ENERGY AND ANGULAR DISTRIBUTIONS FROM DECAYS OF HYPERNUCLEI

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The energy and angular distribution for the decay $He^4_\Lambda \rightarrow He^3 + p + \pi^-$ is calculated in the impulse approximation. It is shown that by including the interaction in the proton — He^3 system one obtains satisfactory agreement with the experimental data. The energy distribution of π^- mesons from the $He^3_\Lambda \rightarrow d + p + \pi^-$ decay is also discussed briefly.

RECENTLY there has been obtained quite extensive experimental material concerning three-particle π^- meson decays of light hypernuclei.[1,2] It appears that the energy and angular distributions for such decays have a whole variety of interesting properties. Thus, for example, in the overwhelming majority of cases the energy of the π^- mesons lies in a narrow range near the upper limit of the energy spectrum; for He^5_Λ and He^4_Λ one observes a marked asymmetry in the angular distribution, etc. According to the proposal of Dalitz, such a situation is related to the presence of a strong interaction in the system of products of the decay of the hypernucleus.

In the present work, we consider the effect of strong interaction on the energy and angular spectra of the decays

$$He^4_\Lambda \rightarrow He^3 + p + \pi^-, \qquad H^3_\Lambda \rightarrow d + p + \pi^-,$$

and we also make a comparison with similar calculations for the decay of He^5_Λ.[3-6]

The amplitude for the decay of the Λ particle in the hypernucleus has the form

$$M = s + pk_0^{-1}(\sigma\mathbf{k}). \tag{1}$$

Here $\mathbf{k}$ is the momentum of the π^- meson, k_0 is its value for the decay of a free Λ particle ($\sim$ 101 Mev/c), s and p are the amplitudes for the decay of the Λ particle in the channels with $l = 0$ and $l = 1$ respectively.

We shall neglect the interaction of the π^- mesons with other decay products, since it is small at the energies considered (for more details concerning this, cf. the paper of Byers and Cottingham[6]), and shall take into account only the interaction in the system proton plus residual nucleus. Corrections related to the deformation of the meson cloud of the Λ particle were discussed by Szymanski[5] and were insignificant for light hypernuclei.

Since the binding energy of the Λ particle in the He^4_Λ and H^3_Λ hypernuclei is small, we may assume that the wave function of the hypernucleus is expressed as a product

$$\psi_i = \psi_A \psi_\Lambda(\rho) \chi_i, \tag{2}$$

where ψ_A is the wave function of the nuclear core, $\psi_\Lambda(\rho)$ is a function describing the motion of the Λ-particle with respect to the center of mass of the nuclear core, χ_i is the spin function.

As was shown by Byers and Cottingham,[6] the energy and angular distributions for three-particle decays of hypernuclei are determined by two nontrivial kinematic variables which can be selected to be k, φ or P_Λ, θ (we shall use the notation of reference 6: φ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}_f$, where $\mathbf{k}_f$ is the relative momentum of the system proton plus residual nucleus; $\mathbf{P}_\Lambda$ is the vector sum of the momenta of the π^- meson and proton; θ is the angle between the vectors $\mathbf{P}_\Lambda$ and $\mathbf{q}$, where $\mathbf{q}$ is the relative momentum of the π^- meson and proton in their center-of-mass system). In the following we shall use both the variables k, φ, as well as P_Λ, θ.

$He^4_\Lambda \rightarrow He^3 + p + \pi^-$ Decay

Using (1) and (2), we find (after averaging over spin states) for the square of the matrix element of the process, as a function of the spin j of the hypernucleus He^4_Λ, the expression

$$|M_{if}|^2 = s^2|I^s|^2 + p^2k^2k_0^{-2}|I^t|^2 \quad \text{for } j = 0;$$

$$|M_{if}|^2 = \tfrac{1}{3}p^2k^2k_0^{-2}|I^s|^2 + (s^2 + \tfrac{2}{3}p^2k^2k_0^{-2})|I^t|^2 \quad \text{for } j = 1. \tag{3}$$

Here

$$I^{t,s} = \int \psi^{t,s*}(\rho) e^{-i3\mathbf{k}\rho/4} \psi_\Lambda(\rho)\, d\rho,$$

$$\psi^{t,s}(\rho) = \frac{1}{2}\sum_{l=0}^{\infty} i^l(2l+1)[h_l^{(1)}(k_f\rho) + \exp(-2i\delta_l^{t,s}) h_l^{(2)}(k_f\rho)] P_l(\cos\gamma), \tag{4}$$

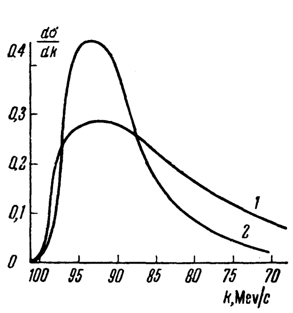

FIG. 1. Energy distribution of π^- mesons in the decay of He^4_Λ: 1 – without interaction in the final state; 2 – including interaction.

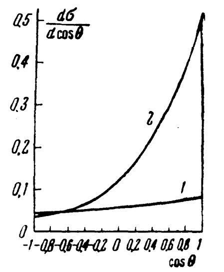

FIG. 2. Angular distribution of π^- mesons from the decay of He^4_Λ. The notation is the same as for Fig. 1.

$h_l^{(1)}$ and $h_l^{(2)}$ are spherical Hankel functions of the first and second kind.

The problem of finding the phases $\delta_l^{t,s}$ for scattering of protons on He^3 nuclei was considered by Bransden and Robertson,[7] who started from a potential for the interaction between the two nucleons without including spin-orbit or tensor forces. As the comparison of the results of their calculations with experimental data[8] showed, in the case of Serber forces one obtains satisfactory agreement with experiment for proton energies up to 10 Mev. We shall therefore use the phase shifts calculated by Bransden and Robertson for Serber-type interaction, and shall include only states with $l = 0$ and $l = 1$, since phase shifts with $l \geq 2$ are small at the energies considered here.

For the function $\psi_\Lambda(\rho)$ we shall choose the expression

$$\psi_\Lambda(\rho) \sim \frac{1}{\rho}(e^{-\alpha\rho} - e^{-\beta\rho}).$$

By means of a variational procedure similar to that used by Derrick,[9] we find that $\beta = 3.8\alpha$. The Coulomb interaction is included in the same way as in the work of Tang.[3] In accordance with the result of Dalitz,[10] the spin $j = 0$ was chosen for the hypernucleus He^4_Λ.

In Fig. 1 we show the energy spectrum of the π^- mesons (in arbitrary units) which has the form

$$\frac{d\sigma}{dk} \sim \int_{-1}^{+1} |M_{if}(k, \cos\varphi)|^2 k^2 k_f \, d\cos\varphi. \tag{5}$$

Curve 1 was calculated without interaction in the proton – He^3 system, and curve 2 including such interaction. Both curves were calculated on the assumption that $|p/s| = 1$.[10]

As the experimental data[1,2] show, in almost all the decays the momenta of the π^--mesons lie in a narrow range of momenta from 80–100 Mev/c (there are no more than 10% of all the decays outside this interval). The energy spectrum calculated without including final state interaction (curve 1) shows that in this case, for approximately 35% of all the decays, the momenta of the π^- mesons will lie below 80 Mev/c, in contradiction with the experimental data. The spectrum, calculated including such an interaction, shows that in approximately 90% of the cases the momenta of the π^- mesons should lie in this interval, i.e., agreement with the experimental data is satisfactory.

In Fig. 2 we show the angular distribution

$$\frac{d\sigma}{d\cos\theta} \sim \int_0^{P_{\Lambda max}} |M_{if}(P_\Lambda, \cos\theta)|^2 P_\Lambda^2 q \, dP_\Lambda. \tag{6}$$

The comparison of the calculated angular distributions with the experimental data[1,2] clearly shows the presence of strong interaction in the system of decay products. Curve 1 (no interaction) increases very slowly with increasing $\cos\theta$. The number of decays for which $\cos\theta < 0$ is $\sim 45\%$ which sharply contradicts the forward asymmetry observed experimentally in the decay of He^4_Λ. On the other hand, when we include interaction in the proton – He^3 system (curve 2), the number of decays for which $\cos\theta < 0$ is 15%, which is in satisfactory agreement with the experimental data.

We note that the energy and angular distributions presented in Figs. 1 and 2 depends very slightly on the value of the variational parameter β in the wave function $\psi_\Lambda(\rho)$ (cf. the paper of Picasso and Rosati[12]). Including the interaction of the π^- mesons with other decay products also does not essentially change our results.

Several authors,[3,5] especially and in detail Byers and Cottingham,[6] have discussed the energy and angular distribution from the $He^5_\Lambda \to He^4 + p + \pi^-$ decay. Byers and Cottingham[6] came to the conclusion that these distributions (and especially the angular distribution) are essentially determined by the resonance character of the phase shifts for scattering of protons by He^4_Λ. However, our calculations do not confirm the conclusion that the presence of resonances plays such an important role (cf. also reference 5), at least for the decays of light hypernuclei. As we know, the phase shifts for scattering of proton by He^3 do not have resonance character,[7,8] but nevertheless the angular distribution calculated using them for the $He^4_\Lambda \to He^3 + p + \pi^-$ decay has a marked asymmetry, in agreement with experiment.

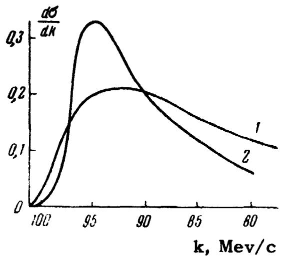

FIG. 3. Energy distribution of π^- mesons from the decay of He^3_Λ. The notation is the same as for Fig. 1.

The $H^3_\Lambda \rightarrow d + p + \pi^-$ Decay

The square of the matrix element of the decay for a spin $j = 1/2$ of the H^3_Λ hypernucleus has the form

$$|M_{if}|^2 = (s^2 + \tfrac{1}{9} p^2 k^2 k_0^{-2}) |I^d|^2 + \tfrac{8}{9} p^2 k^2 k_0^{-2} |I^q|^2. \quad (7)$$

Here

$$I^{d,q} = \int \psi^{d,q*}(\rho) e^{-2i\mathbf{k}\rho/3} \psi_\Lambda(\rho) d\rho. \quad (8)$$

The function $\psi^{d,q}(\rho)$ is obtained from (4) by replacing the phase shifts $\delta_l^{t,s}$ by the phases $\delta_l^{d,q}$, which were calculated by Massey.[11] For the function $\psi_\Lambda(\rho)$ we chose the expression[12]

$$\psi_\Lambda(\rho) \sim e^{-\gamma\rho}, \qquad \gamma = 0.75. \quad (9)$$

Using (7) — (9) we computed the energy spectrum of the π^- mesons, which is shown in Fig. 3. Curve 2, computed including interaction in the proton-deutron system, shows that $\sim 80\%$ of all the decays are contained in the interval 85 — 100 Mev/c, which is in satisfactory agreement with experimental data.[13]

Thus the results of our computations show that the effect of final-state interaction essentially determines the shape of the energy spectrum and angular distribution for decays of hypernuclei, even when this interaction is described by nonresonant phase shifts.

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