*RADIATION OF A PARTICLE MOVING ACROSS THE INTERFACE OF TWO MEDIA WITH ACCOUNT OF MULTIPLE SCATTERING**

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Submitted to JETP editor February 2, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 332-336 (August, 1960)

The spectral intensity distribution of the electromagnetic radiation emitted by a charged particle moving across the interface between two media is derived taking the effect of multiple scattering into account.

It was shown previously[1] that the transition radiation emitted by an ultra-relativistic particle in the direction of its motion can be greatly affected by multiple scattering in the region where the radiation originates. In the present article, formulas are given for the intensity of the electromagnetic radiation emitted by a particle entering or leaving a medium. The calculation is based on a simplified derivation of the transition-radiation equations, and on the method of taking into account the influence of multiple scattering on bremsstrahlung developed by Gol'dman.[2]

1. DERIVATION OF THE TRANSITION RADIATION FORMULAS

We shall demonstrate a simple method of deriving the formulas for the spectral intensity distribution of the transition radiation in certain interesting cases. As is well known, the energy radiated by a particle in the solid angle element $d\Omega$ in the frequency range $d\omega$ is given by the formula[3]

$$d\mathcal{E}_{\mathbf{n}\omega} = c\,|\,\mathbf{H}_\omega\,|^2 R^2 d\Omega d\omega, \quad (1)$$

$$\mathbf{H}_\omega = e\frac{i\omega e^{i\omega R/c}}{2\pi c^2 R}\int_{-\infty}^{+\infty} e^{i\,(\omega t - \mathbf{k}\mathbf{r}(t))}\,[\mathbf{n}\times\mathbf{v}\,(t)]\,dt. \quad (2)$$

where R is the distance from the point of collision to the point of observation, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the coordinate and the velocity of the particle respectively, and $\mathbf{k} = \mathbf{n}\omega/c$ is the wave vector of the emitted photon.

If the particle moves uniformly in a straight line in vacuum from $-\infty$ to $+\infty$, then the expression obtained by integrating Eq. (2) from $-\infty$ to 0 cancels the expression obtained by integrating from 0 to $+\infty$ and, as a result, no radiation is produced. If, in its motion, the particle goes from one medium into another (motion along the z axis, with the plane $z = 0$ separating the media), symmetry is lost and radiation appears.

It is evident that, at the interface of the media, radiation will be partially reflected and partially transmitted into the second medium. This radiation can easily be calculated and identified as due to the transition effect if it propagates in each of the media, after suitable refraction or reflection, from the point in which the trajectory intersects the interface between the media.

The transition radiation produced in such a process is directed both forward and backward at very small angles to the direction of motion of the particle. One can therefore expect such a calculation to yield a correct result for the case of an ultra-relativistic particle. Definite restrictions are then imposed on the angles and frequencies of the emitted radiation in each specific case. In particular, if vacuum is one of the media then, for the radiation into the vacuum, correct results are obtained for all frequencies since, in this case, $\vartheta \sim \sqrt{1-\beta^2}$.[4,5] For the "medium — medium" case, the formulas are applicable to radiation in the forward direction with frequencies greater than $\sqrt{\sigma}$, for which $\vartheta \sim \sqrt{\sigma}/\omega$ (reference 6) ($\sigma = 4\pi Ne^2/m$, $\epsilon = 1 - \sigma/\omega^2$).

It should also be noted that the wave vector $\mathbf{k}$ in Eq. (2) should be replaced in such problems, by $\mathbf{K}$, where $\mathbf{K}$ is a wave vector with a transverse component κ and a longitudinal component λ ($\lambda^2 = \omega^2\epsilon/c - \kappa^2$). As has been shown in reference 5, only such a choice of $\mathbf{K}$ permits a smooth transition of the fields at the interface of the media.

Let us now actually derive the transition radiation formulas. We shall first consider the case where the second medium is vacuum, and deter-

*Editor's note: Analogous qualitative results have been obtained independently by V. E. Pafomov, and are described in an article submitted to the editor on February 18, 1960.

mine the radiation in that medium. We should use the vacuum value[6] of κ, equal to $\omega \sin \vartheta / c$, where ϑ is the angle between the direction of radiation and the z axis. Radiation emitted in the first medium will not all penetrate into the second. Accordingly, the magnetic field intensity in the first medium should be multiplied by the Fresnel coefficient for normal incidence $2\sqrt{\epsilon}/(1+\sqrt{\epsilon})$.[7] As a result, we have

$$H_\omega = e\frac{e^{i\omega R/c} v \sin\vartheta}{2\pi c^2 R}\left\{\frac{1}{1-\beta\sqrt{\varepsilon-\sin^2\vartheta}}\frac{2\sqrt{\varepsilon}}{1+\sqrt{\varepsilon}} - \frac{1}{1-\beta\cos\vartheta}\right\}. \quad (3)$$

Substituting Eq. (3) into Eq. (1), we obtain

$$d\mathscr{E}_{n\omega} = \frac{e^2\sin^2\vartheta}{4\pi^2 c}\left(\frac{\sqrt{\varepsilon}-1}{(1-\beta\cos\vartheta)(1-\beta\sqrt{\varepsilon-\sin^2\vartheta})}\right)^2 d\Omega d\omega, \quad (4)$$

i.e., a formula identical with that given in references 5 and 6 for the ultra-relativistic case.

If the first medium is vacuum, then the radiation emitted in it will be partially reflected from the interface of the medium. It is therefore necessary to multiply the field intensity by the Fresnel reflection coefficient for normal incidence $(1-\sqrt{\epsilon})/(1+\sqrt{\epsilon})$. The radiation emitted in the second medium may be neglected. Substituting $\kappa = \omega \sin\theta / c$, where θ is the angle between the direction of propagation of the radiation and the negative direction of the z axis, we obtain the following expression for the backward radiation:

$$d\mathscr{E}_{n\omega} = \frac{e^2\sin^2\theta}{4\pi^2 c}\frac{1}{(1-\beta\cos\theta)^2}\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2 d\Omega d\omega, \quad (5)$$

i.e., the Ginzburg-Frank[4] formula for the ultra-relativistic case.

Finally, let us consider the "medium — medium" case and find the radiation in the second medium with frequency $\omega \gg \sqrt{\sigma}$. We then have $\kappa = (\omega/c) \times \sqrt{\epsilon_2}\sin\vartheta$, and the Fresnel transmission coefficient can, with good accuracy, be taken as unity. As a result we obtain the formula

$$d\mathscr{E}_{n\omega} = \frac{e^2\sin^2\vartheta}{4\pi^2 c}\left(\frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{(1-\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\vartheta})(1-\beta\sqrt{\varepsilon_2}\cos\vartheta)}\right)^2 d\Omega d\omega, \quad (6)$$

which is identical with the usual formula for the forward transition radiation for $\beta \sim 1$ and $\omega \gg \sqrt{\sigma}$.

2. EFFECT OF MULTIPLE SCATTERING

It was shown by Pomeranchuk and the author[1] that, for particles moving with velocities close to that of light, multiple scattering influences only the forward transition radiation with frequencies $\omega \gg \sqrt{\sigma}$. It is therefore sufficient to assess the influence of multiple scattering on transition radiation within the framework of the approach described in Sec. 1. In addition, such an approach is convenient because the corresponding expressions are of a form suitable for the derivation of the bremsstrahlung formulas, taking multiple scattering into account, whether the particle moves in an unbound homogeneous medium[8,9] or whether it moves from a vacuum into a medium.[2] The latter case, in which the additional radiation originating near the medium boundary is considered, in addition to the bremsstrahlung with the effects of multiple scattering, has a direct bearing on our problem. This radiation is generated, because the particle moved in the vacuum to the left of the boundary without undergoing scattering.

Let the particle move from the vacuum into the medium in which it begins to undergo multiple scattering. According to the preceding section, in order to take transition radiation into account, it is necessary, in the derivation of Gol'dman,[2] to substitute for the wave vector $\mathbf{k}$ the quantity $\mathbf{K}_1$ $(\kappa_0\omega\sqrt{\epsilon}\sin\vartheta/c,\ \omega\sqrt{1-\epsilon\sin^2\vartheta}/c)$ in vacuum, and the quantity $\mathbf{K}_2(\mathbf{k}\sqrt{\epsilon})$ in the medium, where κ_0 is the unit vector in the plane perpendicular to the z axis.

In the frequency range $\omega \gg \sqrt{\sigma}$, it can be shown that the radiation associated with the transition of the particle through the medium boundary is given by the expression

$$E_\omega = \frac{e^2}{\pi c}\operatorname{Re}\left\{2\int_0^\infty \frac{dz}{1+z}\int_0^\infty (1+i)s' e^{-(1+i)(s'x+sz\,\mathrm{th}\,x)}dx + \int_0^\infty\left(\coth x - \frac{1}{x}\right)(1-(1+i)s'x)e^{-(1+i)s'x}dx - \ln\frac{s'}{s} - 2\right\}, \qquad s = \tfrac{1}{4}(1-\beta^2)\sqrt{\omega/q},$$

$$s' = \tfrac{1}{4}(1-\beta^2+\sigma/\omega^2)\sqrt{\omega/q}, \qquad q = (c/4L)(E_s/E)^2, \quad (7)$$

where L is the radiation length and $E_s = 21$ Mev. Obviously, Eq. (7) includes both transition radiation and the radiation produced as the result of multiple scattering, i.e., bremsstrahlung. It is easy to obtain the following asymptotic expressions from Eq. (7):

$$E_\omega = \frac{e^2}{\pi c}\ln\frac{1}{s} \qquad (s' \ll 1); \quad (8)$$

$$E_\omega = \frac{e^2}{\pi c}\left[\frac{s'+s}{s'-s}\ln\frac{s'}{s} - 2\right] \quad (s' \gg 1). \quad (9)$$

As in other processes involving multiple scattering (see, e.g., the review by Feĭnberg[10]), one finds that there exists an energy $E_{cr} = (\mu c^2/E_s)^2 \times L\sqrt{\sigma}\mu c$ such that, for $E < E_{cr}$, multiple scattering does not affect the transition radiation and, consequently, there is also no additional bremsstrahlung radiation due to the existence of the boundary. To ascertain this, let us write the expression for s' in the following form (for $\omega \ll \sqrt{\sigma}E/\mu c^2$):

$$s' = \frac{1}{2}\sqrt{\frac{E_{cr}}{E}}\left(\frac{\sqrt{\sigma}\,E/\mu c^2}{\omega}\right)^{3/2}, \tag{10}$$

and for $\omega \gg \sqrt{\sigma}E/\mu c^2$ in the form

$$s' = \frac{1}{2}\sqrt{\frac{E_{cr}}{E}}\left(\frac{\omega}{\sqrt{\sigma}\,E/\mu c^2}\right)^{1/2}. \tag{11}$$

If $E < E_{cr}$, then it follows from Eqs. (10) and (11) that s' is always greater than unity, and the radiation is described by Eq. (9)

$$E_\omega = \frac{2e^2}{\pi c}\left[\left(\frac{1}{2} + \frac{\omega^2(1-\beta^2)}{\sigma}\right)\ln\left(1 + \frac{\sigma}{\omega^2(1-\beta^2)}\right) - 1\right], \tag{12}$$

i.e., the usual formula for the transition radiation. In the limiting cases of frequencies smaller and greater than $\sqrt{\sigma}E/\mu c^2$, we have, correspondingly,

$$E_\omega = \frac{e^2}{\pi c}\ln\frac{\sigma}{(1-\beta^2)\omega^2}, \tag{13}$$

$$E_\omega = \frac{e^2}{6\pi c}\left(\frac{\sigma}{(1-\beta^2)\omega^2}\right)^2. \tag{14}$$

For $E > E_{cr}$ and $\omega < \sqrt{\sigma}E/\mu c^2$, it is convenient to introduce a characteristic frequency $\omega_1 = (\sigma E/E_s)^{2/3}(L/c)^{1/3}$. It can easily be seen that $s' \gg 1$ for $\omega \ll \omega_1$, and the radiation is given by Eq. (13); i.e., there is an agreement with the conclusion of reference 1 concerning the fact that, for $E > E_{cr}$ and up to frequencies close to $\sim \omega_1$, the particle emits photons according to the usual formulas for transition radiation. For $\omega \gg \omega_1$, we have $s' \ll 1$, and, from Eq. (8), we find

$$E_\omega = \frac{e^2}{\pi c}\ln\left(\frac{2}{\sqrt{1-\beta^2}}\frac{E_s}{\mu c^2}\sqrt{\frac{c}{L\omega}}\right). \tag{15}$$

In considering the frequencies $\omega \gg \sqrt{\sigma}E/\mu c^2$, it is convenient to introduce another characteristic frequency $\omega' = (c/L)(E_sE/\mu^2c^4)^2$. For $\omega \ll \omega'$, $s' \ll 1$, and the radiation is described by Eq. (15). On the other hand, for $\omega \gg \omega'$ we have $s' \gg 1$, and Eq. (14) follows.

The energy E_{cr} in lead is equal to 400 Mev for electrons and 2.7×10^9 Bev for protons; in carbon, these energies are 7.6 Bev and 5×10^{10} Bev, respectively.

Thus, if for $E > E_{cr}$ the usual transition radiation mechanism* begins to break down at $\omega > \omega_1$ because of multiple scattering, then the multiple scattering itself leads to the appearance of a "transition" bremsstrahlung and, as a result, we have the radiation described by Eq. (15), which already extends up to frequencies $\sim \omega' > \sqrt{\sigma}E/\mu c^2$. The region in which this radiation originates is of the order of $c/\sqrt{\omega q}$ (reference 2).

*As in reference 1, by "transition radiation" is meant the radiation due to a transition of a uniformly moving charge from one medium into another.[4,11]

By means of analogous calculations, it can be shown that the radiation originating in the exit of a charged particle from a medium into a vacuum is, for frequencies $\omega \gg \sqrt{\sigma}$, described by formulas identical to those given above.

The author is indebted to I. I. Gol'dman for helpful comments.

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Translated by H. Kasha
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