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EFFECT OF WEAK INTERACTIONS ON THE ELECTROMAGNETIC PROPERTIES OF FERMIONS

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Self-mass effects and the singularities of behavior of fermions in a magnetic field, brought about by the universal Fermi interaction, are considered in the first order in the constant G_F. It is shown that parity nonconservation in weak interactions does not induce parity nonconservation in the field mass. An expression describing the effect of weak interactions on the electromagnetic properties of the fermions is found. The expression is proportional to the external current and is $\sim 10^{-2}$ times the similar expression obtained by taking into account the vacuum terms in electrodynamics.

IN the present note we consider the electromagnetic properties of fermions, brought about by the universal Fermi interaction, in the first order in e and G. The corresponding effects can, in principle, be verified experimentally in scattering processes and, by the same token, serve as a test for the existence of the universal interaction.

As is well known, writing of the Lagrangian of the Fermi interactions in the form of a simple product of two currents results in specific terms of the form

$$[p,\ n]^2 = \bar{p}\gamma_\mu(1+\gamma_5)\,n\cdot\bar{n}\gamma_\mu(1+\gamma_5)\,p,$$

$$[\nu,\ \mu]^2 = \bar{\nu}\gamma_\mu(1+\gamma_5)\,\mu\cdot\bar{\mu}\gamma_\mu(1+\gamma_5)\,\nu$$

and so forth. In particular, Zel'dovich[1] directed attention to the significance of these terms in scattering.

Let us first consider the term of local Fermi interaction of nucleons (the generalization to the case of other fermions presents no difficulties). Limiting ourselves to phenomenological account of the renormalization of the pseudovector current, due to the strong interaction, we can write the term $[p, n]^2$ in the Lagrangian in the form

$$L = \frac{G}{\sqrt{2}}\{[\bar{p}\gamma_\mu(1+\lambda\gamma_5)\,n,\ \bar{n}\gamma_\mu(1+\lambda\gamma_5)\,p]_+ + \zeta\,[\bar{p}\gamma_\mu\gamma_5 n,\ \bar{n}\gamma_\mu\gamma_5 p]_+\}, \qquad (1)$$

in the absence of renormalization (the main case considered)

$$\lambda = 1, \qquad \zeta = 0. \qquad (2)$$

Condition (2) can be satisfied for leptons; however, in this case ζ is never strictly equal to zero, because of electromagnetic effects. In general, (1) should also include terms with derivatives, but in the limit of a slowly varying electromagnetic field such an inclusion does not change the basic results obtained.

Making use of the relations put forth by Firz,[2] and taking commutation properties into account, we can rewrite Eq. (1) in the form*

$$L = -\frac{G}{\sqrt{2}}\Big\{\frac{(1+\lambda)^2+\zeta}{4}\,[\bar{p}\gamma_\mu(1+\gamma_5)\,p\cdot\bar{n}^c\gamma_\mu(1+\gamma_5)\,n^c + \bar{n}\gamma_\mu(1+\gamma_5)\,n\cdot\bar{p}^c\gamma_\mu(1+\gamma_5)\,p^c] + \frac{(1-\lambda)^2+\zeta}{4}$$
$$\times[\bar{p}\gamma_\mu(1-\gamma_5)\,p\cdot\bar{n}^c\gamma_\mu(1-\gamma_5)\,n^c + \bar{n}\gamma_\mu(1-\gamma_5)\,n\cdot\bar{p}^c\gamma_\mu(1-\gamma_5)\,p^c]$$
$$+\frac{(1-\lambda^2)-\zeta}{2}\,[\bar{p}(1+\gamma_5)\,p\cdot\bar{n}^c(1-\gamma_5)\,n^c + \bar{p}^c(1-\gamma_5)\,p^c\cdot\bar{n}(1+\gamma_5)\,n]\Big\}. \qquad (3)$$

The terms of the Lagrangian (3) have the following general form:

$$K\bar{\psi}_1 O^i\psi_1\cdot\bar{\psi}_2 O^i\psi_2, \quad O^i = (1\pm\gamma_5), \quad (1\pm\gamma_5)\gamma_\mu, \qquad (4)$$

where K is some coefficient. Here and below, summation is implied over the omitted vector indices of O^i. Now, repeating the reasoning carried out in the second half of our earlier paper,[3] we can show that the addition to the mass operator for particle 1, brought about by the presence in the Lagrangian of a component of the form (4), has, in first order in K, the form

$$\Delta M^1(x,\ x') = iKO^i\,\mathrm{Sp}\,[G^2(x,\ x')\,O^i]\,\delta(x-x'), \qquad (5)$$

*The following definition is employed: $\psi^C = \gamma_2 C\psi^*$, $C\gamma_\mu = \gamma^*_\mu C$, $C^+ = C^{-1}$; the asterisk denotes the complex conjugate.

where $G^2(x, x')$ is the causal propagation function of particles 2, which depends on the external field in the general case. Starting out from (5), one can write down the contribution to the operator of the field mass, for example, of the neutron, induced by the interaction (3). We have

$$\Delta M^n(x, x') = -\frac{G}{\sqrt{2}}\Big\{\frac{(1+\lambda)^2+\zeta}{2}\gamma_\mu(1+\gamma_5) \times i\,\mathrm{Sp}\,[G^p(x, x')\gamma_\mu(1+\gamma_5)] + \frac{(1-\lambda)^2+\zeta}{2}\gamma_\mu(1-\gamma_5)\,i\,\mathrm{Sp}\,[G^p(x, x')\gamma_\mu(1-\gamma_5)] - \frac{(1-\lambda^2)-\zeta}{2}[(1+\gamma_5)\,i\,\mathrm{Sp}\,G^p(x, x')(1-\gamma_5) + (1-\gamma_5) \times i\,\mathrm{Sp}\,G^p(x, x')(1+\gamma_5)]\Big\}\,\delta(x-x'), \qquad (6)$$

where $G^p(x, x')$ is the proton propagation function. In the absence of an external field, the first two components of (6) yield a zero result because of the invariance conditions, while the remaining component leads to the diverging field mass

$$\Delta M^n = \frac{G}{\sqrt{2}\,(2\pi)^2}[(1-\lambda^2)-\zeta]\int\frac{(dp)\,M^p}{p^2+(M^p)^2} \qquad (7)$$

or, if we introduce a cutoff in M,

$$\Delta M^n = GM^2[(1-\lambda^2)-\zeta]\,M^p.$$

The most important result is the fact that the field mass, in the case of a weak interaction, is the same for particles of different helicity i.e., parity nonconservation is not induced for free particles. This conclusion is not changed when the form of the Lagrangian (1) is made more accurate for strongly interacting particles, or when derivatives are introduced into (1).

We now consider the presence of an external electromagnetic field. In the expressions given for the field mass, it is necessary to substitute propagation functions that depend on the electromagnetic field. Here, taking account of the fact that actually $x = x'$ in (6), of the condition of gauge invariance, and of the equalities

$$\mathrm{Sp}\,\sigma_{\mu\nu} = \mathrm{Sp}\,\gamma_\alpha\sigma_{\mu\nu} = \mathrm{Sp}\,\gamma_5\gamma_\alpha\sigma_{\mu\nu} = 0,$$

$$\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu),$$

we come to the conclusion that the static moment* induced by weak interactions is equal to zero (for a constant electromagnetic field $F_{\mu\nu}$).

*Only the tensor Fermi interaction with particle 2, of the form $G'(\psi_1\gamma_\mu\gamma_\nu\psi_2\cdot\bar{\psi}_2\gamma_\mu\gamma_\nu\psi_1)$. could lead to an anomalous static magnetic moment μ_F^1 of particle 1. In this case, $\mu_F^1 = \frac{G'M_2M_1}{2\pi^2}\ln\frac{M}{M_2}\cdot\frac{e}{2M_1}$, where M is the cutoff parameter. Absence of a similar induced moment could serve as one of the indirect proofs of the exclusion of the tensor interaction.

In the case of a variable electromagnetic field, the terms induced by the weak interaction [1, 2][2] and dependent on the field do not vanish in the equations for the fermions 1 that interact with charged fermions 2. Starting out from (6), and making use of the expansion of the Green's function of the fermion in powers of the external electromagnetic field, which was given in the work of Karplus and Klein,[4] we obtain the result (after simple calculations) that a component

$$\Big\{\frac{eG_FM_2}{\sqrt{2}(4\pi)^2}[(1+\lambda+\zeta)+2\lambda\gamma_5]\int_{M^{-2}}^{\infty}\frac{dS}{S}\,e^{-M_2^2S}\int_{-1}^{1}\frac{(1-v^2)\,dv}{2} \times\int(2\pi)^{-2}d^4k\;e^{ikx}\exp\Big[-S\frac{k^2(1-v^2)}{4}\Big]\,k_\alpha\gamma_\beta F_{\alpha\beta}(k)\Big\}\psi_1(x). \qquad (8)$$

appears in the Dirac equation for particle 1. Here M is the cutoff parameter; as $M \to \infty$, we obtain a logarithmic divergence. It is important that (8) always depends weakly on the choice of M.

For a more definite representation of the size of the effect, we put down the approximate value of the quantity in curly brackets in (8) keeping only the term with the lowest field derivative for the case of weakly interacting particles (2):

$$\Big\{\Big\}_1 = \frac{-5\sqrt{2}}{3(4\pi)^2}\,G_FM_2M_1\,\frac{e}{2M_1}(1+\gamma_5)\gamma_\alpha j_\alpha(x),$$

$$j_\alpha(x) = (2\pi)^{-2}\int d^4k\;e^{ikx}k_\beta F_{\alpha\beta}(k). \qquad (9)$$

The existence of an effect corresponding to the appearance of the effective kinematic moment and described by the expression (8), can be judged by the neutrino (for $1 = \nu$, $2 = \mu$, e) or neutron ($1 = n$, $2 = p$) behavior in a variable electromagnetic field. In the latter case we must separate the component of the effect that corresponds to violation of spatial parity [see the second term in the square brackets in (8)]. It is also interesting that, according to (8), ν_e and ν_μ ought to possess essentially different properties in the electromagnetic field.

In conclusion, we note that the presence of the universal nonlinear self-action of the fermions should lead to the appearance of an expression of the form (8), where G_F is replaced by a nonlinear constant and the particles 1 and 2 become identical.

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