ASYMMETRY OF $\pi^+ \to \mu^+ \to e^+$ DECAY IN A MAGNETIC FIELD

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The dependence of spatial asymmetry in $\pi^+ \to \mu^+ \to e^+$ decay on magnetic field strength has been measured in the range from 0 to 17 koe. It is shown that this dependence can be explained by the Paschen-Back effect for muonium upon the assumption of additional depolarization due to charge exchange toward the end of the μ^+ range or to electron exchange after stopping. For a NIKFI-R emulsion the minimum value of the asymmetry coefficient is -0.09 ± 0.01 (in zero field); the maximum value is -0.29 ± 0.01 (in strong fields). The 10% difference between -0.29 ± 0.01 and the value -0.33 predicted by the V-A theory greatly exceeds the experimental error. No three-electron decays were detected among the 340,000 $\pi^+ \to \mu - e$ decays examined.

1. INTRODUCTION

THE discovery of parity nonconservation in $\pi \to \mu \to e$ decay has led to the experimental study of the new hydrogen-like atom muonium, which is a stable system consisting of a μ^+ and an electron. This system can be formed when the velocity of a μ^+ as it slows down is comparable with the velocities of atomic electrons. With further slowing down the μ^+, like a slowed-down proton, repeatedly loses and captures an electron (charge exchange) and finally reaches thermal velocity in either a free or a bound state. A sufficiently slow μ^+ can no longer acquire or lose an electron through ionization; its interaction with electrons is then confined to electron exchange with atoms of the medium. The initial formation of muonium and its subsequent charge exchanges and exchanges with unpolarized electrons may account for the experimentally observed depolarization of μ^+ in various substances. For the purpose of testing these ideas experimentally we investigated the effect of a static magnetic field on the polarization of a μ^+; this had previously been studied in references 1—4. Since the suggested depolarization mechanism is the interaction between the μ^+ and electron magnetic moments in the muonium ground state (hyperfine splitting), a strong magnetic field which destroys the coupling between the two magnetic moments (the Paschen-Back effect) should also oppose depolarization and thus conserve the asymmetry of the angular distribution of positrons from $\mu^+ \to e^+$ decay. The quantum-mechanical theory of this effect[5] shows that enhanced polarization P with increasing field strength is represented by

$$P = 3a = \xi(0.5 + 0.5\, x^2/(1 + x^2)), \tag{1}$$

where $x = H/H_0$ is the ratio of the applied magnetic field to the field $H_0 = 1580$ oe, which is the average field due to the μ^+ magnetic moment at the electron orbit in muonium, and ξ is an asymmetry parameter which represents the degree of longitudinal μ^+ polarization in $\pi^+ \to \mu^+$ decay and which is three times as large as the asymmetry coefficient a in $\pi \to \mu \to e$ decay.[6]

It follows from this formula that in fields of the order of tens of kilooersteds ($x^2 \gg 1$) there is practically no depolarization ($P \to \xi$), and that in weak magnetic fields ($x^2 \sim 0$) polarization should amount to half of its maximum value ξ.

The present work investigates the dependence of asymmetry in $\pi^+ \to \mu^+ \to e^+$ decays within NIKFI-R emulsion on magnetic fields ranging from 0 to 20 koe. We measured the asymmetry coefficients in emulsions which were carefully shielded ($H = 0$) and in fields of 54, 110, 206, 420, 680, 1300, 1900, 2500, 3500, 5100, 6300, 14,000 and 17,000 oe parallel to the plane of the emulsion. The emulsions were placed in a double soft-iron shield for protection from the magnetic field existing in the synchrocyclotron room. The vertical magnetic field at the center of the shielded space was $< 10^{-2}$ oe. Weak fields of 54 — 420 oe were excited by an air-core coil of 16 cm inside diameter and 60 cm length placed inside the double cylindrical magnetic shield. Fields of 1900 — 6300 oe were excited by a coil of 14 cm inside diameter surrounded by a heavy closed core of iron. The field within the emulsion was uniform to within 1%. Irradiation in fields of 14 and 17 koe required the use of a special electromagnet, for which we are indebted to I. I. Gurevich and B. A. Nikol'skiĭ.

2. IRRADIATION AND SCANNING

Pellicle stacks consisting of 50 to 100 NIKFI-R emulsions 400μ thick and measuring 10 × 10 or 10 × 15 cm^2 were irradiated in a π^+ beam produced by the synchrocyclotron of the Joint Institute for Nuclear Research. The absorbers which followed the collimator were calculated to cause stopping of the π^+ mesons in the middle of the pellicle stack.

In the case of a magnetic field it is more convenient to analyze the data with the usual angular correlation formula $dN \sim (1 + a\cos\vartheta)\, d\cos\vartheta$ replaced by

$$dN \sim (1 + a\cos\gamma\cos\beta)\, d\cos\gamma\, d\cos\beta, \qquad (2)$$

where γ and β are the angles formed by the μ^+ and electron paths, respectively, with the magnetic field direction.[4] Asymmetry was then measured as follows. A scale based on two mutually perpendicular lines was placed in the field of view of the micrometer eyepiece; the magnetic field direction bisected the right angle between the lines. The observers recorded the $\pi^+ \to \mu^+ \to e^+$ decays contained entirely in a given emulsion layer when the projections of the angles γ and β on the emulsion plane lay in the interval 0 ± 45° (first quadrant of the scale) or 180 ± 45° (third quadrant) while the vertex of the $\mu \to e$ decay was at least 50μ from the emulsion or glass surface. 99,442 $\pi^+ \to \mu^+ - e$ decays satisfying these criteria were registered in different magnetic fields. N_f will denote the number of decays for which the projections of γ and β lie in the same (first or third) quadrant of the ocular scale, while N_b denotes the number of decays for which the angular projections lie in opposite quadrants. The asymmetry coefficient is then given by

$$a = K(N_f - N_b)/(N_f + N_b).$$

K = 1.57 if we assume in first approximation, correct to within a few percent, that all μ^+ tracks lie in the plane of the emulsion while the electron tracks are distributed isotropically with respect to the vertical direction.

Table I gives the measured values of N_f and N_b and the corresponding calculated asymmetry coefficients a for the magnetic fields H that were used. The value K = 1.57 was subjected to a 3% correction to allow for the actual distributions of μ^+ tracks and of positron tracks around the vertical direction.[2]

TABLE I

H, oe	N_f	N_b	$N = N_f + N_b$	$-a$
$<10^{-2}$	7363	8270	15 633	0.093±0.013
54	3005	3477	6 482	0.117±0.020
110	3133	3592	6 725	0.110±0.020
206	2897	3247	6 144	0.093±0.020
420	2778	3317	6 095	0.142±0.020
680	3921	4718	8 639	0.149±0.017
1300	4052	4830	8 882	0.142±0.017
1900	2214	2692	4 906	0.155±0.022
2500	3246	4200	7 446	0.206±0.020
3500	2854	3589	6 443	0.185±0.020
5100	2619	3339	5 958	0.196±0.020
6300	2384	3319	5 703	0.265±0.021
14 000	1333	1848	3 181	0.262±0.028
17 000	2977	4228	7 205	0.282±0.018

Note. The value of a for $H < 10^{-2}$ oe was obtained in two runs, in the first of which the entire angular distribution was measured, while in the second run the measurements were performed as for $H \neq 0$. The given value is the weighted average for the two runs.

3. DISCUSSION

Table I and the figure show the values obtained for the asymmetry coefficient in different magnetic fields. We shall first consider the asymmetry in zero field and in very strong fields, both of which are especially important for an analysis of the data.

In zero field we obtained a(0) = − 0.093 ± 0.013, which agrees with the value − 0.092 ± 0.018 given in reference 1 for the same NIKFI-R emulsion.

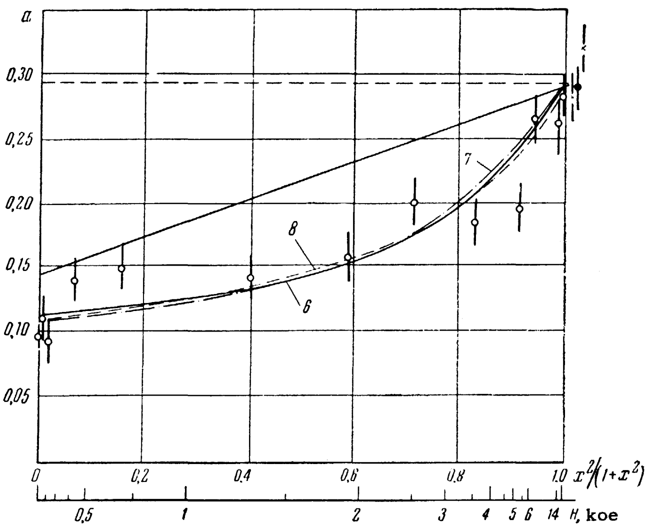
Our value for a maximum field of 17,000 oe is − 0.282 ± 0.018. Table II shows our values and all values given in the literature for strong fields.

TABLE II

Source	H, koe	$-a$
Our data	17	0.282±0.018
Reference 7	25	0.290±0.013
Reference 8	20	0.280±0.020
	27	0.320±0.200

Combining these consistent data, we obtain a = − 0.292 ± 0.009. It follows from (1) that the value of a in strong fields ($H \gg H_0$) gives directly $\xi = 3a = 0.88 \pm 0.03$, which is denoted by a dashed line in the figure.

The straight line in the figure shows the dependence of a on $x^2/(1 + x^2)$ according to (1). The experimental data disagree completely with this formula and with the hypothesis that a increases linearly; therefore another depolarization mechanism must exist in addition to that described by (1). As noted above, this mechanism may be either charge exchange or electron exchange between muonium and the medium.



Dependence of asymmetry coefficient a on $x^2/(1 + x^2)$, where $x = H/H_0$. Curves 6, 7, and 8 were calculated by means of (6), (7) and (8); o – present data, ● – from reference 7, × – from reference 8. In order to avoid crowding the figure for $x \sim 1$, the values of a for H = 20, 25 and 27 koe are shown beyond the line drawn for $x^2/(1 + x^2) = 1$.

Since the electrons of the medium are unpolarized, every electron exchange entails further "dilution" of the remaining polarization, and a few exchanges are sufficient to greatly reduce or even eliminate μ^+ polarization in weak fields. The following polarization formula given by Sens et al.[9] is a simple extension of (1) to the case in which all n exchanges occur in a time that is longer than the hyperfine splitting time but considerably shorter than the μ^+ lifetime:

$$P = \xi\,[(1 + 2x^2)/2\,(1 + x^2)]^n. \qquad (3)$$

When n = 1 this formula goes over into (1), and when $x^2 \rightarrow \infty$ we have $P \rightarrow \xi$ as in the case of (1).

A. M. Perelomov has communicated to us privately the following polarization formula, based on the assumption that electron exchanges occur throughout the entire μ^+ lifetime:

$$P = \xi\,(1 - {}^1/_2\Delta^2)\,(1 + W/\Delta^2)^{-1}, \qquad (4)$$

where $\Delta^2 = 1 + x^2$ and W is the number of exchange collisions during the mean μ^+ lifetime. As in the case of (3) this formula goes over into (1) for W = 0 and polarization is entirely restored in strong fields.

Ferrell et al.[10] considered the additional depolarization mechanism to be charge exchange while an emitted μ^+ is still fast enough to lose and capture electrons several (n) times. If each capture lasts an average time τ equal to the mean muonium lifetime (in units of $\hbar/\Delta E = 3.6 \times 10^{-11}$ sec) their formula becomes

$$P = \xi\,(1 - 0.5\,/\,(1 + \tau^{-2} + x^2))^n \qquad (5)$$

with limiting transitions similar to those of the preceding formulas.

The analysis of our data must take into account the fact that a fraction of the μ^+ stoppings occur in gelatin, where depolarization is either weak or absent, and the remainder in AgBr crystals, which are responsible for most of the depolarization.[11] Accordingly we have $f = (0.093 \pm 0.013)/(0.292 \pm 0.009) = 0.32$ as the ratio between the number of stoppings in gelatin and in AgBr. For the purpose of analyzing the data we obtain

$$P = 3a = \xi\,[f + (1 - f)\,(1 - 0.5/(1 + \tau^{-2} + x^2))^n]. \qquad (6)$$

A similar analysis can be performed for the field dependence of the asymmetry coefficient that is represented by (3) and (4), which are based on the assumption of electron exchange as the cause of additional depolarization. The collision process is then characterized by the single parameter W [Eq. (7)] or n [Eq. (8)]. The second parameter that we can obtain in this case is $k = \rho\,(1 - f)$, where $(1 - f)$ is the relative number of stoppings in AgBr and ρ is the probability of muonium formation in these stoppings.

The formulas corresponding to (3) and (4) are

$$P = \xi\left[(1 - k) + k\left\{\frac{1 + 2x^2}{2\,(1 + x^2)}\right\}^n\right], \qquad (7)$$

$$P = \xi\,[(1 - k) + k\,(1 - 1/2\Delta^2)\,(1 + W/\Delta^2)^{-1}]. \qquad (8)$$

The method of least squares was used to compare the experimental data with (6), (7), and (8), and to determine the corresponding values of the parameters. The results are shown in Table III

TABLE III

Formula	Parameters	χ^2_{min}	p,%
(6)	$\tau = 1{,}3;\ n = 7$	21	7
(7)	$W = 3{,}3;\ k = 0.7$	18	15
(8)	$n = 6;\quad k = 0.65$	18	15

and in the figure. The first column of the table indicates the formulas used in the analysis, the second column gives the optimum values of the corresponding parameters, the third column gives the values of $\chi^2_{min} = \Sigma N_i\,(\Delta a_i/1.57)^2$ corresponding to these parameters, and the fourth column gives the probabilities $p\,(\chi^2 > \chi^2_{min})$.

Satisfactory agreement is found in all three instances. Thus a few charge exchanges in a path ending or a few electron exchanges after stopping can account for the measured field dependence of the asymmetry coefficient. The difference between the values of χ^2_{min} is too small to permit any choice among the three possibilities.

4. CONCLUSIONS

1. The asymmetry coefficient increases from $a = -0.09 \pm 0.01$ in zero field to $a = -0.29 \pm 0.01$ in fields of 17 — 27 koe.

2. The measured field dependence of the asymmetry coefficient in the range 0 — 17 koe does not agree with the hypothesis that the asymmetry depends linearly on $x^2/(1 + x^2)$, which would follow from (1) for the polarization when the Paschen-Back effect occurs in muonium.

3. The data are in good agreement with the hypothesis that μ^+ depolarization occurs through muonium formation if we assume either that several electron exchanges occur after stopping or that a few charge exchanges occur toward the end of the slowing-down process. Our data do not permit us to make a distinction between the two mechanisms for additional "dilution" of the initial polarization. Electronic measurements[9] indicate that the time during which the depolarization occurs is considerably shorter than 10^{-6} sec. This suggests that (7) is to be favored over (8), although both formulas agree equally well with our data. The charge-exchange mechanism[10] also insures very rapid depolarization and its great advantage over the electron-exchange mechanism lies in its universality — charge exchange of muonium may occur in all substances, whereas electron exchange is far from being everywhere possible.

4. The observed asymmetry coefficient $a = -0.29 \pm 0.01$ in the maximum magnetic fields is 10% smaller than the value $a = -1/3$ predicted by the V-A theory of weak interactions.[12] This discrepancy is larger than the experimental error and cannot be accounted for by systematic errors. Its cause lies either in additional 10% depolarization in an emulsion that is not restored by a magnetic field (the mechanism of which is not known) or in the failure of the V-A interaction to account for asymmetry in $\mu \rightarrow e$ decay.

5. In the course of the present and earlier work[2] we have examined about 340,000 $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decays, and have hunted especially for three-electron μ^+ decays according to the schemes $\mu^+ \rightarrow 3e$ or $\mu \rightarrow e + \bar{\nu} + \nu + \gamma$ with internal conversion of a gamma ray into a pair (a few decays of this second type were observed in references 13 and 14). Although these decays should be observable with efficiency close to unity we did not find a single reliable instance, while a few apparent cases of the second type can be accounted for by chance superposition of a slow-electron track.

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[1] Gurevich, Kutukova, Mishakova, Nikol'skiĭ, and Surkova, JETP **34**, 280 (1958), Soviet Phys. JETP **7**, 195 (1958).

[2] Vaĭsenberg, Rabin, and Smirnit-skiĭ, JETP **36**, 1680 (1959), Soviet Phys. JETP **9**, 1197 (1959).

[3] Orear, Harris, and Bierman, Phys. Rev. **107**, 322 (1957).

[4] Barkas, Giles, Heckman, Inman, and Smith, Phys. Rev. **107**, 911 (1957).

[5] R. A. Ferrell and F. Chaos, Phys. Rev. **107**, 1322 (1957).

[6] T. Konishita and A. Sirlin, Phys. Rev. **108**, 844 (1957).

[7] G. Lynch and J. Orear, Preprint; Lynch, Orear, and Rosendorf, Preprint.

[8] Ali-Zade, Gurevich, Dobretsov, Nikol'skiĭ, and Surkova, JETP **36**, 1327 (1959), Soviet Phys. JETP **9**, 940 (1959).

[9] Sens, Swanson, Telegdi, and Yovanovitch, Phys. Rev. **107**, 1465 (1957).

[10] Ferrell, Lee, and Pal, Preprint.

[11] R. A. Swanson, Phys. Rev. **112**, 580 (1958).

[12] R. A. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[13] Gurevich, Nikol'skiĭ, and Surkova, JETP **37**, 318 (1959), Soviet Phys. JETP **10**, 225 (1960).

[14] J. Lee and N. P. Samios, Phys. Rev. Letters **3**, 55 (1959).

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51