THE DECAY $\Sigma^0 \to \Lambda^0 + 2\gamma$ *AND THE MAGNETIC MOMENT OF THE* Σ^0 *HYPERON*

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The possibility of determing the magnetic moment of the Σ^0 hyperon by comparing the decays $\Sigma^0 \to \Lambda^0 + 2\gamma$ and $\Sigma^0 \to \Lambda^0 + \gamma$ is examined.

BESIDES the known decay

$$\Sigma^0 \to \Lambda^0 + \gamma \tag{1}$$

there must also occur the decay

$$\Sigma^0 \to \Lambda^0 + 2\gamma. \tag{2}$$

The present paper is devoted to an estimate of the probability of the decay (2). As will be shown below, the ratio of the probabilities of the decays (2) and (1) is very small ($\sim 10^{-6}$) and depends strongly on the values of the magnetic moments of the Λ and Σ hyperons. The value of the magnetic moment of the Λ^0 hyperon will apparently be determined in the near future by measuring the spin flip of the Λ^0 hyperon in a strong magnetic field. This method cannot be applied in the case of the Σ^0 hyperon, however, because of the short lifetime of the latter. Using isotopic invariance, one can find the magnetic moment of the Σ^0 hyperon from the known relation $2\mu_{\Sigma_0} = \mu_{\Sigma^+} + \mu_{\Sigma^-}$, which was first obtained by Marshak, Okubo, and Sudarshan,[1] if the magnetic moments of the Σ^- and Σ^+ hyperons are known. Below we shall discuss the possibilities of a direct measurement of the magnetic moment of the Σ^0 hyperon using the decay (2).*

The process (1) is described by the Feynman graph of Fig. 1; the circle in this graph represents the totality of the virtual strong interactions. The matrix element corresponding to this graph has the form

$$M_1 = \mu \bar{u}_2 \hat{\varepsilon} \hat{k} O u_1, \tag{3}$$

where the coefficient μ may be called the magnetic moment of the transition $\Sigma \to \Lambda$; u_1 and u_2 are the spinors of the Σ and Λ hyperon, respectively; ϵ and k are the four-vectors of the polarization and the momentum of the photon ($k = k_0\gamma_0 - \mathbf{k}\boldsymbol{\gamma}$, where the γ are the Feynman matrices). The form of the operator O depends on

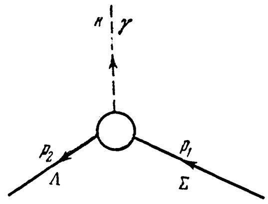

FIG. 1

the relative parity of the Σ and Λ hyperons, which we shall denote by $P_{\Sigma\Lambda}$. If $P_{\Sigma\Lambda} = -1$, we have $O = \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$; if $P_{\Sigma\Lambda} = +1$, we have $O = 1$. The probability of the decay (1), calculated with the help of the matrix element M_1, is independent of $P_{\Sigma\Lambda}$ and is equal to*

$$w_1 = 4\mu^2\Delta^3, \tag{4}$$

where $\Delta = m_{\Sigma^0} - m_{\Lambda^0} \approx 76$ Mev (here and in the following we neglect terms of order Δ/m_Σ compared to unity). Assuming that $\mu = e/2m_\pi$, where m_π is the mass of the π meson, we obtain from formula (4) $\tau = 1/w_1 \approx 4 \times 10^{-21}$ sec; if we choose $\mu = e/m_p$, then $\tau \approx 4.5 \times 10^{-20}$ sec.

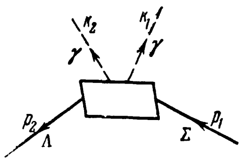

FIG. 2

Let us now turn to the decay (2). It is described, in its most general form, by the graph of Fig. 2. The box in this graph represents symbolically the set of all graphs which give a contribution to the decay (2). In the following we shall consider only a few of these graphs (Fig. 3, a and b) which are believed to give the most important contribution.

The emission of one of the photons in these graphs is, as before, due to the interaction of Fig. 1 (transition $\Sigma \to \Lambda$); the emission of the other photon is connected with the magnetic mo-

*In this connection it is also of interest to investigate the internal bremsstrahlung which accompanies the production of Σ^0 hyperons.

*We use a system of units in which $\hbar = c = 1$ and $e^2 = 1/137$. In the usual units $1/m_\pi = 4.7 \times 10^{-24}$ sec.

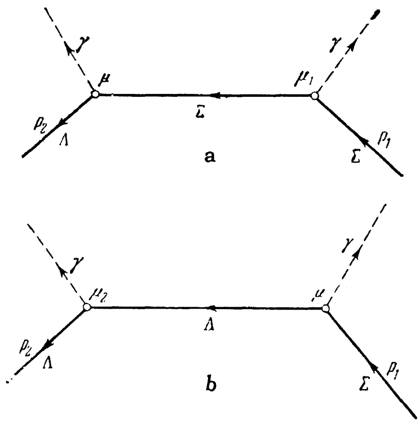

FIG. 3

ment of the Σ^0 hyperon μ_1 (Fig. 3a) and the magnetic moment of the Λ^0 hyperon μ_2 (Fig. 3b).

The graphs of Figs. 3a and 3b differ from the remaining graphs symbolically shown in Fig. 2 in that the energy dependence of the emitted photons is different. This is connected with the presence of pole terms* in the graphs of Fig. 3 which give contributions which are inversely proportional to the energy of the photon ($\sim 1/\Delta$). The remaining graphs contain, besides the term $\sim 1/\Delta$, a term of order $\sim R_0$, where R_0 is a measure of the size of the radiating region ($1/m_\pi \gtrsim R_0 \gtrsim 1/m_p$). If the mass difference of the Σ and Λ hyperons were so small in comparison with the mass of the π meson that the inequality $R_0\Delta \ll 1$ were fulfilled, the graphs 3a and 3b would, then, definitely give the most important contribution. In actuality, $\Delta/m_\pi \approx 1/2$, and the question whether the graphs 3a and 3b are the basic ones must really be decided by putting in specific numbers. This conclusion is confirmed by the computation of the contribution of the graph of Fig. 4 with a radiating region of an effective size of the order $\sim 1/m_\pi$ (see Appendix II).

The matrix element corresponding to the sum of the graphs 3a and 3b, symmetrized with respect to the photons, is equal to

$$M_2 = \mu\mu_1 \bar{u}_2 O \left[\hat{e}_2\hat{k}_2 \frac{1}{\hat{p}_1 - \hat{k}_1 - m_1} \hat{e}_1\hat{k}_1 + \hat{e}_1\hat{k}_1 \frac{1}{\hat{p}_1 - \hat{k}_2 - m_1} \hat{e}_2\hat{k}_2\right] u_1 + \mu\mu_2 \bar{u}_2 \left[\hat{e}_2\hat{k}_2 \frac{1}{\hat{p}_2 + \hat{k}_2 - m_2} \hat{e}_1\hat{k}_1 + \hat{e}_1\hat{k}_1 \frac{1}{\hat{p}_2 + \hat{k}_1 - m_2} \hat{e}_2\hat{k}_2\right] O u_1. \quad (5)$$

Here k_1, k_2 and e_1, e_2 are the momenta and the polarization vectors of the photons.

The total probability for the decay (2), calculated with the help of M_2, is equal to (see Appendix I)

*For a detailed discussion of the role of the pole terms in the emission of quanta of small energy, cf. the paper of Low.[2]

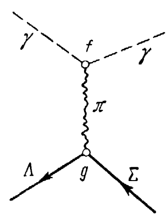

FIG. 4

$$w_2^- = (2\Delta^5/5\pi)\,\mu^2 (\mu_1^2 + \mu_2^2 + \tfrac{2}{3}\mu_1\mu_2), \quad (6)$$

for $P_{\Sigma\Lambda} = -1$, and equal to

$$w_2^+ = (14\Delta^5/45\pi)\,\mu^2 (\mu_1^2 + \mu_2^2 - \tfrac{6}{7}\mu_1\mu_2), \quad (7)$$

for $P_{\Sigma\Lambda} = +1$.

If we include only the contributions from the graphs 3a and 3b, we obtain for the ratio of the probabilities of the decays (2) and (1)

$$w_2^-/w_1 = (\Delta^2/10\pi)(\mu_1^2 + \mu_2^2 + \tfrac{2}{3}\mu_1\mu_2) \text{ for } P_{\Sigma\Lambda} = -1; \quad (8)$$

$$w_2^+/w_1 = (7\Delta^2/90\pi)(\mu_1^2 + \mu_2^2 - \tfrac{6}{7}\mu_1\mu_2) \text{ for } P_{\Sigma\Lambda} = +1. \quad (9)$$

We see that this ratio does not depend on μ at all, and depends weakly on the relative parity of the Σ and Λ hyperons.

It follows from (8) and (9) that the value of μ_{Σ^0} can be determined from the ratio w_2/w_1, if the relative parity $P_{\Sigma\Lambda}$ and the value of the magnetic moment of the Λ^0 hyperon, μ_2, are known (two solutions are possible here).

Let us consider a crude model in which the Σ^0 hyperon is represented by a Λ^0 hyperon with a π^0 meson rotating about it. In this model $\mu_1 = \mu_2$ for $P_{\Sigma\Lambda} = -1$ and $\mu_1 = -\mu_2$ for $P_{\Sigma\Lambda} = +1$. Substituting this in (8) and (9), we obtain

$$w_2^-/w_1 = 4\Delta^2\mu_1^2/15\pi, \quad (10)$$

$$w_2^+/w_1 = 2\Delta^2\mu_1^2/9\pi. \quad (11)$$

If we now assume that $\mu_1 = e/m_p$, we find

$$w_2/w_1 \approx 5 \cdot 10^{-6}. \quad (12)$$

As we shall see presently, even this small ratio is apparently an overestimate.

For this purpose let us estimate the expected value of the magnetic moments of the Λ^0 and Σ^0 hyperons. Both particles have vanishing normal magnetic moments. We are therefore concerned only with their anomalous magnetic moments due to charged virtual particles.

It is well known that the anomalous magnetic moments of baryons are in general equal to the sum of an isoscalar and an isovector term:

$$\mu = \mu_S + T_3\mu_V. \quad (13)$$

Thus we have for nucleons

$$\mu_p = \mu_S + \mu_V = 1{,}79 \text{ nucl. magnetons},$$

$$\mu_n = \mu_S - \mu_V = 1{,}91 \text{ nucl. magnetons},$$

which leads to $\mu_S = -0.06$ nucl. magnetons and $\mu_V = 1.85$ nucl. magnetons; therefore $\mu_S/\mu_V \sim 3\%$. It can be easily shown that only the isoscalar part is different from zero for Λ^0 and Σ^0 hyperons. In the case of the Σ^0 hyperon this is seen by setting $T_3 = 0$ in (13); in the case of the Λ^0 hyperon it is an obvious consequence of the fact that the isospin of the Λ hyperon is zero. If we assume that $\mu_S \ll \mu_V$ for the hyperons as well as for the nucleons, we should expect that the magnetic moments of the Λ^0 and Σ^0 hyperons are about an order of magnitude smaller than the anomalous magnetic moments of the Σ^+ and Σ^- hyperons, and the latter should, according to formula (13), have about equal magnitude and opposite sign.

The inequality $\mu_S \ll \mu_V$ follows readily from the assumption that the main contribution to the anomalous magnetic moments of the nucleons is due to the virtual π mesons. This assumption is quite realistic, since the π mesons are the lightest among the strongly interacting particles, and it is these which determine the size of the cloud of virtual particles surrounding the nucleon. Since the interaction of the π mesons with the electromagnetic field is of the isotopic vector type, we obtain $\mu_S = 0$, $\mu_V \neq 0$ in the approximation in which the electric charges of all virtual particles except the π mesons are "excluded." In order of magnitude we should have $\mu_V \lesssim e/2m_\pi$. The approximation in which only the electric charges of the π mesons (and of the Σ hyperons, whose interaction with the electromagnetic field is also of the isotopic vector type) are "included" will be called the isovector approximation.

This approximation was first considered by Katsumori;[3] he formulated his results in the form of a "mirror theorem," according to which

$$\mu_p + \mu_n = 0, \quad \mu_{\Xi^-} + \mu_{\Xi^0} = 0,$$

$$\mu_{\Sigma^-} + \mu_{\Sigma^+} = 2\mu_{\Sigma^0} = 0, \quad \mu_\Lambda = 0.$$

It is easily seen that in the isovector approximation μ, the magnetic moment of the transition $\Sigma \rightarrow \Lambda + \gamma$, is different from zero, unlike μ_{Λ^0} and μ_{Σ^0}. In the isovector approximation the decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ is therefore allowed, while the decay $\Sigma^0 \rightarrow \Lambda^0 + 2\gamma$ is forbidden (since $\mu_\Lambda = \mu_{\Sigma^0} = 0$). It may turn out that this forbiddenness comes only from the special form of the graphs 3a and 3b and that there exist other graphs which give a non-vanishing contribution to the matrix element of the decay (2) in the isovector approximation. If this were so, then our starting assumption that the graphs 3a and 3b are the most important ones would be incorrect. It can be easily shown, however, that not only the graphs 3a and 3b, but also the most general graph 2, gives a vanishing contribution in the isovector approximation. Indeed, the isotopic structure of the matrix element corresponding to the graph 2 has in the isovector approximation the form

$$\langle \bar{0} | 1 | n \rangle \langle \bar{n} | 1 | 1 \rangle, \tag{14}$$

where $\langle \bar{f} | 1 | i \rangle$ is the matrix element for the transition from the state i to the state f due to an interaction with T = 1; $|1\rangle$ and $\langle \bar{0}|$ designate the isospin of the initial (Σ) and final (Λ) states; n is the isospin of the intermediate states. The matrix element $\langle \bar{0} | 1 | n \rangle$ is different from zero only if n = 1; but for n = 1 the matrix element $\langle \bar{n} | 1 | 1 \rangle$ vanishes, since $T_3 = 0$ for the initial and final states. The decay $\Sigma^0 \rightarrow \Lambda^0 + 2\gamma$ is therefore strictly forbidden in the isovector approximation.

Let us now turn to the estimate of the ratio w_2/w_1. On the basis of the above-mentioned isospin considerations we may conclude that this ratio should be of order 10^{-7}. Such a small value for the ratio w_2/w_1 makes it unlikely that the decay $\Sigma^0 \rightarrow \Lambda^0 + 2\gamma$ can be observed with present experimental techniques.

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APPENDIX I

Let us compute the spectra of the secondary particles in the decay (2). Summing over the spins of the photons and the Λ^0 hyperon and averaging over the spin of the Σ^0 hyperon in the standard way, we obtain for the differential probability of the decay (2)

$$w_2 = \frac{2}{(2\pi)^3 m_1} \overline{|M_2|^2} \delta^4 (p_1 - p_2 - k_1 - k_2) \frac{d\mathbf{k}_1}{\omega_1} \frac{d\mathbf{k}_2}{\omega_2} \frac{d\mathbf{p}_2}{E_2}, \tag{I.1}$$

$$\begin{aligned} \overline{|M_2|^2} = {} & \mu^2\mu_1^2 \Big[(p_1k_1)(p_2k_1) + (p_1k_2)(p_2k_2) - m_1(m_1 \pm m_2)(k_1k_2) \\ & + \frac{m_1^3(m_1 \pm m_2)(k_1k_2)^2}{2(p_1k_1)(p_1k_2)} \Big] + \mu^2\mu_2^2 \Big[(p_1k_1)(p_2k_1) + (p_1k_2)(p_2k_2) \\ & - m_2(m_2 \pm m_1)(k_1k_2) + \frac{m_2^3(m_2 \pm m_1)(k_1k_2)^2}{2(p_2k_1)(p_2k_2)} \Big] \\ & + \mu^2\mu_1\mu_2 \Big\{ \mp [2(p_1k_1)(p_1k_2) + 2(p_2k_1)(p_2k_2) \\ & - (k_1k_2)(m_1 \mp m_2)^2] + m_1m_2(k_1k_2) \Big[(p_1k_1)(p_1k_2) \\ & + (p_2k_1)(p_2k_2) - \frac{1}{2}(k_1k_2)(m_1^2 + m_2^2) \Big] \\ & \times \Big[\frac{1}{(p_1k_1)(p_2k_1)} + \frac{1}{(p_1k_2)(p_2k_2)} \Big] \Big\}, \end{aligned} \tag{I.2}$$

where the upper signs correspond to the case $P_{\Sigma\Lambda} = +1$ and the lower signs to the case $P_{\Sigma\Lambda} = -1$.

The further calculations will be carried out with an accuracy up to order Δ/m. The photon spectrum has the form

$$w(\omega)\,d\omega = 4\mu^2\pi^{-1}\left[\mu_1^2 + \mu_2^2 + \tfrac{2}{3}\mu_1\mu_2\right] \times \left[\omega^2 + (\Delta - \omega)^2\right]\omega(\Delta - \omega)\,d\omega, \quad \text{(I.3)}$$

for $P_{\Sigma\Lambda} = -1$, and

$$w(\omega)\,d\omega = 4\mu^2\pi^{-1}\{[(\mu_1^2 + \mu_2^2)(\omega^2 + (\Delta - \omega)^2) - \tfrac{2}{3}\omega(\Delta - \omega)] + \mu_1\mu_2[\tfrac{2}{3}(\omega^2 + (\Delta - \omega)^2) - 4\omega(\Delta - \omega)]\}\omega(\Delta - \omega)\,d\omega, \quad \text{(I.4)}$$

for $P_{\Sigma\Lambda} = +1$.

The angular distribution of the photons is given by

$$w(\cos\vartheta)\,d\cos\vartheta = (\mu^2\Delta^5/5\pi)\left[\mu_1^2 + \mu_2^2 + \mu_1\mu_2\left(1 + \tfrac{4}{3}\cos\vartheta - \cos^2\vartheta\right)\right]d\cos\vartheta, \quad \text{(I.5)}$$

if $P_{\Sigma\Lambda} = -1$, and by

$$w(\cos\vartheta)\,d\cos\vartheta = (\mu^2\Delta^5/15\pi)\left[(\mu_1 + \mu_2)^2(2 + \cos^2\vartheta) - \mu_1\mu_2(1 + 3\cos^2\vartheta)\right]d\cos\vartheta, \quad \text{(I.6)}$$

if $P_{\Sigma\Lambda} = +1$.

The spectrum of the Λ hyperons has the form

$$w^-(x)\,dx = \frac{\mu^2\Delta^5}{\pi}\left[(\mu_1^2 + \mu_2^2)\left(1 + \frac{x^2}{3}\right) + 2\mu_1\mu_2\left(-5 + \frac{17}{3}x^2 + \frac{2(1 - x^2)}{x}\ln\frac{1 + x}{1 - x}\right)\right]x^2dx, \quad \text{(I.7)}$$

for $P_{\Sigma\Lambda} = -1$, and

$$w^+(x)\,dx = \frac{\mu^2\Delta^5}{\pi}\left[(\mu_1^2 + \mu_2^2)\left(-1 + \frac{7}{3}x^2 + \frac{(1 - x^2)^2}{x}\ln\frac{1 + x}{1 - x}\right) + 2\mu_1\mu_2\left(-5 + \frac{13}{3}x^2 + \frac{2(1 - x^2)}{x}\ln\frac{1 + x}{1 - x}\right)\right]x^2dx, \quad \text{(I.8)}$$

for $P_{\Sigma\Lambda} = +1$. Here $x = p/\Delta$ (p is the momentum of the Λ hyperon).

Integrating formulas (I.3) to (I.8) we easily obtain the formulas (6) and (7) quoted in the text.

APPENDIX II

Let us now estimate the contribution of the graph 4 to the probability of the decay $\Sigma \to \Lambda + 2\gamma$.

This graph describes the process (2) as going through the decay of a virtual π^0 meson. If $P_{\Sigma\Lambda} = +1$, the virtual π^0 meson is in a P state and the contribution of the graph 4 is negligibly small. Therefore, we consider only the case $P_{\Sigma\Lambda} = -1$. The matrix element corresponding to the graph 4 is

$$M_2' = 4\pi i\frac{fg}{m_\pi}\bar{u}_2u_1\left[(k_1 + k_2)^2 - m_\pi^2\right]^{-1}\varepsilon_{\alpha\beta\gamma\delta}\,e_{1\alpha}e_{2\beta}k_{1\gamma}k_{2\delta}, \quad \text{(II.1)}$$

where m_π is the mass of the π meson, g is the constant of the strong $\Sigma\Lambda\,\pi$ coupling, and the dimensionless constant f characterizes the decay of the π^0 meson ($\pi^2f^2m_\pi\tau_0 = 1$, where τ_0 is the lifetime of the π^0 meson). If both M_2' and M_2 [formula (5)] are included, we obtain for w_2^-, instead of formula (6),

$$w_2^- = \frac{2\Delta^5}{5\pi}\mu^2\left(\mu_1^2 + \mu_2^2 + \frac{2}{3}\mu_1\mu_2\right) + \frac{32\pi}{105}f^2g^2\frac{\Delta^7}{m_\pi^6}C_1 - \frac{8}{90}fg\,\mu(\mu_1 - \mu_2)\frac{\Delta^6}{m_\pi^3}C_2, \quad \text{(II.2)}$$

The coefficients C_1 and C_2 are corrections which take account of the term $(k_1 + k_2)^2$ together with m_π^2 in the π-meson Green's function:

$$C_1 = \frac{105}{16\lambda^7}\left[\frac{2}{3}\lambda^3 - 5\lambda + \frac{5 - 4\lambda^2}{(1 - \lambda^2)^{1/2}}\tan^{-1}\frac{\lambda}{(1 - \lambda^2)^{1/2}}\right] = 1.6. \quad \text{(II.3)}$$

$$C_2 = \left[\frac{45}{8\lambda^5}\int_0^\lambda\frac{\ln[1 - \lambda^2 + z^2]}{z^2 - \lambda^2}dz - \frac{15}{8\lambda^2} - \frac{45}{8\lambda^4}\right] = 1.2, \quad \text{(II.4)}$$

where $\lambda = \Delta/m_\pi$.

Choosing $\tau_0 = 10^{-16}$ sec, $\mu \sim e/m_\pi$, and $\mu_\Lambda \sim \mu_{\Sigma^0} \sim e/10m_p$, we find that the contribution of the graph 4 becomes important when $g \approx 1$.

[1] Marshak, Okubo, and Sudarshan, Phys. Rev. 106, 599 (1957).

[2] F. E. Low, Phys. Rev. 110, 974 (1958).

[3] H. Katsumori, Progr. Theor. Phys. 18, 375 (1957).

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76