EFFECT OF MULTIPLE SCATTERING ON THE PROPER FIELD OF A FAST CHARGED PARTICLE

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It is shown that due to the effect of multiple scattering the proper field of a fast charged particle passing through a medium decreases at a higher rate with increasing distance from the axis of motion than in the vacuum. This effect leads to the emission of radiation when the particle passes from one medium to another and affects the direct pair interaction.

1. Gol'dman[1] (see also reference 2) has shown that an ultrarelativistic electron entering a dense medium emits radiation with a spectrum which lies precisely in the frequency region of the quanta whose emission in the slowing down of a particle in an infinite medium is hindered by multiple scattering. Garibyan[3] established even earlier that the transition of a charged particle through the interface of two media with different dielectric properties is accompanied by the emission of radiation of a different type due to the readjustment of the proper field of the particle.

In the present paper it will be shown that the emission of the transition radiation predicted by the theory of Gol'dman is also connected with a readjustment of the proper field of the particle, which, however, is due to the effect of multiple scattering. This problem will be considered in Sec. 2, where we shall compute the number of quanta per unit interval of the spectrum which are equivalent to the proper field of the particle moving in the medium [formula (6)]. It will also be shown [formula (9)] that the number of equivalent photons in the vacuum, as determined by the well-known Weizsäcker-Williams formula,[4] agrees with good accuracy with the number of transition quanta added to the number of photons which are equivalent to the field of the particle in the medium.

At distances from the point of entry into the medium which are smaller than the cascade unit of length, where the transition quanta have not yet succeeded in being absorbed, the cross section for the direct creation of pairs will of course be proportional to the number of equivalent photons in the vacuum. Far away from the point of entry, at places where the transition radiation does not penetrate, the number of pairs created directly by the field of the particle will become proportional to the number of equivalent photons in the medium. A detailed review of this very reasonable situation will be given in Sec. 3.

Finally, in Sec. 4, we shall investigate the process of pair creation by a particle passing through a medium for the case where the distance at which the radiation field is formed is comparable with the cascade unit. Here it becomes impossible to separate out the cross section for the direct creation of pairs, and the total number of pairs at a depth much larger than the cascade unit is determined by formula (19).

2. To determine the number of photons equivalent to the proper field of the particle in the medium we obtain first an expression for the number of pairs in the first cascade unit created by a fast electron passing through the medium with energy $p_0 \gg 1$, and separate out the pairs which are created directly by the field of the particle. The number of the latter, as is well known,[4] is proportional to the number of photons connected with the proper field of the particle.

We shall start from the expression for the probability for bremsstrahlung per unit length of path in an infinite medium, derived by Migdal.[5,6] This expression has the following form for $k \ll p_0$:*

$$W_r(p_0, k) = \frac{e^2 k}{2\pi^2}\int_0^\infty f(t)\,dt,$$

$$f(t) = \operatorname{Re} e^{i\omega t}\int (\xi\eta)\, w(\xi, t, \eta)\, d\xi d\eta, \qquad (1)$$

where w is the Fourier component of the usual distribution function of the trajectories, $\omega = k/\sqrt{\epsilon}$ is the energy of a quantum with momentum k, $\epsilon = 1 - \omega_0^2/\omega^2$ is the dielectric constant of the medium, $\omega_0^2 = 4\pi n Z e^2$, and n is the number of nuclei per unit volume.

*We use a system of units in which $\hbar = m = c = 1$.

When the emitted bremsstrahlung quanta are absorbed they create pairs, which can, however, also be created directly by the field of the particle itself. To account for this effect it is sufficient to introduce under the integral sign in formula (1) the factor $\exp[-n\sigma t/2]$, which corrects for the absorption of quanta in the radiation process itself. The expression $n\sigma$ is the total cross section for the creation of pairs by a γ quantum with energy k per unit length of path in the medium. This procedure is equivalent to introducing an imaginary part of ϵ due to the absorption of γ quanta through pair creation. It can be shown that the corresponding change in the real part is negligibly small.

We thus obtain the following expression for the total number of pairs created per unit length of path in an infinite medium:

$$W_p(k, p_-)(n\sigma)^{-1}\widetilde{W}_r(p_0, k)\,dk\,dp_-;$$

$$\widetilde{W}_r = \frac{e^2k}{2\pi^2}\int_0^\infty e^{-n\sigma t/2} f(t)\,dt, \qquad (2)$$

$$W_p(k, p_-)\,dp_- = \frac{1}{3kL}\left[1 + 2\frac{p_-^2 + (k-p_-)^2}{k^2}\right]dp_-. \qquad (3)$$

Here $W_p(k, p_-)$ is the probability for pair creation by a γ quantum per unit length of path with an electron energy between p_- and $p_- + dp_-$, and L is the radiation length. If we denote the important time interval in the integral (1) by

$$\tau_k = \min\left\{2p_0^2/k, \sqrt{p_0^2L/2\pi k\cdot 137}\right\}$$

we can expand the factor $\exp[-n\sigma\tau/2]$ in formula (2) in a series, provided that $n\sigma\tau_k \ll 1$. Then Eq. (2) takes the form

$$\widetilde{W}_r \approx W_r + n\sigma q(k), \qquad q(k) = -\frac{e^2k}{4\pi^2}\int_0^\infty t\,dt f(t). \qquad (4)$$

Comparing (2) and (4), we see that $W_p q$ is the cross section for direct pair creation, which means that $q(k)\,dk$ is the number of γ quanta in the spectral interval k, k + dk which are equivalent to the proper field of the particle.

Methods for calculating expressions of the type (2) or (4) have been discussed by various authors[1,2,5] These expressions have the feature that they contain logarithmic divergences for large radiation angles, because our method becomes invalid when the angular width of the radiation exceeds the average angle between the directions of emission of the particles of the pair. The latter is of the order of magnitude of k^{-1}. If we cut off the integration over angles at $\eta \sim k^{-1}$, we obtain

$$q(k) = \frac{e^2}{\pi k}\left\{\ln\frac{\varkappa p_0^2}{k^2(1+\delta)} - 1 + \mathrm{Re}\int_0^\infty\left(\coth x - \frac{1}{x}\right)(1-\mu' x)e^{-\mu' x}dx\right\}, \qquad (5)$$

where $\kappa \approx 1$ and

$$\mu' = 2(1-i)s', \quad s' = s(1+\delta), \quad \delta = \omega_0^2 p_0^2/k^2,$$
$$s \approx 1.37\cdot 10^3\sqrt{kL/p_0^2}.$$

Here k, p_0, and ω_0 are in mc^2 units, and L is in centimeters.

The integral in formula (5) reduces to zero when $s' \gg 1$ and is approximately equal to $\log s'$ when $s' \ll 1$. For arbitrary values of s and δ, formula (5) can therefore be replaced by the simple approximate formula

$$q(k) \approx \frac{2e^2}{\pi k}\ln\left[\frac{\varkappa p_0}{k}\sqrt{\frac{s}{1+s(1+\delta)}}\right], \qquad (6)$$

the inaccuracy of which shows up in the presence of the unknown factor $\kappa \sim 1$ in the argument of the logarithm. This factor will be omitted in the following.

As is known,[4] the number of photons in the spectral interval k, k + dk which are equivalent to the field of an electron moving in the vacuum with energy $p_0 \gg 1$ is given by $q_0(k)\,dk$ with

$$q_0(k) \approx \frac{2e^2}{\pi k}\ln\frac{\rho_{max}}{\rho_{min}} = \frac{2e^2}{\pi k}\ln\frac{p_0}{k}, \qquad (7)$$

where $\rho_{min} \sim \lambda_c = \hbar/mc = 1$ and $\rho_{max} \sim p_0/k$ is the maximal distance from the axis of motion at which the Fourier component of the field of the particle is still appreciably different from zero. Comparing (6) and (7), we see that, due to the effects of the polarization of the medium and of multiple scattering, the field of the particle differs from zero only at distances of order $\rho'_{max} = (p_0/k)\sqrt{s/(1+s')}$. The same result can also be obtained directly through a study of the important values in the expression for the Fourier component of the field of the particle in the medium.

If the particle makes a transition from the medium to the vacuum, radiation should be emitted as a consequence of the readjustment of the field of the particle. It has been shown earlier[1,2] that the number of quanta of the transition radiation in the spectral interval k, k + dk can be written in the approximate form

$$N(k)\,dk \approx \frac{2e^2}{\pi k}dk\ln\frac{p_0}{k}\sqrt{\frac{1+s(1+\delta)}{s}}. \qquad (8)$$

Comparing expressions (6) — (8), we find that

$$q_0(k) \approx N(k) + q(k). \qquad (9)$$

This result has important consequences for the theory of the direct pair creation.

3. If the transition radiation occurs at distances of order $\tau_k \ll 1/n\sigma$, one can assume that at time t the quanta of the transition radiation create

$$W_p(k, p_-)\, N(k) \frac{1-e^{-n\sigma t}}{n\sigma}\, dk\, dp_-$$

pairs with total energy k, with electron energies between p_- and $p_- + dp_-$.

The number of pairs created directly by the field of the particle in time t is equal to

$$W_p(k, p_-)\, q(k)\, t\, dp_-\, dk.$$

For $t \ll 1/n\sigma$, the number of pairs created by the field of the particle is given by the expression

$$W_p[q + N]\, t dk\, dp_- \approx W_p q_0\, t dk\, dp_-. \qquad (10)$$

If $t \gg 1/n\sigma$, the quanta of the transition radiation are completely absorbed, and the number of pairs created per unit length of path directly by the field of the particle will be equal to $q(k) W_p dk dp_-$, where $q(k)$ is given by formula (6). If here $10^6 L \gg p_0 \gg 10^3\sqrt{L} \equiv p_1$ (L is measured in centimeters and p_0 in units of mc^2), the total cross section for the direct creation of pairs is given by the expression

$$\sigma_\pi = \frac{7}{9L137\pi} \ln \frac{p_0^2}{p_1} \ln p_1, \qquad (11)$$

which is proportional only to the first power of the logarithm of the energy.

If $p_0 > 10^6 L$ and

$$\tilde{s} \approx 1.37\cdot 10^3 [kL/p_-(k-p_-)]^{1/2} \leqslant 1,$$

we must take account of the effect of multiple scattering on the pair creation by γ quanta.[6,7] Here the average angle between the directions of emission of the particles is widened and reaches the value $k^{-1}\tilde{s}^{-1/2}$. In the integration over η in formula (4) we must therefore take $k^{-1}[(1+\tilde{s})/\tilde{s}]^{1/2}$ as the upper limit. We then find

$$q(k) \approx \frac{2e^2}{\pi k} \ln \left[\frac{p_0}{k} \sqrt{\frac{s\,(1+\tilde{s})}{\tilde{s}\,(1+s')}}\right] \approx \frac{e^2}{\pi k} \ln \frac{1+\tilde{s}}{1+s'}. \qquad (12)$$

Using the result of reference 7 for the cross section for direct pair creation per unit length of path in an infinite medium, we obtain the expression

$$d\sigma_\pi = \frac{\xi(\tilde{s})\, dk\, dp_-}{3\pi\, 137\, k^2 L}\left[G(\tilde{s}) + 2\,\frac{p_-^2 + (k-p_-)^2}{k^2}\,\Phi(\tilde{s})\right] \ln \frac{1+\tilde{s}}{1+s'}, \qquad (13)$$

where G and Φ are the functions introduced by Migdal,[5,6] and

$$\xi(\tilde{s}) = \min\left\{2,\ \frac{\ln\left(190\, Z^{-1/3}\sqrt{(1+\tilde{s})/\tilde{s}}\right)}{\ln 190\, Z^{-1/3}}\right\}.$$

Formula (13) differs from the formula quoted in reference 7 by a factor in the argument of the logarithm which takes account of the effect of multiple scattering on the proper field of the particle. It describes the direct creation of pairs at large distances from the point of entry of the particle into the medium.

The results of reference 7 remain valid near the boundary. For the total number of pairs at an arbitrary depth t we find the expression*

$$\frac{\xi(\tilde{s})}{3kL}\left[G(\tilde{s}) + 2\,\frac{p_-^2 + (k-p_-)^2}{k^2}\,\Phi(\tilde{s})\right]$$
$$\times \left\{\frac{e^2}{\pi k}\left[t \ln \frac{p_0^2 s(1+\tilde{s})}{k^2 \tilde{s}(1+s')} + \frac{1-e^{-n\sigma t}}{n\sigma} \ln \frac{1+s'}{s}\right]\right.$$
$$\left. + \frac{1}{n\sigma}\left[t - \frac{1-e^{-n\sigma t}}{n\sigma}\right] \frac{4\xi(s)}{3kL}\,\Phi(s)\right\} dk\, dp_-. \qquad (14)$$

4. All the results mentioned above are valid only if the absorption of the quanta in the process of radiation itself is small. If this condition is not fulfilled, expansion (4) becomes incorrect and it becomes generally impossible to separate out the bremsstrahlung cross section.† The inequality $n\sigma\tau_k \gg 1$ is equivalent to the condition $p_0^2/k \gg 10^{12} L$ (L in cm) and can be satisfied only for very soft quanta at reasonable electron energies. We shall therefore neglect in the following the effect of multiple scattering on the creation of pairs by γ quanta.

Carrying out the calculations, we find from formula (2)

$$\tilde{W}_r = \frac{e}{16\pi k} \operatorname{Re}\left\{\left[\frac{n\sigma}{2} - \frac{ik(1+\delta)}{p_0^2}\right]\left[\frac{1}{\mu'+\nu} - \psi\left(1 + \frac{\mu'+\nu}{2}\right)\right.\right.$$
$$\left.\left. + \ln \frac{\mu p_0^2}{4k^2}\right]\right\}, \qquad (15)$$

where

$$\nu = 2(1+i)\, r, \qquad r \approx 4\cdot 10^{-8}\sqrt{p_0^2/kL},$$

and ψ is the logarithmic derivative of the γ function.

For $r \ll 1$ or $s' \gg r$ formula (15) reduces to (4), while for $r \gg 1$ or $s' \ll r$ (15) leads to

$$\tilde{W}_r = n\sigma \frac{e^2}{\pi k} \ln \frac{p_0^2 s}{k^2 r}. \qquad (16)$$

If L is measured in centimeters and k in units of mc^2, we have $p_0^2 s/k^2 r \approx 10^{11} L/k$.

Under these conditions we must also change the expression for the number of transition quanta. For this purpose it is sufficient to introduce in the expression for ϵ in formula (16) of the author's

*The last term in the curly brackets describes the contribution from the bremsstrahlung quanta.

†This was first pointed out by Landau and Pomeranchuk.[8]

paper[2] an imaginary part due to the absorption of γ quanta through pair creation. We then obtain

$$N = \frac{e^2}{\pi k} \operatorname{Re} \left\{ 2(\mu' + \nu) \times \int_0^\infty \frac{dz}{1+z} \int_0^\infty \exp\left[-(\mu'+\nu)\tau - \mu z \tanh \tau\right] d\tau + \int_0^\infty \left(\coth \tau - \frac{1}{\tau}\right) [1 - (\mu'+\nu)\tau] \times \exp\left[-(\mu'+\nu)\tau\right] d\tau - 2 - \ln \frac{\mu'+\nu}{\mu} \right\}. \quad (17)$$

For $r \gg 1$ and $r \gg s'$ we find from (17)

$$N = \frac{e^2}{\pi k} \ln \frac{r}{s}, \quad (18)$$

and for the total number of pairs with energy k at the depth $t \gg 1/n\sigma$ we obtain the expression

$$n\sigma \frac{e^2}{\pi k}\left[t \ln \frac{p_0^2 s}{k^2 r} + \frac{1}{n\sigma} \ln \frac{r}{s}\right]. \quad (19)$$

The region of validity of expressions (16), (18), and (19) is determined by the inequalities $r \gg 1$ and $r \gg s'$. In order for these two conditions to be fulfilled simultaneously, we must have

$$\lambda_C\, p_0^2/100\, L \gg k \gg \omega_0^2\, L/\lambda_C, \quad (20)$$

where $\lambda_C = \hbar/mc$, and p_0, k, and ω_0 are measured in the units mc^2. The inequality (20) can be fulfilled only if $p_0 \gg p_2 = 10\omega_0 L/\lambda_C$. For lead $p_2 = 7.5 \times 10^{12}$ ev, and for aluminum $p_2 = 1.6 \times 10^{14}$ ev.

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