SOVIET PHYSICS JETP VOLUME 12, NUMBER 1 JANUARY, 1960

EXCITATION OF STANDING SPIN WAVES IN A FILM

M. I. KAGANOV

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor February 15, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 158-162 (July, 1960)

The surface resistance (impedance) of a metallic ferromagnetic film is calculated, with allowance for the spatial dispersion of the magnetic susceptibility. The influence of the skin effect and of the boundary conditions for the magnetic moment on the excitation curve of standing spin waves is examined.

SUCCESS has been achieved recently in observing a number of resonance peaks on the magnetic energy absorption curve of thin ferromagnetic films.[1] These peaks are correctly treated as evidence of the excitation of standing spin waves. Since the experiments were performed in a uniform magnetic field, Kittel,[2] to explain the observed phenomenon, proposed that because of large surface anisotropy energy, the magnetic moment at the surface coincides with its equilibrium value. In other words, according to Kittel the alternating part of the magnetic moment at the surface vanishes. Starting with this assumption, Kittel determined the frequencies of oscillation of the magnetic moment; these made it possible to determine from the excitation curve (the dependence of the energy absorption on the applied field) the magnitude of the exchange interaction, which enters into the spectrum of characteristic frequencies of the film.

In the present communication, the surface resistance of a ferromagnetic film is calculated. In this calculation the finite conductivity σ of the film is taken into account, and also the effect of the boundary conditions for the magnetic moment upon the character of the surface resistance (and by the same token on the excitation curve) is discussed.

The constant magnetic field H_0 is assumed to be directed along an axis of easiest magnetization, which is perpendicular to the film. (We choose this axis as z axis, with origin in the middle of the film; the thickness of the film is 2d.)

The linearized equation for determination of the alternating part of the magnetic moment **m** $(m_x, m_y, 0)$ has, under these conditions, the form

$$\left\{\omega - i\lambda \frac{H_e}{M_0} + i\lambda\alpha \frac{\partial^2}{\partial z^2} - gH_e + gM_0\alpha \frac{\partial^2}{\partial z^2}\right\} m^- = - gM_0\left(1 + \frac{i\lambda}{gM_0}\right) h^-. \qquad (1)$$

Here $h^- = h_x - ih_y$, where **h** is the alternating magnetic field; $H_e = H_0 + \beta M_0$, where β is the anisotropy constant; M_0 is the equilibrium magnetic moment of unit volume; ω is the frequency of the applied field; λ is the relaxation constant in the Landau-Lifshitz[3] equation of motion of the magnetic moment $(\lambda < 0)$; α is the exchange interaction constant (in order of magnitude, α is equal to $(\Theta_c/\mu M_0) a^2$, where Θ_c is the Curie temperature, μ is twice the Bohr magneton, and a is the lattice constant); g is the gyromagnetic ratio $(g > 0)$. We mention furthermore that the field inside the film is connected with the magnetic field outside it by the relation $H_0 + 4\pi M_0 = H_{ext}$. We limit ourselves to writing down the equation for m^- and h^- alone, since the circular electromagnetic wave with right-hand polarization (h^+) does not resonate with magnetic moment rotation for any value of the magnetic field H_0. The questions that interest us hereafter will be ones connected with resonance absorption of energy.

Since we wish to take account of the skin effect, it is necessary to supplement Eq. (1) with Maxwell's equations, which after elimination of the electric field can be written*

$$\partial^2 h^-/\partial z^2 + 2ih^-\delta^{-2} + 8\pi\, im^-\delta^{-2} = 0, \qquad \delta = c/\sqrt{2\pi\sigma\omega}. \qquad (2)$$

To Eqs. (1) and (2) must be added the boundary conditions. Besides the usual electrodynamic conditions (continuity of the tangential components of

*Equation (2) was obtained on the assumption that there is a normal skin effect. This seems to be a completely justifiable assumption, since the conductivity of a film as a rule is appreciably smaller than the conductivity of the metal in bulk. Formally, the use of Eq. (2) is limited by the condition $l \ll d$, where l is the mean free path of the electrons. If this condition is not satisfied, but the opposite limiting case occurs, then evidently it is necessary to replace σ in the final formulas by $\sigma_{eff} \approx \sigma d/l$.

the electric and magnetic field vectors), it is necessary to formulate boundary conditions for the magnetic moment. Following Kittel,[2] we shall assume that the alternating magnetic moment vanishes at the film surface, i.e.,

$$m^- |_{z=\pm d} = 0. \tag{3}$$

We shall be interested in the excitation of magnetic moment oscillations by a symmetric field. Therefore a solution of (1) and (2) may be sought in the form of a Fourier cosine series

$$m^-(z) = \sum_{n=0}^{\infty} m_n \cos k_n z,$$

$$h^-(z) = h^-(d) + \sum_{n=0}^{\infty} h_n \cos k_n z, \quad k_n = (n + {}^1/_2)\pi/d. \tag{4}$$

The values of k_n are so chosen as to satisfy the boundary conditions (3).

On substituting the series (4) in (1) and (2), we easily find

$$h_n = \frac{2h^-(d)(-1)^n}{\pi(n + {}^1/_2)} \times \frac{\omega - \omega_n^{(a)}(1 + i\lambda/gM_0)}{[\omega - \omega_n(1 + i\lambda/gM_0)](\delta^2 k_n^2/2i - 1) + 4\pi g M_0(1 + i\lambda/gM_0)}.$$

Here $\omega_n = gH_e + gM_0\alpha k_n^2 = gH_e + gM_0\alpha v_n^2/d^2$, $\omega_n^{(a)} = gB_e + gM_0\alpha v_n^2/d^2$, $v_n = k_n d$, $B_e = H_e + 4\pi M_0 = H_{ext} + \beta M_0$.

From the last expression and from (4) it is easy to determine the value of $\partial h^-/\partial z$ at $z = d$:

$$\left.\frac{\partial h^-}{\partial z}\right|_{z=d} = -\frac{2h^-(d)}{d} \times \sum_{n=0}^{\infty} \frac{\omega - \omega_n^{(a)}(1 + i\lambda/gM_0)}{[\omega - \omega_n(1 + i\lambda/gM_0)](\delta^2 k_n^2/2i - 1) + 4\pi g M_0(1 + i\lambda/gM_0)}. \tag{5}$$

From the equation $\text{curl}\,\mathbf{h} = (4\pi\sigma/c)\mathbf{E}$ we find that

$$E^- = -(ic/4\pi\sigma)\partial h^-/\partial z.$$

Therefore the surface impedance for waves with left-handed circular polarization is equal to

$$\zeta^- = -\frac{ic}{4\pi\sigma h^-(d)} \left.\frac{\partial h^-}{\partial z}\right|_{z=d}.$$

From this and from the expression (5) we have

$$\zeta^- = -\frac{2\omega d}{c\pi^2} \sum_{n=0}^{\infty} \frac{1}{(n+{}^1/_2)^2} \times \frac{\omega - \omega_n^{(a)}(1 + i\lambda/gM_0)}{[\omega - \omega_n(1 + i\lambda/gM_0)](1 - 2i/\delta^2 k_n^2) + (8\pi i g M_0/\delta^2 k_n^2)(1 + i\lambda/gM_0)}. \tag{6}$$

Knowing the surface resistance of the film, we can determine the amount of energy absorbed by a film of area 1 cm^2 in one second. In fact, from the expression for the Poynting vector $\mathbf{W} = (c/4\pi)[\mathbf{E}\times\mathbf{h}]$ we easily find*

$$W_z = \frac{c}{32\pi}\{\text{Im}\,\zeta^-|h^-(d)|^2 - \text{Im}\,\zeta^+|h^+(d)|^2\}.$$

From this it is clear that the power loss per unit volume of the film is

$$Q = \frac{c}{32\pi d}\{\text{Im}\,\zeta^-|h^-(d)|^2 - \text{Im}\,\zeta^+|h^+(d)|^2\}. \tag{7}$$

By comparison of formula (7) with the usual expression for the volumetric energy loss, it is possible to determine the imaginary part of the effective magnetic susceptibility ($\mu''_\pm$).

The expression (6) for the surface impedance becomes considerably simpler if the conductivity of the film approaches zero (i.e., if $\delta \to \infty$):

$$\zeta^- = -\frac{2\omega d}{c\pi^2} \sum_{n=0}^{\infty} \frac{1}{(n+{}^1/_2)^2} \frac{\omega - \omega_n^{(a)}(1 + i\lambda/gM_0)}{\omega - \omega_n(1 + i\lambda/gM_0)}. \tag{8}$$

Neglect of spatial dispersion, i.e., of the dependence of ω_n on the "wave vector" k_n, is possible for sufficiently thick specimens, when the line width $\Delta\omega \approx |\lambda| H_e/M_0$ is considerably larger than the distance between resonance frequencies (we are of course concerned only with the first few frequencies, since on account of the factor $(n + {}^1/_2)^{-2}$ the intensity of the remote absorption lines, with $n \gg 1$, is extremely small):

$$2\pi^2 \frac{\Theta_c}{\hbar} \frac{a}{d}(n+1) \ll |\lambda| \frac{H_e}{M_0}.$$

Thus on setting $n = 9$, we get the condition for neglect of spatial dispersion in the form

$$d \gg 20\pi^2 \frac{\Theta_c/\hbar}{|\lambda|} \frac{M_0}{H_e} a. \tag{9}$$

In other words, for films whose thickness satisfies the condition (9) a single resonance maximum should be observed, at frequency

$$\omega_r = gH_e = g(H_{ext} - 4\pi M_0 + \beta M_0).$$

Under these conditions

$$\zeta^- = -\frac{\omega d}{c} \frac{\omega - \omega_a(1 + i\lambda/gM_0)}{\omega - \omega_r(1 + i\lambda/gM_0)}, \quad \omega_r = gH_e, \quad \omega_a = gB_e.$$

Experiments by observation of resonance on standing spin waves are usually conducted on very thin films ($d \sim 10^{-6}$ cm). Therefore δ is always $\gg d$. By taking account of this, we can write the expression (6) in the following form:

$$\zeta^- \approx -\frac{2\omega d}{\pi^2 c} \sum_{n=0}^{\infty} \frac{1}{(n+{}^1/_2)^2} \times \frac{\omega - \omega_n^{(a)}(1 + i\lambda/gM_0)}{\omega - \omega_n(1 + i\lambda/gM_0) + (2i/\delta^2 k_n^2)(\omega_n^{(a)} - \omega)}, \tag{10}$$

*We note that in the absence of gyrotropy, $\zeta^- = i\zeta$ and $\zeta^+ = -i\zeta$.

here we have omitted from the denominator terms containing the products $(\lambda/gM_0)(\delta k_n)^{-2}$. It is clear from the expression (10) that the skin effect increases somewhat the line width but does not shift the resonance frequency appreciably. In this connection it should be mentioned that the first few frequencies (small n) are widened more than the later ones:

$$\Delta\omega_{\text{skin}} \approx (8/\pi)(d/\delta)^2 gM_0/(n+{}^1\!/_2)^2. \quad (11)$$

The expression (6) and what follows are derived on the assumption that the alternating part of the magnetic moment vanishes at the film surface. If the normal derivative of the alternating part of the magnetic moment is assumed to vanish at this surface,[4] then for $\delta \to \infty$, i.e., for $\sigma \to 0$, a uniform magnetic field will excite only uniform precession. With finite conductivity, $\sigma \neq 0$, the magnetic field inside the film is nonuniform and will excite nonuniform oscillations of the magnetic moment (spin waves). By a procedure similar to the previous one, we can obtain

$$(\zeta^-)^{-1} = -\frac{\omega d}{c}\left\{\frac{\omega-\omega_0(1+i\lambda/gM_0)}{\omega-\omega_0^{(a)}(1+i\lambda/gM_0)} - \frac{2i}{\pi^2}\left(\frac{d}{\delta}\right)^2\sum_{n=1}^{\infty}\frac{1}{n^2}\right.$$
$$\left.\times\frac{\omega-\omega_n(1+i\lambda/gM_0)}{(2i/\delta^2k_n^2)[\omega-\omega_n^{(a)}(1+i\lambda/gM_0)]-[\omega-\omega_n(1+i\lambda/gM_0)]}\right\}. \quad (12)$$

Here

$$\omega_n = gH_e + gM_0\alpha k_n^2, \quad \omega_n^{(a)} = \omega_n + 4\pi M_0, \quad k_n = n\pi/d.$$

For $d \ll \delta$, we have

$$\zeta^- \approx -\frac{\omega d}{c}\frac{\omega-\omega_0^{(a)}(1+i\lambda/gM_0)}{\omega-\omega_0(1+i\lambda/gM_0)+i(d^2/3\delta^2)4\pi M_0}, \quad (13)$$

that is, the electrical conductivity leads to an additional broadening of the fundamental (uniform) oscillation

$$\Delta\omega_{\text{skin}} \approx (d^2/3\delta^2)\,4\pi M_0. \quad (14)$$

A nonuniform magnetic field can excite standing spin waves if there are more complicated boundary conditions than (3). We suppose that at the surface a linear combination of the alternating part of the moment and of its normal derivative vanishes:

$$b\partial\mathbf{m}/\partial z_n + \mathbf{m} = 0 \qquad (z = \pm d), \quad (15)$$

$\partial/\partial z_n$ denotes differentiation along the outward normal, and b is a constant with the dimensions of length, describing the ratio between exchange forces and anisotropy forces.

On neglecting the conductivity of the film (i.e., on supposing that $\delta \gg d$), we easily obtain an expression for the mean moment excited in the film:

$$\overline{m^-} = -2gM_0(1+i\lambda/gM_0)h^-\sum_{n=1}^{\infty}\frac{\sin^2 v_n}{v_n^2(1+\sin 2v_n/2v_n)}$$
$$\times\frac{1}{\omega-\omega_n(1+i\lambda/gM_0)}, \quad (16)$$

where the v_n's are the roots of the dispersion equation

$$\cot v = qv, \qquad q = b/d, \quad (17)$$

whose solution is easily obtained graphically. If $q = 0$, $v_n = (n - {}^1\!/_2)\pi$, and we have the case treated earlier [cf. formula (8)]. If $q = \infty$, only the uniform oscillation is excited ($v_1 = 0$):

$$\overline{m^-} = -\frac{gM_0(1+i\lambda/gM_0)}{\omega-\omega_0(1+i\lambda/gM_0)}h^-.$$

When q differs from ∞ (but is not equal to zero), all the oscillations are excited, but their amplitudes decrease with order faster than in the case $q = 0$. We give an approximate expression for $q \gg 1$:

$$\overline{m^-} \approx -gM_0(1+i\lambda/gM_0)h^-\left\{\frac{1}{\omega-\omega_0(1+i\lambda/gM_0)}\right.$$
$$\left.+\frac{2}{q^2\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\frac{1}{\omega-\omega_n(1+i\lambda/gM_0)}\right\}. \quad (18)$$

In this case, obviously, the amplitude of the zero-order oscillation is many times larger than of the rest. Furthermore, the amplitude of the subsequent oscillations decreases with order as n^{-4}, whereas for $q = 0$ the amplitude decreases as $(n - {}^1\!/_2)^{-2}$. Comparison of the formulas given with the experimental resonance curves[1] seemingly indicates that the parameter q is small ($q \ll 1$).

In closing, I wish to thank V. Bar'yakhtar for very helpful discussions.

[1] M. H. Seavey and A. Tannenwald, Phys. Rev. Lett. 1, 168 (1958).

[2] C. Kittel, Phys. Rev. 110, 1295 (1958).

[3] L. Landau and E. Lifshitz, Physik. Z. Sowjetunion 8, 153 (1935).

[4] W. S. Ament and G. T. Rado, Phys. Rev. 97, 1558 (1955).

Translated by W. F. Brown, Jr.
30