PROBABILITIES OF ROTATIONAL GAMMA TRANSITIONS OF TYPE E2 AND QUADRUPOLE MOMENTS OF DEFORMED NUCLEI WITH K = 1 *AND* ½

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Expressions are given for B(E2) and Q for cases where these quantities depend not only on the direct matrix element, but also on the cross matrix element.

THE quadrupole moments Q of deformed nuclei and the reduced probabilities B for γ transitions of type E2 between levels of a rotational band may have anomalous values for the case where K = 1 or ½ (K is the projection of the total angular momentum on the nuclear symmetry axis). For K = 1 and ½ the quantities Q and B(E2) depend not only on the intrinsic quadrupole moment $Q_0 \equiv \langle\chi_K|\hat{Q}|\chi_K\rangle$ but also on the cross matrix element $\langle\chi_{-K}|\hat{Q}|\chi_K\rangle$ ($\hat{Q}$ is the quadrupole moment operator, χ_K is the function characterizing the internal state of the nucleus).

The anomaly in the values of Q and B(E2) in nuclei with K = 1 and ½ is similar to the well-known anomaly in the magnetic moments μ and reduced probabilities for rotational transitions, B(M1), in nuclei with K = ½, and is due to the equivalence of positive and negative directions of the nuclear axis.

The quadrupole moment of a nucleus with angular momentum I and projection K = 1 can be written as follows:

$$Q = Q_0 \frac{3 - I(I+1)[1 + (-1)^I 3b_0]}{(I+1)(2I+3)}. \qquad (1)$$

In particular, for the ground state of the rotational band where I = K = 1,

$$Q = Q_0(1 + 6b_0)/10. \qquad (2)$$

The coefficient b_0 characterizes the ratio of the matrix elements $\langle\chi_{-1}|\hat{Q}|\chi_1\rangle / \langle\chi_1|\hat{Q}|\chi_1\rangle$. The reduced probabilities for E2 γ transitions between levels of a rotational band with K = 1 can be expressed in terms of the same parameters Q_0 and b_0. For transitions with I + 1 → I,

$$B(E2) = \frac{5}{16\pi} e^2 Q_0^2 (C^{IK}_{I+1K;\,20})^2 [1 - (-)^{I-K}(I+1) b_0]^2, \qquad (3)$$

For transitions with I + 2 → I,

$$B(E2) = \frac{5}{16\pi} e^2 Q_0^2 (C^{IK}_{I+2K;\,20})^2 [1 + (-)^{I-K} b_0]^2, \qquad (4)$$

where $C^{\cdot\cdot\cdot}_{\cdot\cdot\cdot}$ are Clebsch-Gordan coefficients.

In the case of K = ½, the quadrupole moment of the ground state of the rotational band I = K = ½ is identically equal to zero. The expressions for the reduced probabilities B(E2) for transitions between levels of a rotational band with K = ½ have the same form as (3) and (4), and differ only in the values of I and K. The coefficient b_0 characterizes the ratio $\langle\chi_{-1/2}|Q|\chi_{1/2}\rangle / \langle\chi_{1/2}|Q|\chi_{1/2}\rangle$.

The magnitude and sign of the coefficient b_0 are determined by the internal state of the nucleus. In particular cases it may turn out that the cross matrix element is small compared to Q_0. However, there is no basis for assuming that $b_0 \ll 1$ for all nuclei. Therefore the measurement of a single value for B(E2) in nuclei with K = 1 and ½ is insufficient for determining the intrinsic quadrupole moment and the deformation parameter of the nucleus.

There are a considerable number of deformed nuclei known at present which have states with K = 1 or ½. But the corresponding experimental data are available only for five nuclei with K = ½. These data are given in the table. There we also give the theoretical values calculated on the assumption that $b_0 = 0$. Agreement of experi-

Nucleus	Ratio	Theory for $b_0 = 0$	Experiment
Yb^{171}	$\frac{B(E2, 1/2 \to 5/2)}{B(E2, 1/2 \to 3/2)}$	1.50	1.49 [1]
Tm^{169}	$\frac{B(E2, 5/2 \to 3/2)}{B(E2, 5/2 \to 1/2)}$	0.28	0.31 [2,3]
W^{183}	$\frac{B(E2, 5/2 \to 3/2)}{B(E2, 5/2 \to 1/2)}$	0.28	0.52 [4,5]
U^{235}	$\frac{B(E2, 5/2 \to 3/2)}{B(E2, 5/2 \to 1/2)}$	0.28	0.16 [6,7]
Pu^{239}	$\frac{B(E2, 5/2 \to 3/2)}{E(E2, 5/2 \to 1/2)}$	0.28	1.04 [8,9]

mental and theoretical values occurs only for Yb^{171} and Tm^{169}. For the other nuclei one observes deviations, i.e., b_0 and consequently also $\langle \chi_{-K} | \hat{Q} | \chi_K \rangle$ are different from zero. However, the precision of the available experimental data is low, so that accurate measurements of Q and B(E2) for nuclei with K = 1 and $1/2$ would have considerable interest.

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