reaction at 430 Mev incident meson energy.[8]

The mean value of $\sigma(\pi^+ + p \to \pi^+ + \pi^+ + n) + \sigma(\pi^+ + p \to \pi^+ + \pi^0 + p)$ was estimated[9] for the energy range 300 — 575 Mev. It was found to be equal to (1.8 ± 0.6) mb. It follows then from Eq. (4a) that $A_3 = (1.6 \pm 0.5)$ mb and that the contribution of the term containing A_3 to the total cross sections $\sigma(\pi^- + p \to n + \pi^+ + \pi^-)$, $\sigma(\pi^- + p \to n + \pi^0 + \pi^0)$, and $\sigma(\pi^- + p \to p + \pi^0 + \pi^-)$ is of the order of 0.3, 0.05, and 0.2 mb respectively, i.e., is small. Neglecting it, we obtain an additional relation between the total cross sections

$$\sigma(\pi^- + p \to n + \pi^+ + \pi^-) + 4\sigma(\pi^- + p \to p + \pi^0 + \pi^-) = 3\sigma(\pi^- + p \to n + \pi^0 + \pi^0). \quad (11)$$

It follows from reference 8 that the mean value of $\sigma(\pi^- + p \to p + \pi^+ + \pi^-)$ in the energy range under consideration is of the order of 2 — 2.5 mb. Since $A_3 \sim 1.6$ mb and $A_{13} \le \sqrt{A_1 A_3}$, one finds easily from Eq. (4a) that the mean value of $\sigma(\pi^- + p \to p + \pi^0 + \pi^-) \le 2$ mb in the investigated energy range.

This fact is confirmed by the results of Zinov and Korenchenko[10] who measured the quantity $2\sigma(\pi^- + p \to n + \pi^+ + \pi^-) + 0.7\,\sigma(\pi^- + p \to p + \pi^0 + \pi^-)$ (see Fig. 2). From their data, as well as those of reference 8, it follows that the cross section $\sigma(\pi^- + p \to p + \pi^0 + \pi^-)$ cannot be large.

The available experimental data are not sufficient to establish how well the proposed model describes the two-meson production process in meson-nucleon collisions in the energy range under consideration. The agreement between the experimental and the theoretical angular distributions of π^+ mesons and the total cross sections does not provide a decisive proof for the validity of the model because of the large number of unknown parameters a^T_{jL} (six constants). It should be mentioned, however, that the model yields a correct increase of the cross sections with the energy, which depends only on the variation of I_1 with the energy but is independent of the constants a^T_{jL}. The model also predicts an increase of the coefficients b_1 and b_2 with the energy in the expressions (10) for the angular meson distribution, which is in agreement with experiment.

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APPENDIX

We give here the values of α_T and β_T for the various processes:

I. $\pi^+ + p \to n + \pi^+ + \pi^+$

$$\alpha_{1/2} = \beta_{1/2} = 0, \quad \alpha_{3/2} = \beta_{3/2} = \sqrt{2/15}.$$

II. $\pi^+ + p \to p + \pi^0(1) + \pi^+(2)$

$$\alpha_{1/2} = \beta_{1/2} = 0, \quad \alpha_{3/2} = -2/\sqrt{15}, \quad \beta_{3/2} = \sqrt{3/5}.$$

III. $\pi^- + p \to n + \pi^-(1) + \pi^+(2)$

$$\alpha_{1/2} = 1/\sqrt{3}, \quad \beta_{1/2} = 1/3\sqrt{3},$$
$$\alpha_{3/2} = -\sqrt{2/15}, \quad \beta_{3/2} = \frac{2}{3}\sqrt{2/15}.$$

IV. $\pi^- + p \to n + \pi^0 + \pi^0$

$$\alpha_{1/2} = \beta_{1/2} = -2/3\sqrt{3}, \quad \alpha_{3/2} = \beta_{3/2} = -\frac{1}{3}\sqrt{2/15}.$$

V. $\pi^- + p \to p + \pi^-(1) + \pi^0(2)$

$$\alpha_{1/2} = -\frac{1}{3}\sqrt{2/3}, \quad \beta_{1/2} = \frac{1}{3}\sqrt{2/3},$$
$$\alpha_{3/2} = -1/3\sqrt{15}, \quad \beta_{3/2} = 4/3\sqrt{15}. \quad (A1)$$

The expressions for the cross sections (8) — (10) contain the constants $A_1, A_2, \ldots E_1, E_2$. If we are interested in the cross section for the meson denoted in (A1) by π (1), the constants are

$$A_1 = D = \frac{2}{5}\sum_{TT'} \alpha_T \alpha_{T'} \operatorname{Re}(a^{T*}_{1/2\,0} a^{T'}_{1/2\,0}),$$

$$B_1 = \sum_{TT'} \alpha_T \alpha_{T'} \left\{\frac{1}{3}\operatorname{Re}(a^{T*}_{1/2\,1} a^{T'}_{1/2\,1}) + \frac{2}{3}\operatorname{Re}(a^{T*}_{3/2\,1} a^{T'}_{3/2\,1})\right\},$$

$$C = \sum_{TT'} \beta_T \beta_{T'} \left\{-2\sqrt{\frac{2}{15}}\operatorname{Re}(a^{T*}_{1/2\,1} a^{T'}_{3/2\,0}) + \frac{4}{5\sqrt{3}}\operatorname{Re}(a^{T*}_{3/2\,1} a^{T'}_{3/2\,0})\right\},$$

$$E_1 = \sum_{TT'} \alpha_T \alpha_{T'} \left\{ -\frac{4}{3}\sqrt{\frac{2}{5}}\,\mathrm{Re}\,(a^{T*}_{1/2\,1} a^{T'}_{3/2\,1}) - \frac{2}{15}\,\mathrm{Re}\,(a^{T*}_{3/2\,1} a^{T'}_{3/2\,1}) \right\},$$

$$E_2 = \sum_{TT'} \beta_T \beta_{T'} \left\{ -\frac{2}{3}\sqrt{\frac{2}{5}}\,\mathrm{Re}\,(a^{T*}_{1/2\,1} a^{T'}_{3/2\,1}) - \frac{8}{15}\,\mathrm{Re}\,(a^{T*}_{3/2\,1} a^{T'}_{3/2\,1}) \right\}. \tag{A2}$$

A_2 and B_2 can be obtained from the expressions for A_1 and A_2 by substituting β_T for α_T.

For the meson denoted in (A1) by π (2), the constants $A_1, \ldots, E_2$ are obtained from the corresponding constants in Eq. (A2) by substituting β_T for α_T. Thus, e.g., the cross section for the π^- meson in the $\pi^- + p \to n + \pi^+ + \pi^-$ reaction is obtained by substituting into Eq. (A2) the values $\alpha_{1/2} = 1/\sqrt{3}$, $\beta_{1/2} = 1/3\sqrt{3}$, $\alpha_{3/2} = -\sqrt{2/15}$, $\beta_{3/2} = \frac{2}{3}\sqrt{2/15}$, and the cross section for the π^+ meson is obtained by substituting into Eq. (A2) the values $\alpha_{1/2} = 1/3\sqrt{3}$, $\beta_{1/2} = 1/\sqrt{3}$, $\alpha_{3/2} = \frac{2}{3}\sqrt{2/15}$, $\beta_{3/2} = -\sqrt{2/15}$.

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Translated by H. Kasha
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Errata

Volume	No.	Author	page	col.	line	Reads	Should read
10	5	Bogachev et al.	872	1	21	±0.3 cm	±0.7 cm
11	6	Gol'danskii et al.	1229	r	Eq. (13)	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$
			1331	r	4	$\ldots + \frac{1}{4} + \frac{\gamma_a}{2}$	$\ldots + \frac{1}{4}\cos \quad + \frac{\lambda}{2}$
12	2	Moroz and Fedorov	210	1	Eq. (7)	$\ldots \frac{\sin k_0 x_0}{k_0} e^{i\mathbf{kx}} d^3k,$	$\ldots \frac{ik_0 \delta (k^2)}{\vert k_0 \vert} e^{ikx}\, d^4k.$
			212	1	Eq. (39)	$\ldots = 4\pi\hbar c \ldots$	$\ldots = -4\pi\hbar c \ldots$
			212-3	r-1	Eqs. (44) and (39)	$\ldots + \frac{1}{2} iel\, \nabla_k \psi_4 (x) \ldots$	$\ldots + \frac{1}{2} iel \nabla_k \psi_4'' (x) \ldots$
			213	r	Eq. (51), line 2	$\ldots \frac{iel}{2} \int \nabla_m \psi_4 (x) \ldots$	$\ldots \frac{iel}{2} \int \nabla_m \psi_4'' (x) \ldots$
			213	r	Eq. (53)	$\ldots e^{-ik_0 \vert x_0 - x_0' \vert} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} \frac{d^3k}{k_0}.$	$\ldots e^{ik(x-x')} \frac{d^4k}{2\pi i (k^2 - i\varepsilon)}$
12	3	Nikishov	530	1	Eq. (10)	—	$\mu^{(2)} = \frac{1}{2\beta_2\zeta} \ln\left[\frac{y_1 - 1}{y_1 + 1} \cdot \frac{-y_2 - 1}{-y_2 + 1}\right]$
			533	r	Fig. 4	The dashed curve of Fig. 4 has been incorrectly calculated (corrections to μ^+ scattering on electrons). Its value ranges from −6 to −8.	
12	1	Anisovich	72, 75		Eqs. (4a), (4b), (11)	$\sigma (\pi^+ + p \to n + \pi^+ + \pi^+)$	$2\sigma (\pi^+ + p \to n + \pi^+ + \pi^+)$
	5	,,	948		Eq. (6)	$\sigma (\pi^- + p \to n + \pi^0 + \pi^0)$	$2\sigma (\pi^- + p \to n + \pi^0 + \pi^0).$