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INELASTIC FINAL-STATE INTERACTIONS AND NEAR-THRESHOLD SINGULARITIES

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It is shown that non-monotonic energy variations can occur in the energy spectrum of particle a from a reaction of the type A + B → a + C + D in the neighborhood of the threshold for the reaction C + D → E + F. As an example, we analyze the spectrum of K mesons from the reaction N + N → Λ + N + K in the region of the energy of the Λ − N pair close to the threshold for the process Λ + N → Σ + N. For the process p + p → Λ + N + K we find the energy spectrum of the K mesons when the incident nucleons are unpolarized, and the polarization of the baryons when the incident nucleons are polarized.

We discuss the non-monotonic energy variations in the spectra of particles for some other reactions. In the Appendix we analyze the production of Y − K pairs in np collisions and discuss the case of a scalar K particle.

1. INTRODUCTION

It is known that in processes of production of particles an interaction between two of the particles formed affects the energy spectrum and angular distribution of the third particle. In certain cases, the effect of final-state interaction can be separated from the primary mechanism for production of the particles. This occurs when the effective radius for the primary interaction is much less than the radius of interaction of a pair of particles in the final state. In addition, if the interaction of the pair of particles with other emerging particles is weak, the interaction of the two particles in the final state can be characterized by a two-particle scattering length.

The theory of final-state interaction was applied by Migdal,[1] Brueckner and Watson,[2] and Paruntseva[3] to meson production in NN collisions. Recently Henley[4] and Feldman and Matthews[5] applied it to the analysis of the reaction

$$N + N \rightarrow Y + N + K. \qquad (1)$$

They showed that the energy spectrum of the K mesons is strongly distorted by the effect of the YN interaction.

Karplus and Rodberg[6] generalized the theory of final-state interaction to the case where the strong interaction in the final state can lead to an inelastic process.

In the present paper we shall show that in the neighborhood of the threshold for production of the Σ hyperon certain anomalies occur in the energy spectrum of the K particles formed together with the Λ hyperons. They are a new example of near-threshold anomalies which have been extensively studied in recent years.[7]

In addition to the cross section for the new inelastic process, the shape and appearance of near-threshold anomalies depend on the spin and parity of the particles. The study of these anomalies with sufficient accuracy can enable us to determine properties of the produced particles. On the assumption that the final state of reaction (1) is described by singlet and triplet s waves of the Y-N system, we analyze in the second section of the present paper the kinematics of the reaction and obtain expressions for the energy spectrum of the K mesons and the polarization of the Λ particles and nucleons when the incident beam of nucleons is polarized.

In the third section, starting from the unitarity of the S matrix and the analyticity of the reaction amplitude, we give a general formulation of the theory of inelastic final-state interactions.

In Sec. 4 we consider local near-threshold anomalies in the energy spectrum of K mesons in the reaction N + N → Λ + N + K in the neighborhood of the threshold for formation of the Σ hyperon.

In conclusion, we mention some other similar processes and discuss the possible generalization of the method developed here to these processes.

2. KINEMATICS. PHENOMENOLOGICAL ANALYSIS.

We introduce Jacobi coordinates in the final state of the three-particle system:

$$\mathrm{R} = \frac{M_N \mathbf{r}_N + M_Y \mathbf{r}_Y + M_K \mathbf{r}_K}{M_N + M_Y + M_K}, \qquad \boldsymbol{\rho} = \mathbf{r}_K - \frac{M_N \mathbf{r}_N + M_Y \mathbf{r}_Y}{M_N + M_Y},$$
$$\mathbf{r} = \mathbf{r}_N - \mathbf{r}_Y, \tag{2}$$

where M_N, M_Y and M_K are respectively the masses of the nucleon, hyperon and K meson; $\mathbf{r}_N$, $\mathbf{r}_Y$ and $\mathbf{r}_K$ are their coordinates. The momenta conjugate to $\mathbf{R}$, $\boldsymbol{\rho}$, and $\mathbf{r}$ will be $\mathbf{p}_R$, $\mathbf{p}_Y$ and $\mathbf{q}$ respectively. The total energy E in the new variables is equal to (c.m.s.)

$$E = p_Y^2/2m_Y + q^2/2\mu + M_K + M_Y - M_N; \tag{3}$$

$$M = M_N + M_Y + M_K, \qquad m_Y = \frac{M_N M_Y}{M_N + M_Y},$$
$$\mu = \frac{M_K (M_N + M_Y)}{M_K + M_N + M_Y}. \tag{4}$$

The phase volume of the final state is expressed in terms of p_Y and q as follows:

$$dJ = m_Y p_Y d\Omega_Y q^2 dq\, d\Omega_q, \tag{5}$$

where $d\Omega_y$ and $d\Omega_q$ are the solid angles for the momenta $\mathbf{p}_Y$ and $\mathbf{q}$ respectively.

To be specific, we consider the reaction

$$p + p \to \Lambda + p + K^+ \tag{6}$$

below the threshold of the reaction

$$p + p \to \Sigma^0 + p + K^+. \tag{7}$$

The admissible energy for the final state of reaction (6) in the c.m.s. does not exceed 80 Mev, so that we may assume that the particles which are formed are in an s state.

Let us represent the S matrix element in the form

$$\langle \Lambda p K^+ | S | pp \rangle = -2\pi i \delta (E_i - E_f) \langle \Lambda p K^+ | T | pp \rangle. \tag{8}$$

If the K meson is a pseudoscalar particle, the spin structure of the T matrix has the form

$$\langle \Lambda p K | T | pp \rangle = A_\Lambda (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \mathbf{k})$$
$$+ B_\Lambda \{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2, \mathbf{k}) + i([\boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2]\mathbf{k})\}$$
$$+ C_\Lambda \{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2, \mathbf{k}) - i([\boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2]\mathbf{k})\}, \tag{9}$$

where $\boldsymbol{\sigma}$ is the spin matrix, $\mathbf{k}$ is a unit vector along the direction of the incident proton: A_Λ, B_Λ, and C_Λ are scalar functions of the total energy E and the relative momentum P_Λ of the $\Lambda-N$ pair. Since there are two identical particles in the initial state, the elements of the T matrix must be antisymmetrized with respect to the two initial protons. It can be shown that this results in $B_\Lambda = 0$.

The expression for the cross section for reaction (6) with unpolarized particles has the form

$$\frac{d\sigma}{d\Omega_\Lambda d\Omega_q dT} = (2\pi)^4 \frac{E}{2(E^2 - 4M_N^2)^{1/2}}$$
$$\times (2m_\Lambda \mu)^{3/2} [T(T_{max} - T)]^{1/2}$$
$$\times [|A_\Lambda + C_\Lambda|^2 + |A_\Lambda - C_\Lambda|^2 + 2|C_\Lambda|^2], \tag{10}$$

where $T = q^2/2\mu$ is the kinetic energy of the K meson with respect to the center of mass of the $\Lambda-N$ system.

If the protons in the initial state are polarized (with polarization vector $\mathbf{P}$), the polarization vector of the Λ particle in the final state, $\mathbf{P}_\Lambda$, will be

$$\mathbf{P}_\Lambda [|A_\Lambda + C_\Lambda|^2 + |A_\Lambda - C_\Lambda|^2 + 2|C_\Lambda|^2] = 2[|A_\Lambda + C_\Lambda|^2$$
$$- |C_\Lambda|^2](\mathbf{kP})\mathbf{k} + [|A_\Lambda - C_\Lambda|^2 - |A_\Lambda + C_\Lambda|^2]\mathbf{P}. \tag{11}$$

The expression for the polarization of the nucleon in the final state differs from (11) by the sign in front of C_Λ.

3. ELASTIC FINAL-STATE INTERACTION

Let us look at the unitarity condition

$$\langle \Lambda p K | T - T^+ | pp \rangle$$
$$= 2\pi i \sum_n \langle \Lambda p K | T | n \rangle \langle n | T^+ | pp \rangle \delta (E_i - E_n), \tag{12}$$

where $|n\rangle$ is a possible intermediate state lying on the same energy surface as the initial state. Let us assume that in the region of energy considered the imaginary part of the T matrix is related mainly to strong interaction in the $\Lambda-p$ system. Then we may neglect on the right side of (12) all intermediate states except for ΛpK states, and approximately replace $\langle \Lambda pK|T|\Lambda'p'K'\rangle$ by $\langle \Lambda p|T|\Lambda'p'\rangle \langle K|K'\rangle$. This means that we are neglecting the interaction between the K meson and the $\Lambda-p$ pair.

In the low energy region the matrix element $\langle \Lambda p|T|\Lambda'p'\rangle$ is equal to

$$\langle \Lambda N | T | \Lambda' N' \rangle$$
$$= (4\pi^2 p_\Lambda m_\Lambda)^{-1} \left[\tfrac{1}{4}(3 + \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\alpha_3 + \tfrac{1}{4}(1 - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\alpha_1\right], \tag{13}$$

where

$$\alpha_3 = e^{i\delta_3}\sin\delta_3, \qquad \alpha_1 = e^{i\delta_1}\sin\delta_1, \tag{14}$$

and δ_1 and δ_3 are the scattering phases in the singlet and triplet states respectively.

Using all these assumptions and taking account of invariance under time reversal, we find from (12)

$$\operatorname{Im} A_\Lambda = \frac{\operatorname{Re}\alpha_3}{1-\operatorname{Im}\alpha_3}\operatorname{Re} A_\Lambda = \frac{\operatorname{Im}\alpha_3}{\operatorname{Re}\alpha_3}\operatorname{Re} A_\Lambda = \tan\delta_3 \operatorname{Re} A_\Lambda,$$

$$\operatorname{Im} C_\Lambda = \tan\delta_1 \operatorname{Re} C_\Lambda,$$

$$A_\Lambda = (1+i\tan\delta_3)\operatorname{Re} A_\Lambda \approx (1+ia_3p_\Lambda)\operatorname{Re} A_\Lambda,$$

$$C_\Lambda = (1+i\tan\delta_1)\operatorname{Re} C_\Lambda \approx (1+ia_1p_\Lambda)\operatorname{Re} C_\Lambda. \qquad (15)$$

From (15) we see that for $\delta \to 0$, i.e., in the absence of final-state interaction, the quantities A_Λ and C_Λ are real functions.

In the energy region we are considering, the matrix elements of the reaction matrix are functions of two quantities: E — the total energy, and ω — the total energy of the $\Lambda - p$ system. If all the singularities of the amplitude are associated with physical processes, then A_Λ and C_Λ as analytic functions of ω and E are representable in the form

$$(p_\Lambda a)^{-1} e^{i\delta(\omega)} \sin\delta(\omega) f(\omega) F_\Lambda(E), \qquad (16)$$

where $f(\omega)$ is an entire function which, for small values of the energy, can be replaced by a constant.

Thus we finally approximate A_Λ and C_Λ by expressions

$$A_\Lambda = \frac{1}{p_\Lambda a_3} e^{i\delta_3}\sin\delta_3 \cdot A_\Lambda^0, \qquad C_\Lambda = \frac{1}{p_\Lambda a_1} e^{i\delta_1}\sin\delta_1 \cdot C_\Lambda^0, \qquad (16')$$

where a_3 and a_1 are the triplet and singlet Λp-scattering lengths in the s state, while A_Λ^0 and C_Λ^0 can be regarded approximately as real functions of the total energy E alone. Consequently, taking account of the unitarity of the S matrix and the analyticity of the reaction amplitude leads directly to the main result of the theory of final-state interaction (cf., for example, the paper of Gribov[8]).

By using (16) the expressions for the reaction cross section and the polarization of the Λ particles can be represented as

$$\frac{d\sigma}{dT} = (2\pi)^4 \frac{E}{2(E^2-4M_N^2)^{1/2}} (4\pi)^2 (2m_\Lambda\mu)^{3/2} [T(T_{max}-T)]^{1/2}$$

$$\times\left[2\frac{\sin^2\delta_3}{(p_\Lambda a_3)^2}|A_\Lambda^0|^2 + 4\frac{\sin^2\delta_1}{(p_\Lambda a_1)^2}|C_\Lambda^0|^2\right], \qquad (17)$$

$$\mathbf{P}_\Lambda\left[\frac{\sin^2\delta_3}{(p_\Lambda a_3)^2}|A_\Lambda^0|^2 + 2\frac{\sin^2\delta_1}{(p_\Lambda a_1)^2}|C_\Lambda^0|^2\right] = \left[\frac{\sin^2\delta_3}{(p_\Lambda a_3)^2}|A_\Lambda^0|^2\right.$$

$$\left. + 2A_\Lambda^0 C_\Lambda^0 \frac{\sin\delta_1\sin\delta_3\cos(\delta_1-\delta_3)}{p_\Lambda^2 a_3 a_1}\right](\mathbf{kP})\mathbf{k}$$

$$- 2A_\Lambda^0 C_\Lambda^0 \frac{\sin\delta_1\sin\delta_3\cos(\delta_1-\delta_3)}{p_\Lambda^2 a_3 a_1}\mathbf{P} \qquad (18)$$

$p_\Lambda^2 = 2m_\Lambda(T_{max} - T)$. If we change the sign in front of C_Λ^0 on the right of equation (18), we obtain the expression for the polarization of the recoil nucleons. Expressions (17) and (10) can be considered as a generalization of the results of Henley, who neglected the dependence of the reaction matrix on spin.

From (17) and (18) we see that the investigation of the energy spectrum of K mesons and, in particular, of the polarization of Λ particles and nucleons is very important for the determination of the Λp-scattering lengths.

4. INELASTIC INTERACTION. NEAR-THRESHOLD SINGULARITIES.

As the energy is increased, the Σ channel is opened, and we may expect a change in the spectrum of K mesons and other quantities for the ΛKp channel. In this case, in the unitarity condition (8), we must consider as a possible intermediate state the state $|\Sigma NK\rangle$. We shall restrict ourselves to interaction in s-states.

As in the preceding section we assume that*

$$\langle \Lambda NK|T|\Sigma N'K'\rangle \approx \langle \Lambda N|T|\Sigma N'\rangle\langle K|K'\rangle,$$

and use the fact that

$$\langle \Lambda N|T|\Sigma N\rangle = [4\pi^2 p_\Lambda^{1/2} p_\Sigma^{1/2} m_\Lambda^{1/2} m_\Sigma^{1/2}]^{-1}$$

$$\times\left[\tfrac{1}{4}(3+\sigma_1\sigma_2)\beta_3 + \tfrac{1}{4}(1-\sigma_1\sigma_2)\beta_1\right], \qquad (19)$$

where the indices Λ and Σ denote quantities in the corresponding channels, while

$$p_\Sigma = [2m_\Sigma(E'-T)]^{1/2}, \qquad E' = E - M_\Sigma + M_\Lambda. \qquad (20)$$

Assuming that there are no bound states of the p-Σ system, we represent the energy dependence of β_3 and β_1 in the low-energy region in the form

$$\beta_3 = b_3 p_\Sigma^{1/2}, \qquad \beta_1 = b_1 p_\Sigma^{1/2}, \qquad (21)$$

if the internal parities of Σ and Λ are the same.

The influence of the Σ channel shows itself for the Λ channel not only as an additional term in the unitarity condition (8), but also as an additional term in the matrix element of the Λp scattering matrix proportional to p_Σ:

$$\alpha_3 = \alpha_3^0 + ic_3 p_\Sigma, \qquad \alpha_1 = \alpha_1^0 + ic_1 p_\Sigma, \qquad (22)$$

where

$$c_{1,3} = (p_\Lambda/4\pi)\sigma_{1,3}^{\Sigma,\Lambda}, \qquad (23)$$

and $\sigma_{2j}^{\Sigma,\Lambda}$ is the total cross section for the reaction $\Sigma + N \to \Lambda + N$ in the state with angular momentum j.

Using (19)-(23), we find from (8)

$$\operatorname{Im} C_\Lambda = (\operatorname{Im} C_\Lambda)_{p_\Sigma=0} + C'_\Lambda p_\Sigma,$$

$$\operatorname{Im} A_\Lambda = (\operatorname{Im} A_\Lambda)_{p_\Sigma=0} + A'_\Lambda p_\Sigma, \qquad (24)$$

*The inclusion of terms of the type <pp|T⁺|pp> <pp|T|YN′K′>, which are small for this reaction, but are necessary in other cases, complicates the expressions but does not change the fundamental result.

where ($\delta \neq \pi/2$)

$$A'_\Lambda = A^0_\Lambda (p_\Lambda/4\pi)\, \sigma_3^{\Sigma,\Lambda}(p_\Sigma = 0)\ \tan^2 \delta_3 + A^0_\Sigma b_3/\cos^2 \delta_3,$$
$$C'_\Lambda = C^0_\Lambda (p_\Lambda/4\pi)\, \sigma_1^{\Sigma,\Lambda}(p_\Sigma = 0)\ \tan^2 \delta_1 + C^0_\Sigma b_1/\cos^2 \delta_1. \quad (25)$$

The relation (24) is valid when the kinetic energy T of the K meson is less than E′. For T > E′ the production of a real Σ particle becomes impossible, and we must replace p_Σ by ik_Σ, where $k_\Sigma = \sqrt{2m_\Sigma(T - E')}$, T > E′, so that the term which depends linearly on k_Σ appears in the real part of the reaction amplitude.

The presence of terms proportional to $p_\Sigma(T < E')$ and $k_\Sigma(T > E')$ causes the derivative with respect to the energy to become infinite both in the energy spectrum of the K mesons and in the energy dependence of the polarization of Λ particles (and nucleons). The order of magnitude of these anomalies is given by (24) and (25), and their shape depends on the relative sign of A^0_Λ, A^0_Σ, $b_{3,1}$ and δ. All four cases of anomalies which have been discussed in the literature for binary reactions can also occur in this present case.

All of the expressions in Secs. 2, 3, and 4 were given for the production of particles in pp collisions. It is not difficult to generalize them to the case of np collisions. This is done in the Appendix. We also discuss there the case of a scalar K particle.

We note that, in the general case also, the quantities which replace A'_Λ and C'_Λ have terms which are directly related to the final-state interaction, as well as terms which are not caused by it.

We emphasize that the expressions obtained in the present section refer to interaction in an s state of the final system. The relatively large mass difference of the Λ and Σ hyperons makes it difficult to apply the theory of inelastic interaction to the analysis of reaction (1), but this does not change the basic assertion that there is a non-monotonic behavior in the spectrum and the causes for its occurrence.

It was shown earlier[9] that the direct analytic continuation $p_\Sigma \to ik_\Sigma$ can not be carried out when there is a resonance in the neighborhood of the threshold. In this case, it is necessary to make use of dispersion relations. Since the analytic behavior of the reaction amplitude as a function of ω is not known, we have not carried out such an analysis. However, even if such a resonance occurs, we may expect non-monotonic variation with energy for a relative energy of the Λ—N pair equal to the threshold for the new channel.

If Σ and Λ have opposite parities, the first term of the expansion in (22) starts with p_Σ^3 and only the second derivative with respect to the energy becomes infinite. Consequently, the study of threshold anomalies in the energy spectrum of K mesons with sufficiently high accuracy may prove important for determining the relative parity of the Σ and Λ particles.

5. DISCUSSION

Thus, endothermic inelastic interactions of the type C + D → E + F in the final state of the reaction A + B → a + C + D can give rise to non-monotonic variations with energy in the spectrum of the particles a, whose form can be determined from the condition of analyticity and unitarity of the S matrix. To investigate these singularities experimentally requires, of course, good accuracy and high energy resolution, but as a result of discovering them and studying them one can obtain information concerning the interaction of unstable particles, their spins and parities.

Earlier we have treated the production of hyperons and K mesons in NN collisions. We mention various other processes in which similar anomalies can occur whose study may give information concerning the interaction of unstable particles.

In the spectrum of π^+ mesons from the reaction

$$K^- + p \to \Lambda + \pi^- + \pi^+ \quad (26)$$

in the neighborhood of the threshold for

$$\pi^- + \Lambda \to \Sigma^- + \pi^0 \quad (27)$$

there will occur an anomaly whose magnitude and character will be related to K^-p scattering at low energies via the reaction amplitude (27).

In the spectrum of protons from the process for production of π mesons by K mesons

$$K^- + p \to p + \pi^0 + K^- \quad (28)$$

an anomaly may occur for an energy corresponding to the threshold for the reaction

$$\pi^0 + K^- \to \overline{K}^0 + \pi^-, \quad (29)$$

if there exist forces leading to such a reaction.

If one attempts to construct a Lagrangian for the πK interaction and does not consider interactions containing derivatives, the expression obtained

$$L_{int} = g\,(\varphi^i_\pi \cdot \varphi^i_\pi)\,(\varphi^k_\Theta \cdot \varphi^k_\Theta)$$

is invariant with respect to rotation of the isotopic spin of each of the particles, and all processes for K π scattering with charge exchange are forbidden. Under more general assumptions one does not obtain a forbiddenness for reaction (20), so that the observation of a non-monotonic variation with energy in the proton spectrum from reaction (28) would be of interest from the point of view of the study of the symmetry of the π K interaction.

Among reactions in which two π mesons participate, it is interesting to note that in the distribution of nucleons from the reactions

$$\gamma + p \to p + \pi^0 + \pi^0, \qquad \pi^- + p \to n + \pi^0 + \pi^0 \qquad (30)$$

there may occur similar anomalies for a relative energy of the π^0 mesons exceeding 9 Mev, where the charge exchange reaction

$$\pi^0 + \pi^0 \to \pi^- + \pi^+ \qquad (31)$$

becomes possible. Including threshold phenomena in the reaction (31) can have significant effects in the theory of $\pi\pi$ interaction at low energies.

The existence of a threshold in reaction (31) can lead to a non-monotonicity in the spectrum of charged π mesons from τ' decay.

$$\tau^\pm \to \pi^\pm + \pi^0 + \pi^0.$$

Analogously to the reactions (30), in the spectrum of nucleons from the reactions

$$\gamma + p \to p + K^- + K^+, \qquad \pi^- + p \to n + K^+ + K^- \qquad (32)$$

in the neighborhood of the threshold for the reaction

$$K^+ + K^- \to \overline{K}{}^0 + K^0 \qquad (33)$$

there may occur energy anomalies associated with KK interaction. Moreover, in the final state of reaction (33), there is no Coulomb interaction which might mask the non-monotonicity (cf. the paper of Newton and Fonda[10]).

In the spectrum of protons from the reaction

$$p + p \to p + p + \pi^0 \qquad (34)$$

near the threshold for

$$\pi^0 + p \to n + \pi^+$$

and in the spectrum of π^+ mesons from the reaction

$$p + p \to n + p + \pi^+ \qquad (35)$$

near the threshold for

$$n + p \to d + \pi^0$$

there will also be energy non-monotonicities.*

APPENDIX

A. PRODUCTION OF A PSEUDOSCALAR K MESON IN np COLLISION

In np collisions there are two possibilities for production of Λ particles

$$n + p \to \Lambda + p + K^0, \qquad (A.1)$$

$$n + p \to \Lambda + n + K^+. \qquad (A.2)$$

We denote the reaction amplitudes in the singlet and triplet isotopic spin states by T_0 and T_1 respectively. The reaction (A.1) is then described by the amplitude $\frac{1}{2}(T_0 + T_1)$, and the reaction (A.2) by the amplitude $\frac{1}{2}(T_1 - T_0)$. The spin dependence of the isotopic triplet amplitude is given by (9) with $B_\Lambda = 0$, while

$$\langle \Lambda NK \,|\, T_0 \,|\, NN \rangle = B_\Lambda \{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2,\ \mathbf{k}) + i\mathbf{k}\,[\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2]\}. \quad (A.3)$$

Under the assumptions made earlier we can take account of final-state interaction by setting

$$B_\Lambda = B^0_\Lambda (p_\Lambda a_3)^{-1} e^{i\delta_3} \sin \delta_3, \qquad (A.4)$$

where B^0_Λ is a function of the total energy E.

The expressions for the cross sections for production and polarization of the Λ particles have the following forms:

$$d\sigma = (2\pi)^4 \frac{E}{2\,[E^2 - 4M_N^2]^{1/2}} \frac{1}{4} (4\pi)^2 (2m_\Lambda \mu)^{3/2} [T\,(T_{max} - T)]^{1/2} dT$$
$$\times\, [\,|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 + |A_\Lambda - C_\Lambda \mp B_\Lambda|^2 + 2\,|C_\Lambda \mp B_\Lambda|^2], \qquad (A.5)$$

$$\mathbf{P}_\Lambda\,[\,|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 + |A_\Lambda - C_\Lambda \mp B_\Lambda|^2 + 2\,|C_\Lambda \mp B_\Lambda|^2]$$
$$= 2\,[\,|A_\Lambda + C_\Lambda \pm B_\Lambda|^2 - |C_\Lambda \mp B_\Lambda|^2]\,(\mathbf{kP})\,\mathbf{k}$$
$$+ [\,|A_\Lambda - C_\Lambda \mp B_\Lambda|^2 - |A_\Lambda + C_\Lambda \pm B_\Lambda|^2]\,\mathbf{P}. \qquad (A.6)$$

The plus sign in front of B^0_Λ holds for reaction (A.1), and the minus sign for (A.2). From (A.5) and (A.6) it is easy to obtain the "intensity rules":

$$d\sigma\,(np \to \Lambda p K^0) = d\sigma\,(np \to \Lambda n K^+), \qquad (A.7)$$

$$\mathbf{P}_\Lambda\,(np \to \Lambda p K^0) = \mathbf{P}_\Lambda\,(np \to \Lambda n K^+) \quad \text{for } \mathbf{P} \parallel \mathbf{k}. \qquad (A.8)$$

These relations are obtained on the assumption

*The scattering lengths for low energies of the π^0-p system differ from those obtained on the assumption of isotopic invariance because of the presence of non-monotonicities which violate isotopic invariance and are related to the reaction

$$\pi^0 + p \to n + \pi^+.$$

An estimate using dispersion relations gives a correction ~ 5%.

that one need only consider the s wave in the final state. They can be used for an experimental check of this assumption.

B. PRODUCTION OF A SCALAR K MESON IN NN COLLISIONS

In this case

$$\langle \Lambda NK | T_1 | NN \rangle = A_\Lambda, \quad \langle \Lambda NK | T_0 | NN \rangle = B_\Lambda (\boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2); \tag{B.1}$$

$$A_\Lambda = A_\Lambda^0 (p_\Lambda a_1)^{-1} e^{i\delta_1} \sin \delta_1, \quad B_\Lambda = B_\Lambda^0 (p_\Lambda a_3)^{-1} e^{i\delta_3} \sin \delta_3. \tag{B.2}$$

If we introduce

$$f(NN \to \Lambda NK) = \frac{d\sigma(NN \to \Lambda NK) \cdot 8 (E^2 - 4M_N^2)^{1/2}}{dT\, [T(T_{max} - T)]^{1/2} (2\pi)^4 E (4\pi)^2 (2m_\Lambda \mu)^{3/2}},$$

the cross section and polarization of the Λ particles in all three reactions are given by

$$f(pp \to \Lambda p K^+) = | A_\Lambda^0 |^2 (p_\Lambda a_1)^{-2} \sin^2 \delta_1,$$

$$\mathbf{P}_\Lambda (pp \to \Lambda p K) = \mathbf{P};$$

$$f(np \to \Lambda NK) = f(pp \to \Lambda p K^+) + 3 | B_0 |^2 (p_\Lambda a_3)^{-2} \sin^2 \delta_3, \tag{B.3}$$

$$\mathbf{P}_\Lambda (np \to \Lambda NK) = (| A_\Lambda |^2 - | B_\Lambda |^2)(| A_\Lambda |^2 + 3 | B_\Lambda |^2)^{-1} \mathbf{P}. \tag{B.4}$$

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74