DISPERSION RELATIONS AND ANALYSIS OF THE ENERGY DEPENDENCE OF CROSS SECTIONS NEAR THRESHOLDS OF NEW REACTIONS

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The application of dispersion relations to the analysis of the energy dependence of scattering (and reaction) amplitudes near thresholds of new reactions is discussed. General expressions are obtained which characterize the nonmonotonic behavior of forward-scattering amplitudes as functions of the energy. The energy dependence of one of the amplitudes for elastic scattering of γ-ray quanta by deuterons is examined near the threshold for photodisintegration of the deuteron.

1. An examination of the scattering of γ-ray quanta by nucleons near the threshold for pion production[1] has shown that the dispersion relations automatically lead to the appearance of discontinuities of the derivative of the real part of the amplitude, if one takes account of the energy dependence of the reaction cross section near threshold.* Within the framework of the dispersion relations the problem of the appearance of discontinuities of the derivative of the forward scattering amplitude involves the analysis of integrals of the form

$$\frac{k_0^2}{4\pi^2}\mathrm{P}\int\frac{d\omega}{k}\,\frac{\sigma(\omega)}{\omega\mp\omega_0},$$

where the usual notation is used, and the total cross section $\sigma(\omega)$ includes both the elastic scattering cross section $\sigma_s(\omega)$ and the inelastic interaction cross section $\sigma_c(\omega)$.

The behavior of $\sigma_c(\omega)$ near the threshold of the binary reaction

$$a+b\rightarrow c+d \tag{2}$$

involving particles with masses μ (incident), M (target), m and $\mathfrak{M}$ is given by the expression

$$\sigma_c(\omega)=Bq_c/k_c, \tag{3}$$

where q_c and k_c are the momenta before and after the collision (in the c.m.s.) and B is a constant. It is not hard to see that

$$(q_c/k_c)^2=(\omega-\omega_t)(\omega+\omega_t-\delta)/(\omega^2-\mu^2), \tag{4}$$

where $\omega=(k^2+\mu^2)^{1/2}$ is the total energy of the incident particle in the laboratory system (l.s.),

$$\omega_t-\mu=[(\mathfrak{M}+m)^2-(M+\mu)^2]/2M \tag{5}$$

is the threshold energy of the reaction (2), and

$$\delta=\{\mathfrak{M}^2+m^2-\mu^2-M^2\}/M. \tag{6}$$

The application of dispersion relations makes it possible to examine both "local effects" near the very threshold, which in some cases lead to sharp "peaks," "dips," and "steps," and also the general influence of inelastic processes occurring in some energy range on the processes at a given energy. We recall that the unitarity relations for the S matrix make it possible to take into account the influence on a given process of other processes occurring at the same energy.

2. The study of γ-N scattering has shown that there are "local effects" in only two of the six scalar amplitudes needed to describe the transition matrix in this case. The presence of other strongly energy-dependent amplitudes hinders the analysis. For a detailed analysis of the inelastic processes it is necessary to examine the dispersion relations for nonvanishing momentum transfers Q^2, and possibly also double dispersion relations.

In the present paper we confine ourselves to the examination of dispersion relations in the total energy for $Q^2=0$ for a scalar function $A(\omega)$, which is the trace of the scattering matrix,

$$A(\omega)=\mathrm{Sp}\,M(\omega,Q^2=0), \tag{7}$$

and whose imaginary part is related to the total cross section. The contribution of inelastic processes to $D=\mathrm{Re}\,A(\omega)$ is characterized by the two

*The nonmonotonic behavior of the cross section near the threshold has been treated phenomenologically in a diploma research by G. Ustinova, and also by Capps and Holladay.[2]

integrals

$$\frac{k_0^2}{4\pi^2} P \int \frac{d\omega}{k} \frac{\sigma_c^-(\omega)}{\omega - \omega_0}, \quad (8)$$

$$\frac{k_0^2}{4\pi^2} P \int \frac{d\omega}{k} \frac{\sigma_c^+(\omega)}{\omega + \omega_0}, \quad (9)$$

where $\sigma_c^-(\omega)$ is the total cross section of reaction (2) and $\sigma_c^+(\omega)$ is the cross section of the reaction cross-symmetrical to (2).

Being interested in the energy dependence of the real part of the quantity $A(\omega)$ in Eq. (7), let us calculate the integrals

$$\frac{k_0^2}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{k} \frac{\sigma_c^-}{\omega - \omega_0} = \frac{k_0^2 B^-}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{k^2} \frac{[(\omega - \omega_t)(\omega + \omega_t - \delta)]^{1/2}}{\omega - \omega_0}$$

$$= \frac{B^- k_0^2}{4\pi^2} \Pi(\omega_0), \quad (10)$$

$$\frac{k_0^2}{4\pi^2} P \int_{\omega_t}^{\omega_1} \frac{d\omega}{k} \frac{\sigma_c^+}{\omega + \omega_0} = \frac{k_0^2 B^+}{4\pi^2} \Pi(-\omega_0); \quad (11)$$

in these integrals the range of integration is from the threshold ω_t to ω_1, the limit of the region of the s state in σ_c in which Eq. (3) is still valid. If we introduce the notations

$$a(\omega_0) = (\omega_0 - \omega_t)(\omega_0 + \omega_t - \delta),$$

$$a(-\omega_0) = (\omega_0 + \omega_t)(\omega_0 - \omega_t + \delta),$$

$$R = (\omega_1 - \omega_t)(\omega_1 + \omega_t - \delta), \quad (12)$$

it is not hard to show that

$$k_0^2 \Pi(\omega_0) = -\{\psi(\omega_0) - \tfrac{1}{2}(1 + \omega_0/\mu)\psi(\mu)$$

$$- \tfrac{1}{2}(1 - \omega_0/\mu)\psi(-\mu)\}, \quad (13)$$

where

$$\psi(\mu) = \sqrt{-a(\mu)}\left[\frac{\pi}{2} + \tan^{-1}\frac{(2\mu - \delta)(\omega_1 - \mu) + 2a(\mu)}{2\sqrt{-a(\mu)R}}\right],$$

$$\psi(-\mu) = \sqrt{-a(-\mu)}\left[\frac{\pi}{2} - \tan^{-1}\frac{(2\mu + \delta)(\omega_1 + \mu) - 2a(-\mu)}{2\sqrt{-a(-\mu)R}}\right]. \quad (13')$$

For the function $\psi(\omega)$ at the point ω_0 we get the expressions

$$\psi(\omega_0) = \sqrt{-a(\omega_0)}\left[\frac{\pi}{2} + \tan^{-1}\frac{(2\omega_0 - \delta)(\omega_1 - \omega_0) + 2a(\omega_0)}{2\sqrt{-a(\omega_0)R}}\right],$$

$$\omega_0 \leqslant \omega_t; \quad (14)$$

$$\psi(\omega_0) = \sqrt{a(\omega_0)} \ln\left|\frac{2a(\omega_0) + (2\omega_0 - \delta)(\omega_1 - \omega_0) + 2\sqrt{a(\omega_0)R}}{(\omega_1 - \omega_0)(2\omega_t - \delta)}\right|,$$

$$\omega_0 \geqslant \omega_t. \quad (14')$$

The expression for $k_0^2\Pi(-\omega_0)$ is obtained from (13) and (14) by the replacement $\omega_0 \to -\omega_0$.

It follows from (14) and (14′) that the first derivative has a discontinuity at the point ω_0. The resulting energy dependence has the characteristic feature that the derivative is infinite on the side $\omega_0 < \omega_t$ and has a finite value when we approach the point $\omega_0 = \omega_t$ from the region $\omega_0 < \omega_t$.

It might seem that the quantity $k_0^2\Pi(-\omega_0)$ obtained as the result of substituting (3) and (4) in (9) would also have a discontinuity of the derivative at $\omega_0 = \omega_t - \delta$. On changing the sign of ω_0 in (14), however, we easily verify that this is not so. The values of the derivative of $k_0^2\Pi(-\omega_0)$ calculated with approach to $\omega_0 = \omega_t$ from the two sides are identical. Thus although in a relativistic treatment cross-symmetrical inelastic processes indeed contribute to the real part of the scattering amplitude, they do not lead to nonmonotonic energy dependence of the amplitude.

The application of dispersion relations enables us to get detailed information about the magnitude and half-width of the anomaly near the threshold. The half-width ϵ of the drop in the region $\omega_0 < \omega_t$ can be estimated roughly in the following way. Near $\omega_0 = \omega_t$ ($\omega_0 < \omega_t$) the argument of the arc tangent is large; using this fact, we find

$$\psi(\omega_0) = \sqrt{-a}\ \pi. \quad (15)$$

Defining the half-width ϵ by the condition

$$D(\omega_t - \varepsilon) = \tfrac{1}{2} D(\omega_t) \quad (16)$$

and using Eq. (15), we then get

$$D(\omega_t) - (B/4\pi)\sqrt{(\omega_t - \omega_0)(\omega_0 + \omega_t - \delta)} = \tfrac{1}{2} D(\omega_t), \quad (17)$$

from which we have

$$\varepsilon \approx \tfrac{1}{8}(\omega_t - \delta/2)^{-1}[4\pi D(\omega_t)/B]^2. \quad (18)$$

In the limiting case in which the contribution to $D(\omega_t)$ not associated with the expressions (10) and (11) can be neglected,

$$D(\omega_t) = BJ(\omega_t)/4\pi^2, \quad (19)$$

and from Eq. (13) we have the result that

$$J(\omega_t) = \tfrac{1}{2}[(1 + \omega_t/\mu)\psi(\mu) + (1 - \omega_t/\mu)\psi(-\mu)]. \quad (20)$$

3. Let us consider the photoproduction of neutral pions

$$\gamma + p \to p + \pi^0 \quad (21)$$

near the threshold of the reaction

$$\gamma + p \to n + \pi^+. \quad (22)$$

In this energy range it suffices to consider the electric-dipole transition. We denote by E^0 and E^+ the transition elements for neutral and charged mesons, respectively. From the condition of unitarity and the experimental fact that Re $E^0 \approx 0$, we get the result

$$\operatorname{Im} E^0 = \sqrt{2/3}\,(\alpha_3 - \alpha_1)\operatorname{Re} E^+, \qquad (23)$$

where α_3 and α_1 are the phase shifts for π-N scattering. Substituting the experimental data for α_3, α_1, and E^+, we get

$$\operatorname{Im} E^0 = \sqrt{2/3}\,(a_3 - a_1)\, q_0^{1/2} q_+^{1/2} \sqrt{q_+/\nu}\cdot 3{,}3\cdot 10^{-15}\ \text{cm}, \qquad (24)$$

where

$$q_0^{1/2} \approx (\nu^2 - \nu_{0t}^2)^{1/4}, \quad q_+ \approx (\nu^2 - \nu_{+t}^2)^{1/2}$$

(ν is the energy of the photon, and a_3, a_1 are the scattering lengths). The anomalies at the threshold are determined by an integral of the form

$$\mathrm{P}\int_{\nu_{+t}}^{\nu_1} d\nu\,(\nu^2 - \nu_{0t}^2)^{1/4}(\nu^2 - \nu_{+t}^2)^{1/2}/\nu^{1/2}(\nu - \nu_0), \qquad (25)$$

which undoubtedly gives "peak" singularities.

In analogy with this, in the general case we can consider the cross section of the reaction

$$a + b \to c + d \qquad (26)$$

near the threshold of the reaction

$$a + b \to e + f. \qquad (27)$$

If the threshold of reaction (27) is far from that of reaction (26), we can always find an energy range where

$$\operatorname{Im} M(ab \to cd) = M(ab \to ef)\, M^+(ef \to cd) + \ldots = Aq + \ldots; \qquad (28)$$

here A is a weakly varying function of the energy and q is the relative momentum of the system ef. The other terms in the sum (28) are also slowly varying functions of the energy if there are no thresholds of other reactions in the neighborhood. In this case the dispersion integral has the usual form, and we can determine the magnitude and half-width of the "peak" or "dip" in the same way as for the case of scattering.

In the case of such a process as the photoproduction of pions

$$\operatorname{Im} M(ab \to cd) = Aqq_0^{1/2} + \ldots, \qquad (29)$$

since the threshold of the reaction ab → ef is close to that of the reaction ab → cd. In these cases the dispersion integrals are rather complicated and we have not been able to carry out the integration.

4. It thus follows from the conditions of causality and unitarity, together with Eqs. (3) and (4), that the first derivative of the real part of the scattering amplitude has a discontinuity, and that the derivative from the side $\omega_0 < \omega_t$ is infinite. That it is precisely the first derivative that is infinite is due to the form of the relations (3) and (4). The behavior of the cross section for the reaction (2), when its products are in states with nonvanishing angular momentum l, is given by the expression

$$\sigma_c^{(l)} = B^{(l)}\,[(\omega - \omega_t)(\omega + \omega_t - \delta)/(\omega^2 - \mu^2)]^{l+1/2}. \qquad (30)$$

Substitution of (3) in (8) makes the lth derivative infinite.

It is perhaps interesting to note that, unlike the nonrelativistic treatment, this use of the dispersion relations has not required the assumption that the partial amplitudes are analytic. It has turned out that it is enough to use only the analytic character of the scattering amplitude with respect to the total energy, with a bounded value of the momentum transfer, $Q^2 < Q^2_{max}$.

As Baz' has pointed out, the unitarity of the S matrix has the consequence that as the number of channels increases the effect in each channel decreases. An analysis of γ-N scattering near the threshold for photoproduction of pions, for which there are "peak" effects in only two out of six scalar functions, has shown that there is also a smearing of the effect with increase of the spin of the particles.

An important feature of the theory of dispersion relations is the discussion of the convergence of the dispersion integrals at high energies or, what is the same thing, of the number of subtractions. The main calculations in the present paper are made for dispersion relations with one subtraction. In the case of dispersion relations without subtraction one must make the replacement

$$k_0^2 \int \frac{d\omega}{k}\frac{\sigma}{\omega - \omega_0} \to \int \frac{d\omega\, k\sigma}{\omega - \omega_0}. \qquad (31)$$

With sufficiently high experimental accuracy the difference between Eq. (8) and Eq. (31) can give information about the number of subtractions.

Let us note briefly what sort of singularities can appear near the threshold of the reaction

$$a + b \to c + d + f. \qquad (32)$$

By substituting in Eq. (8) the cross section of the reaction (32) in the form

$$\sigma_c = B' k_c^{-1} P_{c\,max}^4 \approx \Gamma(\omega - \omega_t)^2$$

(reaction products in the s state), we get

$$\int_{\omega_t}^{\omega_1} \frac{\sigma d\omega}{\omega - \omega_0} \approx \Gamma \int_{\omega_t}^{\omega_1} \frac{d\omega\,(\omega - \omega_t)^2}{\omega - \omega_0} = \Gamma\Big\{\frac{1}{2}[(\omega_1 - \omega_0)^2 - (\omega_t - \omega_0)^2] + 2(\omega_0 - \omega_t)(\omega_1 - \omega_t) + (\omega_0 - \omega_t)^2 \ln|(\omega_1 - \omega_0)/(\omega_t - \omega_0)|\Big\}, \qquad (33)$$

which leads to a logarithmic infinity in the second derivative of $D(\omega)$ with respect to ω.

For a reaction with four particles in a final s state the quantity $(\omega_0 - \omega_t)^2 \ln|\omega_t - \omega_0|$ is replaced by $(\omega_0 - \omega_t)^5 \ln|\omega_0 - \omega_t|$. Similar behavior of the real part of the scattering amplitude appears at the thresholds of all reactions.

An example of the application of dispersion relations that is well known in the literature is the analysis of the coherent scattering of photons in the Coulomb field of a nucleus[3] (cf. also reference 4). It is not hard to convince oneself that near $\omega = 2m$ ($\gamma \equiv \omega/2m = 1$) — the threshold for production of an electron-positron pair — the real part of the scattering amplitude has an energy dependence of the type $x^k \ln x$ ($x = \gamma - 1$). To see this we have only to examine the expression for the real part of the amplitude:

$$D(\omega) = \frac{Z^2}{m}\left(\frac{e^2}{4\pi}\right)^3 \left\{\frac{1}{\gamma\pi}\left[2C_1(\gamma) - D_1(\gamma)\right] + \frac{\gamma}{27\pi}\left[\left(109 + \frac{64}{\gamma^2}\right)E_1(\gamma) - \left(67 - \frac{6}{\gamma^2}\right)\left(1 - \frac{1}{\gamma^2}\right)F_1(\gamma)\right] - \frac{1}{9\gamma^2} - \frac{9}{4}\right\}, \tag{34}$$

where

$$C_1(\gamma) = \mathrm{Re}\int_0^\gamma \frac{\arcsin x}{x}\cosh^{-1}\left(\frac{\gamma}{x}\right)dx, \quad C_1(1) = 1.62876;$$

$$D_1(\gamma) = \mathrm{Re}\int_0^\gamma \frac{\cosh^{-1}(\gamma/x)}{(1-x^2)^{1/2}}dx, \quad D_1(1) = 1{,}83193;$$

$$E_1(\gamma) = \begin{cases} E(\gamma^{-1}), & \gamma \geqslant 1 \\ \gamma^{-1}E(\gamma) + (\gamma - \gamma^{-1})K(\gamma), & \gamma \leqslant 1 \end{cases};$$

$$F_1(\gamma) = \begin{cases} K(\gamma^{-1}), & \gamma \geqslant 1 \\ \gamma K(\gamma), & \gamma \leqslant 1 \end{cases},$$

and $K(\gamma)$ and $E(\gamma)$ are the complete elliptic integrals of the first and second kinds. As is well known, for $1 - \gamma^2 \ll 1$ [$\Lambda = \ln(4(1-\gamma^2)^{-1/2}$],

$$K(\gamma) = \Lambda + \tfrac{1}{4}(\Lambda - 1)(1-\gamma^2) + \tfrac{9}{64}(\Lambda - \tfrac{7}{6})(1-\gamma^2)^2 + \tfrac{25}{256}(\Lambda - \tfrac{37}{30})(1-\gamma^2)^3 + \ldots,$$

$$E(\gamma) = 1 + \tfrac{1}{2}(\Lambda - \tfrac{1}{2})(1-\gamma^2) + \tfrac{3}{64}(\Lambda - \tfrac{13}{12})(1-\gamma^2)^2 + \ldots, \tag{35}$$

which indeed shows that the dependence is of the form $x^k \ln x$. It is not hard to check that the scattering of light by light near the threshold of the reaction $\gamma + \gamma \rightarrow e^+ + e^-$ is a process, well known in quantum electrodynamics, for which the amplitude is characterized by a "local" anomaly (cf. Figs. 2—4 in the paper by Karplus and Neuman[5])*. The amplitude for the Compton effect near the threshold of the reaction $\gamma + e \rightarrow 2e + e^+$ has the characteristic dependence $x^2 \ln x$.

*Regarding effects of the Coulomb interaction see papers by Baz'[6] and by Fonda and Newton.[7]

5. The elastic scattering of γ rays by deuterons near the threshold for photodisintegration of the deuteron is an example of a process for which use of dispersion relations is necessary for the analysis of the anomaly near the threshold. The nonmonotonic behavior near the threshold in this case comes from the magnetic-dipole disintegration. The electric-dipole disintegration leads to appreciable changes in the energy dependence of the amplitude for elastic γ-d scattering in a certain relatively wide range of energies.

The amplitude for forward elastic γ-d scattering can be represented in the form

$$e_i' T_{ik} e_k = A\mathbf{e}'\mathbf{e} + iB\mathbf{S}[\mathbf{e}'\mathbf{e}] + \tfrac{1}{2}C[(\mathbf{Se})(\mathbf{Se}') + (\mathbf{Se}')(\mathbf{Se})] + \tfrac{1}{2}D[(\mathbf{S}[\mathbf{k}\times\mathbf{e}])(\mathbf{S}[\mathbf{k}\times\mathbf{e}']) + (\mathbf{S}[\mathbf{k}\times\mathbf{e}'])(\mathbf{S}[\mathbf{k}\times\mathbf{e}])]. \tag{36}$$

The cross section for scattering of unpolarized γ rays by unpolarized deuterons then takes the form

$$\sigma_s(0^\circ) = |A + \tfrac{2}{3}(C+D)|^2 + \tfrac{1}{18}|C+D|^2 + \tfrac{2}{3}|B|^2 + \tfrac{1}{3}|D-C|^2, \tag{37}$$

and we have

$$k\sigma_t = 4\pi\,\mathrm{Im}(A + \tfrac{2}{3}C + \tfrac{2}{3}D).$$

By means of the dispersion relation for the quantity $L = A + {}^2\!/_3 C + {}^2\!/_3 D$,

$$\mathrm{Re}\,L(\omega) = -\frac{e^2}{M_d} + \frac{2\omega^2}{\pi}\mathrm{P}\int_{\omega_d}^{\infty}\frac{\mathrm{Im}\,L(\omega')}{\omega'(\omega'^2 - \omega^2)}d\omega', \tag{38}$$

where ω_d is the threshold for photodisintegration of the deuteron, let us examine the effect of inelastic processes on the energy dependence of the real part of the amplitude L. In the calculation of the dispersion integral it is convenient to use the theoretical expressions for the cross sections for photodisintegration of the deuteron (cf., e.g., reference 8).

Let us begin with the examination of the "local effects." The expression for the cross section for magnetic-dipole disintegration is

$$\sigma_c^{(m)} = \frac{2\pi}{3}\frac{e^2}{\hbar c}\left(\frac{\hbar}{Mc}\right)^2(\mu_p - \mu_n)^2 \frac{(\gamma-1)^{1/2}(1 + \sqrt{\varepsilon'/|\varepsilon|})^2}{\gamma[\gamma - 1 + \varepsilon'/|\varepsilon|]}, \tag{39}$$

where $\gamma = \omega/|\epsilon|$, ω being the energy of the photon; $|\epsilon| = 2.22$ Mev and $\epsilon' \sim 70$ kev are the binding energies of the np system in the $^3S_1$ and $^1S_0$ states; and the rest of the notation is as usual. Because of the factor $[\gamma - 1 + \epsilon'/|\epsilon|]^{-1}$ the expression (39) does not admit of the simple analytic continuation

$$\varkappa = \sqrt{\gamma - 1} \to i\,|\varkappa|,$$

since in this case $\gamma\sigma_c^{(m)}$ goes to infinity below the threshold at $|\kappa|^2 = \epsilon'/|\epsilon|$. Substitution of Eq. (39) in the dispersion integral

$$Z_L(\gamma_0) = \varepsilon \frac{\gamma_0^2}{2\pi^2} \mathrm{P} \int_1^\infty \frac{d\gamma\sigma_c^{(m)}(\gamma)}{\gamma^2 - \gamma_0^2}$$

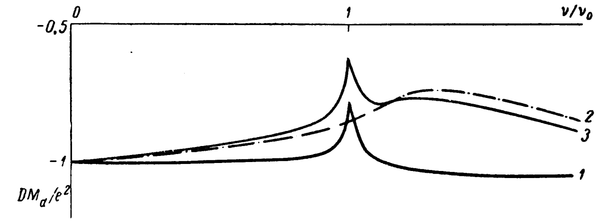
for $\gamma_0 \neq \delta$ gives

$$Z_L(\gamma_0) = \frac{2}{3} \frac{e^2}{2Mc^2} \frac{\varepsilon}{Mc^2} (\mu_p - \mu_n)^2 \left(1 + \sqrt{\varepsilon'/|\varepsilon|}\right)^2$$
$$\times \left\{ \frac{\sqrt{1-\gamma_0}}{\delta - \gamma_0} \theta(1-\gamma_0) + \frac{\sqrt{1+\gamma_0}}{\delta + \gamma_0} - \frac{2}{\delta} - \frac{2\gamma_0^2 \sqrt{\varepsilon'/|\varepsilon|}}{\delta(\delta^2 - \gamma_0^2)} \right\}, \quad (40)$$

where $\delta = 1 - \epsilon'/|\epsilon|$, $\theta(x) = 1$ for $x \geq 1$, and $\theta(x) = 0$ for $x < 1$; for $\gamma_0 = \delta$

$$Z_L(\delta) = \frac{2}{3} \frac{e^2}{2Mc^2} \frac{\varepsilon}{Mc^2} (\mu_p - \mu_n)^2 \left(1 + \sqrt{\varepsilon'/|\varepsilon|}\right)^2$$
$$\times \{\sqrt{1+\delta}/2\delta - 2/\delta + 2\sqrt{\varepsilon'/|\varepsilon|}/3\delta + \sqrt{|\varepsilon|/\varepsilon'}/2\}. \quad (41)$$

The dependence of the quantity $\Delta_L(\gamma_0) = Z_L(\gamma_0)/(e^2/2Mc^2)$ on the γ-ray energy is shown in the diagram (curve 1)



For extreme values of γ_0 we get from Eq. (40)

$$\Delta_L(\gamma_0) = \frac{2}{3} \frac{\varepsilon}{Mc^2} (\mu_p - \mu_n)^2 \left(1 + \sqrt{\frac{\varepsilon'}{|\varepsilon|}}\right)^2$$
$$\times \left\{ \frac{3}{8} + 2\frac{\varepsilon'}{|\varepsilon|} - \sqrt{\frac{\varepsilon'}{|\varepsilon|}} \left(1 + 2\frac{\varepsilon'}{|\varepsilon|}\right) \right\} \gamma_0^2$$
$$\approx \frac{2}{3} \frac{\varepsilon}{Mc^2} (\mu_p - \mu_n)^2 \left(1 + \sqrt{\frac{\varepsilon'}{|\varepsilon|}}\right)^2 \frac{1}{4} \gamma_0^2, \quad \gamma_0 \ll 1; \quad (42)$$

$$\Delta_L(\gamma_0) = -\frac{4}{3} \frac{\varepsilon}{Mc^2} (\mu_p - \mu_n)^2 \left(1 + \sqrt{\frac{\varepsilon'}{|\varepsilon|}}\right)^2, \quad \gamma_0 \gg 1. \quad (43)$$

At the photodisintegration threshold with $1 - \delta = 1/30$.

$$Z_L(1) = 0.24\, e^2 / 2Mc^2.$$

On the side of energies smaller than the threshold energy the half-width of the peak is somewhat smaller than ϵ', i.e., about 50 or 60 kev.

The contribution of the cross section for dipole absorption

$$\sigma_c^{(d)} = 4\pi \frac{e^2}{Mc^2} \frac{\hbar c}{\varepsilon} \frac{(\gamma - 1)^{3/2}}{\gamma^3} \quad (44)$$

at D = Re L is of the form

$$\Delta_p(\gamma_0) = 2Mc^2 Z_p(\gamma_0)/e^2$$
$$= 2\{\gamma_0^{-2}[(1-\gamma_0)^{3/2}\theta(1-\gamma_0) + (1+\gamma_0)^{3/2} - 2] - {}^3/_4\}. \quad (45)$$

The quantity $\Delta_p = \Delta_p(\gamma_0)$ is shown in the diagram by curve 2. In the limiting cases

$$\Delta_p(\gamma_0) = \tfrac{3}{32}\gamma_0^2, \quad \gamma_0 \ll 1; \quad (46)$$
$$\Delta_p(\gamma_p) = -\tfrac{3}{2}, \quad \gamma_0 \gg 1. \quad (47)$$

At the threshold for photodisintegration

$$\Delta_p(1) = 0{,}156.$$

The total effect of the dipole and magnetic-dipole disintegrations on the real part of the amplitude L is shown in the diagram by curve 3. Right at the threshold the effect of photodisintegration leads to a change of the amplitude by about 40 percent.

The contribution of photodisintegration of the deuteron to the polarizability of the deuteron can be seen from Eqs. (42) and (46). Since the value of the cross section for photodisintegration of the deuteron at high energies is larger than the sum of the expressions (39) and (44), the estimates obtained here can be regarded as lower limits on the quantities, although the contribution of high energies is small.

The treatment carried through here for one amplitude of the γ-d scattering can serve as an indication that inclusion of the effects of inelastic processes, and primarily those of the photodisintegration of the deuteron, in the analysis of elastic γ-d scattering can be important over a wide range of energies.

Similar effects must naturally occur also in the scattering of γ rays by heavier nuclei. A study of the elastic scattering of γ rays by nuclei shows[9] that for quite a number of elements the cross section for nuclear scattering of γ rays near the threshold of the reaction (γ, n) is characterized by a peak of considerable height with an energy width of about ±2 Mev, which is evidently due to nonmonotonic effects near the threshold. Further improvement of the accuracy of the experimental data on elastic scattering of γ rays and on the energy dependence of the cross sections of (γ, n) reactions near threshold is necessary for a more reliable analysis of this effect.

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