orientations of the field such a relation is even steeper than in Fig. 2. This indicates that the parameter $\omega\tau$ is not large enough in spite of the fact that the samples were made of very pure tin (an estimate yields $\omega\tau \sim 50$) in which the mean free path of the electrons, is possibly already limited by isotopic inhomogeneity of the metal.[7] Obviously a lowering of the sample temperature is necessary to resolve the resonances and to increase the accuracy of measurements of the resonant field values. However a more important result of these experiments is that, as seen from Fig. 2, the resonant field values do not depend on the temperature, at least, in regions below 3 deg K (an insignificant increase in the resonant fields values is noticed above 3 deg K). This fact permits us to assume that the values of the effective masses found from the cyclotron-resonance spectra actually correspond to definite sections of the Fermi surface. The insufficient value of the parameter $\omega\tau$ could lead to an averaging of the experimentally-obtained effective masses over a considerable region of the Fermi surface, thus greatly reducing the value of these data. A similar partial averaging apparently takes place at temperatures above 3 deg K.

It is appropriate to note the sharp increase in the number of observable resonances in each sequence with decreasing temperature, owing to the increase in the value of $\omega\tau$. This guarantees a substantial increase in the accuracy of measurements of the effective mass at lower sample temperatures.

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*The fundamental theory of cyclotron resonance in an inclined magnetic field is examined by Kaner.[4]

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102

INSTABILITY OF UNIFORM PRECESSION OF MAGNETIZATION

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It is obvious that the occurrence of inhomogeneous magnetization in magnetic systems in a constant external magnetic field leads to a breakdown of the uniform precession. We shall consider therefore that the homogeneous precession is unstable if small fluctuations lead to growing magnetization waves. We shall investigate the stability of the uniform precession of a series of ferromagnetic and ferrimagnetic systems.

1. The change in the magnetization of a ferromagnet can be described by the equation

$$\dot{\mathbf{M}} = -\gamma [\mathbf{M} \times \mathbf{H}_{eff}],$$
$$\mathbf{H}_{eff} = \mathbf{H}_0 + \lambda \mathbf{M} + H_{ex} (l^2 / M) \nabla^2 \mathbf{M} + \mathbf{H}_d, \qquad (1)$$

where $\mathbf{H}_0$ is the external field, λ the constant of the molecular field, H_{ex} the magnitude of the molecular field, l the interatomic distance, and $\mathbf{H}_d$ the demagnetizing field.

If we seek a solution of (1) in the linear approximation in the form of magnetization waves (spin waves) with the frequency ω_k and the wave vector $\mathbf{k}$, then the waves of wavelengths smaller than the dimensions of the system* satisfy the following dispersion relation

$$\omega_k = \gamma [(H_i + H_{ex} l^2 k^2)(H_i + H_{ex} l^2 k^2 + 4\pi M \sin^2\theta)]^{1/2}, \qquad (2)$$

where $H_i = H_0 - 4\pi M N_z$, $N_x = N_y$ and N_z are demagnetizing factors, the field H_0 is along the z axis, and θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$. The nonequilibrium state will be unstable with respect to spin waves if the following relation is satisfied

$$-4\pi M < H_i + H_{ex} l^2 k^2 < 0. \qquad (3)$$

(In this case the frequencies ω_k become imaginary.)

For magnetostatic types of oscillation (for small $\mathbf{k}$)[1] we must replace $\sin^2\theta$ in (2) by

$$\beta_{nmr} = \delta^2 (1 - z_{nmr}^2) / \delta^2 (1 - z_{nmr}^2) + z_{nmr}^2,$$

where δ is the ratio between the major and the minor axes of the ellipsoid, and z_{nmr}^2 is the square of the r-th root of the associated Legendre polynomial $P_n^m(z)$ with $0 < z_{nmr}^2 < 1$.

We must take $\mathbf{k} = (n, m, r)$ and eliminate the terms $H_{ex}l^2k^2$.† It is clear that the oscillations begin to build up when $H_i < 0$.

Formula (2) is correct if the angle between $\mathbf{H}_0$ and $\mathbf{M}$ is close to 0 or π; however, calculations show that instability appears always whenever φ, the angle between $\mathbf{H}_0$ and $\mathbf{M}$, is greater than $\pi/2$, since the spectrum for an arbitrary φ (but not close to $\pi/2$) has the form (for a spherical sample)

$$\omega_k = \gamma\left[\left(H' - \tfrac{1}{2}4\pi M\cos\varphi\sin^2\theta\right)\times\left(H' + \tfrac{1}{2}4\pi M\cos\varphi\sin^2\theta\right)\right]^{1/2},$$

$$H' = H_0 + \left(\cos\varphi + \tfrac{1}{2}\tan\varphi\sin\varphi\right)\left(H_{ex}l^2k^2 - \tfrac{4}{3}\pi M\right) + \tfrac{1}{2}\cos\varphi 4\pi M\sin^2\theta + \tfrac{1}{2}4\pi M\sin\varphi\tan\varphi\cos^2\theta.$$

From the above formula it is clear that it is impossible to have a stable uniform precession of the magnetization of a ferromagnet if the angle between $\mathbf{H}$ and $\mathbf{M}$ is obtuse. In this sense the result of reference 2 on coherent radiation from inverted systems is incorrect when applied to ferromagnets. The same is true of the paper of Morgenthaler,[3] in which the question of ferromagnetic systems (ferrites) especially is discussed.

We notice that the characteristic time of occurrence of spin waves is of the order of $\tau = 1/4\pi\gamma M$.

2. For a ferrimagnet near the compensation point we can find a dispersion relation by using equations analogous to (1) (but with two sublattices). Analysis of this relation shows that the spin waves with $\theta = 0$ build up the fastest; for them

$$\omega_k = \gamma H_0 \pm [2\omega_{ex}K + K^2 + \omega_{ex}^2 l^2k^2]^{1/2},$$

$$K = \tfrac{1}{2}(\gamma_1 - \gamma_2)H_0 + \tfrac{1}{2}(\gamma_1 + \gamma_2)H_a; \qquad \omega_{ex} \approx \gamma H_{ex}.$$

The instability occurs when

$$2\omega_{ex}K + K^2 + \omega_{ex}^2 l^2k^2 < 0. \tag{4}$$

Here γ_1 and γ_2 are the gyromagnetic ratios for the first and the second sublattices, and H_a the effective anisotropy field, which is the same for both sublattices.

We note that in the case of a ferrimagnet at the compensation point the frequency can become complex if condition (4) is fulfilled and $k = 0$, i.e., for uniform precession (with a build-up time smaller than for spin waves). This means that uniform precession predominates in the linear approximation considered here. In reference 4 account is taken of nonlinearity for uniform precession.

3. Suhl[5] has shown that there exists a nonlinear mechanism for the build-up of spin waves as a result of their interaction with the uniform precession. It was shown that for small angles φ, instability appeared if $\sin^2\varphi > \eta/4\pi M\gamma$, where η is the coefficient of relaxation of the spin waves. The characteristic instability time is $\tau \approx (4\pi M\gamma \times \sin^2\varphi - \eta)^{-1}$.

A calculation for a ferromagnet shows that for an arbitrary angle (but not close to $\pi/2$), instability occurs, the spin waves build up exponentially, and $\tau \approx 1/4\pi M\gamma\sin\varphi\tan\varphi$. In making this conclusion it is assumed that there already exists a uniform precession at an angle φ (in general not small) and we investigate its instability. The question considered above will be discussed in the journal Изв. высш. шк., Радиофизика (News of the Colleges, Radiophysics).

*Rough calculations show that the dispersion relation (2) is correct for waves with wavelength ten times shorter than the dimensions of the sample.

†We note that the dependence of the magnetization of the magnetostatic oscillations on the coordinates is not described by the function $e^{i\mathbf{k}\cdot\mathbf{r}}$

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103

THEORY OF THE NONRESONANCE ABSORPTION OF AN OSCILLATING MAGNETIC FIELD BY A FERROMAGNETIC DIELECTRIC

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IN a recently published article[1] we solved the problem of calculating the absorption coefficients of an oscillating magnetic field polarized in a direction perpendicular to the axis of easiest mag-