ON THE GROWTH OF MAGNETOHYDRODYNAMIC WAVES IN A PLASMA STREAM MOVING THROUGH AN IONIZED GAS

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A dispersion equation is obtained which describes the propagation of electromagnetic waves in a plasma stream moving through an ionized gas. The low-frequency case (below the ion gyrofrequency), when the waves degenerate into magnetohydrodynamic waves, is considered in detail. It is shown that the system becomes unstable if the flow velocity exceeds the velocity of Alfven waves in the stream-plus-stationary-plasma system. In this case one of the normal waves builds up with time.

We consider the problem of magnetohydrodynamic waves in a plasma stream moving with a velocity V_0 through a stationary ionized gas in the presence of a uniform external magnetic field H_0. The velocity of the stream, $\mathbf{V}_0$, is everywhere parallel to the lines of force of this field. The plasma in the infinite stream (number density N_s) and the plasma in the stationary medium (number density N_p) are assumed to be fully ionized and quasi-neutral.

The growth of high-frequency electromagnetic waves in ion and electron streams moving through plasmas have been studied in a number of works.[1-4] We investigate under what conditions the system (plasma stream plus stationary plasma) produces magnetohydrodynamic waves which increase in time and, consequently, when the system becomes unstable.

The linearized equations for the electrodynamic processes in this system are[5,6]

$$\frac{\partial \mathbf{j}_s}{\partial t} + (\mathbf{V}_0\nabla)\,\mathbf{j}_s + \nu_{ei}^s\,\mathbf{j}_s + \omega_H\,[\mathbf{j}_s \times \boldsymbol{\tau}] = \frac{e^2 N_s}{m_e}\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}_0 \times \mathbf{h}] + \frac{1}{c}[\mathbf{v}_s \times \mathbf{H}_0]\right), \qquad (1)$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{V}_0\nabla)\,\mathbf{v}_s = \frac{1}{\rho_s c}[\mathbf{j}_s \times \mathbf{H}_0], \qquad (2)$$

$$\frac{\partial \mathbf{j}_p}{\partial t} + \nu_{ei}^p\,\mathbf{j}_p + \omega_H\,[\mathbf{j}_p \boldsymbol{\tau}] = \frac{e^2 N_p}{m_e}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_p \times \mathbf{H}_0]\right), \qquad (3)$$

$$\frac{\partial \mathbf{v}_p}{\partial t} = \frac{1}{\rho_p c}[\mathbf{j}_p \times \mathbf{H}_0], \qquad (4)$$

$$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t}, \qquad (5)$$

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial(\mathbf{j}_p + \mathbf{j}_s)}{\partial t}. \qquad (6)$$

Here, $\mathbf{j}_s$, $\mathbf{v}_s$, and $\rho_s = (m_e + m_i)N_s$ are the electric current density, the velocity, and the mass density of the ionized gas in the stream; $\mathbf{j}_p$, $\mathbf{v}_p$ and ρ_p are the analogous quantities for the stationary plasma; ν_{ei} is the frequency of collisions of electrons with ions; $\omega_H = eH_0/m_ec$ is the gyrofrequency of the electron (m_e is the electron mass); $\mathbf{E}$ and $\mathbf{h}$ are the electric and magnetic field strengths; $\boldsymbol{\tau}$ is a unit vector in the direction of the magnetic field $\mathbf{H}_0$; the subscripts s and p relate to quantities measured in the streaming and stationary plasma, respectively. Equations (2) and (4) are the hydrodynamic equations of a plasma and the equations for the current [(1) and (3)] express the generalized Ohm's law in the moving and stationary plasmas. Equation (6) is easy to derive from Maxwell's equations. In contrast to the equations given in reference 5 and 6, we have retained the total time derivative d/dt in the equations for the currents $\mathbf{j}_s$ and $\mathbf{j}_p$, as this is important in the case we are interested in.

Equations (1) to (6) lead readily to a dispersion relation for plane electromagnetic waves propagated along the flow and, consequently, along the lines of the magnetic field, $\mathbf{H}_0$. We use a cartesian coordinate system with the z axis directed along the magnetic field. We introduce the symbols $E_\pm = E_x \pm iE_y$, $j_{p\pm} = j_{px} \pm ij_{py}$ and similar symbols for j_s, v_s and v_p. By using these symbols and assuming that all the quantities vary as $\exp\{i(\omega t - kz)\}$, we obtain the following dispersion equation:

$$\omega^2 - c^2k^2 = \frac{\omega_{se}^2(\omega - kV_0)}{\omega - kV_0 \pm \omega_H - i\nu_{ei}^s - \omega_H\Omega_H/(\omega - kV_0)} + \frac{\omega_{pe}^2\,\omega}{\omega \pm \omega_H - i\nu_{ei}^p - \omega_H\Omega_H/\omega}, \qquad (7)$$

where

$$\omega_{se} = (4\pi e^2 N_s / m_e)^{1/2},$$

$$\omega_{pe} = (4\pi e^2 N_p / m_e)^{1/2}, \qquad \Omega_H = eH_0 / m_i c.$$

The plus sign corresponds to the ordinary wave and the minus sign to the extraordinary wave. In the absence of a flow ($\omega_{se} = 0$), (7) becomes the well-known equation for electromagnetic waves propagated in a plasma along a magnetic field.[7] If the entire plasma moves, when $\omega_{pe} = 0$, we arrive, neglecting ions, at the equation derived by Twiss.[4]

We consider the usual approximation of magnetohydrodynamics — the case of very low frequencies ($\omega \ll \Omega_H$, $\omega/k \ll c$) — and neglect collisions for simplicity.[7] Furthermore, we assume that $kV_0 \ll \Omega_H$; Eq. (7) then takes on the much simpler forms

$$k^2 = (\omega - kV_0)^2 / V_{As}^2 + \omega^2 / V_{Ap}^2, \tag{8}$$

where we have introduced the Alfven velocity,

$$V_{As}^2 = \omega_H \Omega_H c^2 / \omega_{se}^2 = H_0^2 / 4\pi\rho_s, \; V_{Ap}^2 = H_0^2 / 4\pi\rho_p.$$

The solution to Eq. (8) can be written in the form

$$\frac{\omega}{k} = \frac{V_0 \pm V_A \sqrt{(N_p / N_s)(1 - V_0^2 / V_A^2)}}{1 + N_p / N_s}, \tag{9}$$

where $V_A^2 = V_{As}^2 + V_{Ap}^2$. Thus, if the velocity of the stream exceeds the Alfven velocity ($V_0 > V_A$), waves appear in the stationary-plus-streaming plasma system. These waves build up in time, i.e., the system becomes unstable. This type of magnetohydrodynamic instability is different from the instabilities of plasma streams under electromagnetic perturbations of relatively high frequencies $\omega \geq \Omega_H$.[1-4] We observe that a similar criterion for instability occurs in magnetohydrodynamics when the instability of a tangential discontinuity is considered.[8]

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