# ON THE DECAYS $K^+ \rightarrow \pi^+ + e^+ + e^-$ AND $K^+ \rightarrow \pi^+ + \mu^+ + \mu^-$

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Submitted to JETP editor March 2, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 600-604 (September, 1960)

The hitherto unobserved decays $K^+ \rightarrow \pi^+ + e^+ + e^-$ and $K^+ \rightarrow \pi^+ + \mu^+ + \mu^-$, which may be due to combined electromagnetic and weak interactions, are examined. The absolute values of the probabilities of these decays are determined by the magnitude of the monopole moment of the transition $K \rightarrow \pi$, which cannot be calculated at present. The ratio of the probabilities can be computed and has been found to be $W_\mu/W_e = 0.2$. The $\pi$ meson, electron, and muon spectra have been calculated, and some convenient methods for the treatment of the experimental results are indicated.

IT is known that, unlike the decays

$$K \rightarrow \pi + e + \nu, \qquad K \rightarrow \pi + \mu + \nu, \tag{1}$$

the decays

$$K \rightarrow \pi^+ + e^+ + e^-, \tag{2}$$

$$K \rightarrow \pi^+ + \mu^+ + \mu^-, \tag{3}$$

which have not as yet been observed experimentally, cannot be due to the weak interaction alone. This has to do with the fact that, according to the hypothesis of the universal weak interaction, the interaction contains only charged lepton pairs ($e^+\nu$, $\mu^+\nu$, $\times\, e^-\bar{\nu}$, $\mu^-\bar{\nu}$) and no neutral pairs ($e^+e^-$, $\mu^+\mu^-$, $\times\, \mu^+e^-$, $\nu\bar{\nu}$).

It is easily seen, however, that the decays (2) and (3) are not strictly forbidden even in the present theory and can come about through the combined effect of the weak non-leptonic and electromagnetic interactions. The Feynman graph corresponding to this process is shown in Fig. 1. Here the dotted line represents a virtual photon; the circle represents symbolically the totality of graphs corresponding to the decay of a K meson into a $\pi$ meson and a $\gamma$ quantum. One of these graphs is shown in Fig. 2. The decays (2) and (3) are analogous to the well-known 0-0 conversion transitions in nuclei.

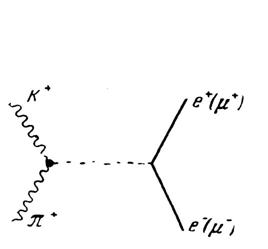


FIG. 1

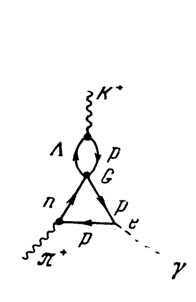


FIG. 2

In order to show how to write down the matrix element corresponding to the graph of Fig. 1, we recall that, in analogy to the 0-0 transition with emission of a real photon, the decay $K^+ \rightarrow \pi^+ + \gamma$ with emission of a real photon is forbidden by gauge invariance. Indeed, the only tensor which can be constructed from the four-momenta of the K meson, p, and of the $\pi$ meson, q, must be proportional to $p_\mu q_\nu$, and thus gives zero when it is multiplied by the tensor of the electromagnetic field $F_{\mu\nu} = k_\nu \epsilon_\mu - k_\mu \epsilon_\nu$ ($\epsilon$ is the photon polarization four-vector, and k is the photon four-momentum):

$$p_\mu q_\nu F_{\mu\nu} = (pk)(k\varepsilon) - k^2(p\varepsilon) = 0.$$

The first term vanishes on account of the transversality of the photon, and the second, because $k^2 = 0$. However, for a virtual photon $k^2 \neq 0$, and the term $k^2(p\epsilon)$ is different from zero.

Keeping these remarks in mind, we write down the matrix element corresponding to the graph of Fig. 1 in the form*

$$M = efGk^2 p_\mu \frac{\varphi_K \varphi_\pi}{k^2} \sqrt{4\pi}\, e\bar{u}\gamma_\mu u = \sqrt{4\pi}\,\alpha G f \bar{u}\hat{p}\, u \varphi_K \varphi_\pi. \tag{4}$$

Here $\alpha = e^2 = 1/137$ ($\hbar = c = 1$); u is the spinor of the leptonic field (e or $\mu$); $\varphi_K$ and $\varphi_\pi$ are the wave functions of the K meson and $\pi$ meson field, respectively, and G is the weak interaction constant ($G = 10^{-5} m_p^{-2}$, where $m_p$ is the proton mass). The vertex part represented in the graph of Fig. 1 by a circle is given by the expression $\varphi_K \varphi_\pi efGk^2 p_\mu$ in formula (4), where f is a dimensionless function of $k^2$ or, what amounts to the same, of the energy

*We note that the decay described by the matrix element (4) conserves parity in contrast to the case of the direct (without participation of a virtual photon) weak interaction of a lepton pair with strongly interacting particles.

of the $\pi$ meson: $k^2 = m_K^2 + m_\pi^2 - 2E_\pi m_K$. It can be expected that this function changes little in the interval $0 < k^2 < (m_K - m_\pi)^2$. Unless the contrary is specifically indicated, we shall therefore assume in the following that this function is constant. The quantity f does not contain the weak and electromagnetic coupling constants G and e, which appear as separate factors in expression (4); in this sense the constant f is therefore of "order unity." Numerically, however, it can be considerably less than unity. Unfortunately, we cannot calculate the value of f owing to the absence of a theory of strong interactions.

If, in analogy to the vertex $K \to \pi + \gamma$, we describe the vertex $K \to 2\pi$ phenomenologically by the amplitude $f_\theta G m_K^2 \varphi_K \varphi_{\pi_1} \varphi_{\pi_2}$ and determine $f_\theta$ by comparing the probability calculated with the help of this amplitude with the experimentally observed probability of the $\theta_1^0$ decay, we obtain $f_\theta^2 \sim 10^{-4}$. This estimate gives us an idea of the possible value of f, although it is not excluded that f can differ from $f_\theta$ by several orders of magnitude. If we neglect the unknown dependence of f on $k^2$, we can calculate the total probabilities of the decays (2) and (3) with the help of the amplitude (4). We find

$$W_e = 0.56\, \overline{W}_0, \qquad (5)$$

$$W_\mu = 0.11\, \overline{W}_0. \qquad (6)$$

Here $\overline{W}_0 = (1/48)\alpha^2 G^2 m_K^5 f^2 \approx 5 \times 10^6 f^2\ \text{sec}^{-1} \approx W_\tau f^2$, where $W_\tau$ is the probability of the $\tau^+$ decay. If f were of order unity, the decays (2) and (3), which should look like anomalous $\tau$ decays in the photoemulsions, would therefore be about as frequent as the normal $\tau$ decays. Up to now about 2000 normal $\tau$ decays have been investigated, but not a single occurrence of the decays (2) or (3) has been observed. This implies that $f^2 < 5 \times 10^{-4}$. The impossibility of a reliable calculation of the absolute probability of the decay (2) has been pointed out earlier by Dalitz,[1] who made an estimate of the order of magnitude of this probability.

It is seen from formulas (5) and (6) that, in contrast to the absolute probability, the relative probability of the decays (2) and (3) does not depend on $f^2$ and is uniquely determined as

$$W_\mu / W_e = 0.2. \qquad (7)$$

We note that the numbers 0.56 and 0.11 in formulas (5) and (6) are obtained by integrating expressions (10) and (9). In the case of the electronic decay the integration is elementary and gives

$$1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x = 0.56, \qquad (8)$$

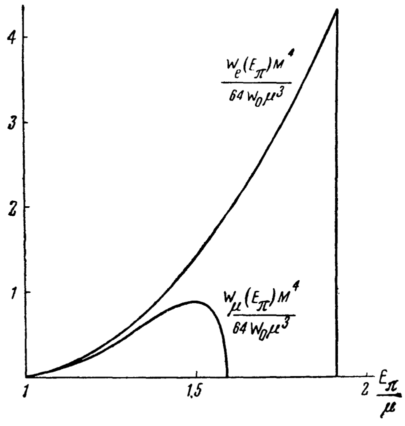


FIG. 3

where $x = \mu/M$ ($\mu$ is the $\pi$ meson mass, and M is the mass of the K meson). In the case of the mesonic decay the integration is done numerically (see Fig. 3).

Let us now consider the spectra of the particles created in the decays (2) and (3). For the decay (3) the spectrum of the $\pi$ mesons has the form

$$W_\mu(E_\pi)dE_\pi = \frac{64\, W_0(E_\pi)}{M^4}\left[1 + \frac{2m^2}{M^2 + \mu^2 - 2E_\pi M}\right] \times \left[\frac{M^2 + \mu^2 - 4m^2 - 2E_\pi M}{M^2 + \mu^2 - 2E_\pi M}\right]^{1/2} (E_\pi^2 - \mu^2)^{3/2} dE_\pi, \qquad (9)$$

where the energy of the $\pi$ mesons varies within the limits

$$\mu \leqslant E_\pi \leqslant (M^2 + \mu^2 - 4m^2)/2M,$$

where m is the mass of the $\mu$ meson. For the decay (2) the spectrum of the $\pi$ mesons is obtained from (9) by going to the limit m = 0:

$$W_e(E_\pi)\, dE_\pi = 64 M^{-4} W_0(E_\pi)(E_\pi^2 - \mu^2)^{3/2} dE_\pi. \qquad (10)$$

In this case the energy of the $\pi$ mesons varies within the limits

$$\mu \leqslant E_\pi \leqslant (M^2 + \mu^2)/2M.$$

The spectra (9) and (10) are shown in Fig. 3. Each of the spectra (9) and (10) contains the unknown function $W_0(E_\pi)$, which can be determined only experimentally.* However, the ratio of the spectra (9) and (10) does not depend on this unknown function and is determined only by quantum electrodynamics. This can be used for the verification of the electromagnetic properties of the muons and electrons up to energies $M - \mu \approx 350$ Mev in the center of mass system of the leptons. It is of special interest to study the ratio of the spectra (9) and (10) in the region of small kinetic energies T of the $\pi$ meson. In this case the energy of the leptons is close to its largest possible value. Then

*We note that the constant $\overline{W}_0$ which appears in the probabilities (5) and (6) is some average value of the function $W_0(E_\pi)$.

$$\frac{W_\mu(T)}{W_e(T)}=\left(1+\frac{2m^2}{(M-\mu)^2}\right)\left(1-\frac{4m^2}{(M-\mu)^2}\right)^{1/2}+O\left(\frac{T}{\mu}\right)\approx 0.95 \quad (11)$$

($m$ is the mass of the muon). Unfortunately, the number of $\pi$ mesons with small energies is exceedingly small, as can be seen from Fig. 3.

The spectra of the muons and electrons in the decays (3) and (2) can be obtained in an explicit form only if it is assumed that $f$ = const, in which case the spectra contain the constant $\overline{W}_0$. Then the spectrum of the muons has the form

$$W_\mu(E_\mu)\,dE_\mu=\frac{96\overline{W}_0}{M^4}\left\{\left[1+\frac{m^2-\mu^2}{Q^2}\right](ME_\mu-2E_\mu^2+m^2)\right.$$
$$\left.-2m^2\right\}\times\left[\frac{Q^2-2m^2-2\mu^2+(m^2-\mu^2)^2/Q^2}{Q^2}\right]^{1/2}$$
$$\times(E_\mu^2-m^2)^{1/2}\,dE_\mu, \quad (12)$$

where $Q^2 = M^2 + m^2 - 2ME_\mu$, and the energy of the muon varies within the limits $m \le E_\mu \le [M^2 + m^2 - (m+\mu)^2]/2M$. The spectrum of the electrons in the decay (2) is obtained from (12) by going to the limit $m = 0$:

$$W_e(E_e)\,dE_e=\frac{96\overline{W}_0}{M^4}\frac{E_e^2(M^2-2ME_e-\mu^2)^2}{M^2(M-2E_e)}dE_e. \quad (13)$$

The shape of the electron and muon spectra is shown in Fig. 4.

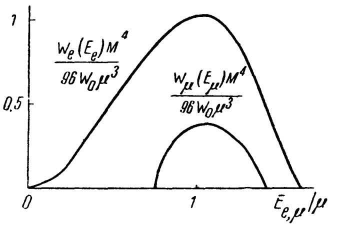


FIG. 4

It is also of interest to consider the simultaneous dependence of the probability on the two variables: the energy of the $\pi$ mesons $E_\pi$ and the absolute value of the energy difference between the leptons $\Delta = |E_1 - E_2|$. In this consideration we can use the experimental data for different $\pi$ meson energies simultaneously. For the electronic decay (2) we then obtain the following spectrum

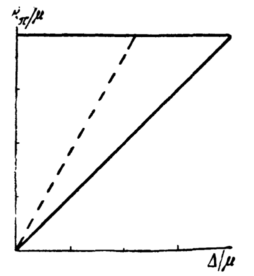


FIG. 5

$$W_e(\Delta,\,E_\pi)\,d\Delta\,dE_\pi=\frac{96W_0(E_\pi)}{M^4}\,[k_\pi^2-\Delta^2]\,d\Delta\,dE_\pi, \quad (14)$$

where $k_\pi^2 = E_\pi^2 - \mu^2$, and the variables are restricted to the region

$$0\leqslant\Delta\leqslant k_\pi, \qquad 0\leqslant k_\pi\leqslant(M^2-\mu^2)/2M,$$

as shown in Fig. 5.

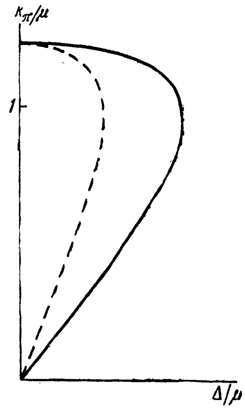


FIG. 6

The spectrum (14) contains the unknown function $W_0(E_\pi)$. However, the determination of this function can be avoided by using the "sliding ray method," which has been proposed earlier[2] for the analysis of the $K_{e3}$ decay. For this purpose we must draw the ray $\Delta = ak_\pi$ in Fig. 5, with $a < 1$. Then the ratio of the number of experimental points lying to the left of the ray over the total number of experimental points does not depend on the unknown function $W_0(E_\pi)$, but only on the quantity $a$:

$$\int_0^{ak_\pi}W_e(\Delta,\,E_\pi)\,d\Delta\Big/\int_0^{k_\pi}W_e(\Delta,\,E_\pi)\,d\Delta=\frac{3}{2}\left[a-\frac{a^3}{3}\right]. \quad (15)$$

For the muonic decay (3) we also obtain the spectrum

$$W_\mu(\Delta,\,E_\pi)\,d\Delta\,dE_\pi=96M^{-4}W_0(E_\pi)\,[k_\pi^2-\Delta^2]\,d\Delta\,dE_\pi. \quad (16)$$

The variables in the spectrum (16) are restricted to the region

$$0\leqslant\Delta\leqslant k_\pi\left[1-\frac{4m^2}{M^2-2ME_\pi+\mu^2}\right]^{1/2},$$

$$0\leqslant k_\pi\leqslant\frac{[(M+\mu)^2-4m^2]^{1/2}\,[(M-\mu)^2-4m^2]^{1/2}}{2M}$$

as shown in Fig. 6. This region differs essentially from the region shown in Fig. 5: for $k_\pi = k_{\pi\max}$ we have $\Delta = 0$. The "sliding ray method," therefore, cannot be directly applied to the spectrum (16).

In conclusion the authors express their deep gratitude to L. D. Landau and I. Yu. Kobzarev for comments, to S. A. Nemirovskaya for carrying out the numerical calculations and to E. D. Zhizhin for pointing out an error in the initial calculations.

[1] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).

[2] I. Yu. Kobzarev, Dissertation, ITÉF AN SSSR (Institute of Theoretical and Experimental Physics, Academy of Sciences, U.S.S.R.) (1959). L. B. Okun', Usp. Fiz. Nauk **68**, 449 (1959), Ann. Rev. Nuc. Sci. **9**, 61 (1959).

Translated by R. Lipperheide
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