WHICH IS HEAVIER, THE K_1^0 MESON OR THE K_2^0 MESON?

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Submitted to JETP editor March 12, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 605-609 (September, 1960)

A method for determining the mass difference of the K_1^0 and K_2^0 mesons is proposed which makes it possible to find not only the absolute value of this difference but also its sign. The method is based on the observation of the interference of the K_1^0 mesons produced in plates of various substances by a beam of K_2^0 mesons.

It is well known that in vacuum the K^0 meson and the $\overline{K}^0$ meson behave like coherent superpositions of time-even (K_1^0) and time-odd (K_2^0) mesons[1,2]:

$$K^0 = (K_1^0 + K_2^0)/\sqrt{2}, \qquad \overline{K}^0 = (K_1^0 - K_2^0)/\sqrt{2},$$
$$K_1^0 = (K^0 + \overline{K}^0)/\sqrt{2}, \qquad K_2^0 = (K^0 - \overline{K}^0)/\sqrt{2}. \quad (1)$$

The K_1^0 and K_2^0 mesons have different properties in relation to weak interactions; the result is that these two mesons have different decay modes and different lifetimes. The time-even meson K_1^0 is short-lived ($\tau_{K_1^0} = (1.00 \pm 0.04) \times 10^{-10}$ sec*) and decays mainly into two mesons (K_{π_2} decay). The time-odd meson K_2^0 is long-lived ($\tau_{K_2^0} = (6.1^{+1.6}_{-1.1}) \times 10^{-8}$ sec), and its main decay modes are K_{π_3}, K_{μ_3}, and K_{e_3}.

As was pointed out in the original paper by Gell-Mann and Pais,[1] the K_1^0 and K_2^0 mesons must have slightly different masses, because of the difference between their weak interactions. If the only transitions permitted in weak interactions are those with $\Delta S = \pm 1$, where S is the strangeness, then this mass difference must be of the order (cf. e.g., the paper by Zel'dovich[4])

$$\Delta m \sim g^2 m_K \sim 1/\tau_{K_1^0} \approx 10^{10}\,\text{sec}^{-1} \approx 10^{-5}\ \text{ev}, \quad (2)$$

where $g^2 \approx 10^{-13}$ is the square of the weak-interaction constant and m_K is the mass of the K meson ($\hbar = c = 1$). If transitions with $\Delta S = \pm 2$ were permitted, the mass difference would be[5]

$$\Delta m \sim g m_K \sim 10^{16}\,\text{sec}^{-1}. \quad (3)$$

Experiment[6] evidently indicates that $\Delta m \sim 10^{10}$ sec^{-1}, and consequently that transitions with $\Delta S = \pm 2$ are forbidden.

Unfortunately, because of the lack of a consistent theory of the strong interactions, even if we assume that transitions with $\Delta S = \pm 2$ are forbidden, we can predict only the order of magnitude of the absolute value of the mass difference Δm. At present we can say nothing definite about the sign of Δm, i. e., about which is heavier, the K_1^0 meson or the K_2^0 meson.

The experiments considered so far by a number of authors also cannot answer this question. For example, there is much discussion of an experiment for the determination of Δm in which one measures the number of $\overline{K}^0$ mesons that appear in a beam originally composed of K^0 mesons only. One can measure the number of $\overline{K}^0$ particles, for example, by placing in the path of the beam plates in which the $\overline{K}^0$ mesons will be captured with the production of hyperons. It is easy to see that in such an experiment one cannot measure the sign of Δm, since the number of $\overline{K}^0$ mesons produced in the time t is an even function of Δm:

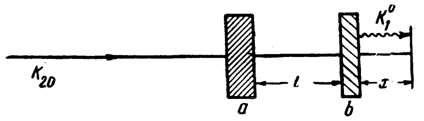
$$w(\overline{K}^0) \sim e^{-\lambda_1 t} + e^{-\lambda_2 t} - 2e^{-(\lambda_1+\lambda_2)t/2}\cos(\Delta m t), \quad (4)$$
$$\lambda_1 = 1/\tau_{K_1^0}, \qquad \lambda_2 = 1/\tau_{K_2^0},$$

t is the characteristic time of the K meson. (sic)

In the present paper we suggest a method for experimentally determining the sign of Δm.

The proposed experiment uses interference phenomena which must occur in a beam of neutral K_2^0 mesons, and will be possible only if $\Delta m \sim 10^{10}$ sec^{-1}.

Let us consider a monochromatic beam of K_2^0 mesons which strikes a target consisting of two thin plates (a and b), which are in general made of different materials and are separated by a certain distance l (see diagram).



It is well known, that K^0 and $\overline{K}^0$ mesons have different interactions with atomic nuclei. We can

*These data are taken from the report by Alvarez at the Kiev Conference of 1959.[3]

describe the passage of the K-meson wave through plate a by introducing indices of refraction n_a for K^0 mesons and $\bar{n}_a$ for $\bar{K}^0$ mesons:

$$n_a = 1 + 2\pi k^{-2} N_a f_a(0), \qquad \bar{n}_a = 1 + 2\pi k^{-2} N_a \bar{f}_a(0). \quad (5)$$

Here k is the momentum of the incident K_2^0 mesons, $f_a(0)$ is the coherent scattering amplitude for K^0 mesons scattered at angle 0^0 by the nuclei of plate a, and $\bar{f}_a(0)$ is the corresponding quantity for $\bar{K}^0$ mesons; N_a is the number of atoms in 1 cm^3. Analogous formulas hold for plate b. After passage of a beam of K_2^0 mesons through a thin plate there is an admixture of K_1^0 mesons in the transmitted wave:

$$K_2^0 = (K^0 - \bar{K}^0)/\sqrt{2} \xrightarrow{a} (1 + ikd_a n_a) K^0/\sqrt{2} - (1 + ik\bar{n}_a d_a)\bar{K}^0/\sqrt{2} = [1 + ikd_a(n_a + \bar{n}_a)/2] K_2^0 + ikd_a(n_a - \bar{n}_a) K_1^0/2, \quad (6)$$

where d_a is the thickness of plate a. If we introduce the notation

$$\tfrac{1}{2} kd_a(n_a - \bar{n}_a) = r_a e^{i\varphi_a}, \quad (7)$$

$$\tan\varphi_a = \mathrm{Im}(f_a(0) - \bar{f}_a(0)/\mathrm{Re}(f_a(0) - \bar{f}_a(0)), \quad (8)$$

$$r_a = \pi N d_a k^{-1}\{[\mathrm{Im}(f_a - \bar{f}_a)]^2 + [\mathrm{Re}(f_a - \bar{f}_a)]^2\}^{1/2} \quad (9)$$

the amplitude of the K_1^0 wave after plate a is of the form

$$K_1^0 \sim ir_a \exp\{i\varphi_a + ik_1 x - \lambda_1' x/2\},$$
$$\lambda_1' = \lambda_1/v_1\gamma_1, \quad k_1 = \sqrt{\omega^2 - m_1^2},$$
$$v_1 = k_1/\omega, \quad \gamma_1 = \omega/m_1. \quad (10)$$

It follows from Eq. (6) that in first approximation we can neglect the weakening of the K_2^0 wave in the thin plate a; similarly, we can neglect the weakening of the K_1^0 wave in plate b. Then the total amplitude of the wave of K_1^0 mesons produced in plates a and b, evaluated at distance x from plate b (see diagram) can be written in the form

$$K_1^0 \sim i[r_a \exp\{i(\varphi_a + k_1 l) - \lambda'_1 l/2\} + r_b \exp\{i(\varphi_b + k_2 l) - \lambda'_2 l/2\}] \times \exp\{ik_1 x - \lambda_1' x/2\}, \quad (11)$$

where φ_b and r_b are quantities analogous to φ_a and r_a, and the factor $\exp\{ik_2 l - \lambda_2' l/2\}$ [where $k_2 = (\omega^2 - m_2^2)^{1/2}$] in the second term is due to the fact that in the interval between plates a and b part of the K_1^0 wave goes over into the form of a K_2^0 wave. In the derivation of the formula (11) we have used the condition $d_a, d_b \ll l$.

For the total probability of K_1^0 decays to the right of plate b we have

$$w(K_{\pi_2}) = r_a^2 e^{-\lambda_1 t_0} + r_b^2 e^{-\lambda_2 t_0} + 2r_a r_b e^{-(\lambda_1+\lambda_2)t_0/2} \cos(\Delta\varphi - \Delta m t_0),$$
$$\Delta\varphi = \varphi_a - \varphi_b, \quad \Delta m = m_1 - m_2, \quad t_0 = l/v\gamma. \quad (12)$$

It can be seen from Eq. (12) that by measuring the number of K_{π_2} decays (in particular, of decays $\theta_1^0 \to \pi^+ + \pi^-$) to the right of plate b for various distances between plates a and b (different values of t_0) one can determine the magnitude and sign of Δm. It is easy to see that the sign of Δm can be determined only if the plates a and b are made of different materials. In fact, if the plates are made of the same substance, then it follows from Eq. (8) that $\Delta\varphi = 0$ and $w(K_{\pi_2})$ is an even function of Δm.

It can be seen from Eq. (12) that it is advantageous for $\Delta\varphi$ to be close to $\pi/2$. It is obvious that in order for $\Delta\varphi$ to have the optimal value the nuclear properties of plates a and b must be very different. Evidently it is desirable to make one plate of a substance with a small atomic number and the other of a substance with a large atomic number.

The formula (12) shows that to determine the sign of Δm from this experiment we must know the sign of $\Delta\varphi$. For this, in turn, we must know the signs and magnitudes of $\mathrm{Re}\, f(0)$ and $\mathrm{Re}\,\bar{f}(0)$ for the plates a and b and the magnitudes of $\mathrm{Im}\, f(0)$ and $\mathrm{Im}\,\bar{f}(0)$ (as is well known, the signs of these latter quantities are always positive). For light nuclei the information we want can be obtained from data on the interactions of K^+ and K^- mesons with these nuclei, if we use the isotopic invariance of the strong interactions.

An analysis of the experimental data on the interaction of K^- mesons with protons indicates that the interaction of K^- mesons with nuclei at energies $\lesssim 100$ Mev must evidently be of the nature of an attraction, i.e., $\mathrm{Re}\,\bar{f}(0) > 0$. This follows from the fact that the real part of the amplitude for the scattering of a K^- meson by a nucleon is positive both in the state with T = 1 and also in the state with T = 0 (T is the isotopic spin).

For K^+ mesons the situation is less definite: it is known that the amplitude for the scattering of K^+ by a nucleon in the state with T = 1 is negative (repulsion), but the data on the interaction of the K^+ meson with the neutron is so incomplete that no definite conclusion can be drawn about the sign and magnitude of the amplitude with T = 0. Therefore so far we can say nothing about the sign of $\mathrm{Re}\, f(0)$.*

*If the amplitude with T = 0 is small, as the experiments evidently indicate, we may suppose that $\mathrm{Re} f(0) < 0$.

It follows from Eq. (12) that the number of K_2^0 mesons that must be sent through the pair of plates a and b in order to observe one K_1^0 decay to the right of plate b is of the order of magnitude of r_b^{-2}.

If in the expression (9) for r we neglect f(0) in comparison with $\overline{f}(0)$, we get

$$r_b^2 \approx d^2\pi^2 N^2 k^{-2} |f_{\overline{K}^0}(0)|^2. \quad (13)$$

Assuming that plate b is made of copper and is of thickness d = 2 mm, and taking

$$\sigma_{\overline{K}^0}(\mathrm{Cu}) = \pi r_0^2 A^{2/3} \approx 10^{-24}\mathrm{cm}^2, \qquad 4\pi |f_{\overline{K}_0}(0)|^2 \sim \sigma_{\overline{K}_0}$$

for K mesons of energy 40 Mev, we get $r_b^{-2} = 4 \times 10^5$. If we use the fact that the decay $\theta_1^0 \to \pi^+ + \pi^-$ comprises two thirds of all decays of K_1^0 mesons, we find that to observe one decay $\theta_1^0 \to \pi^+ + \pi^-$ one must send about 600,000 K_2^0 mesons through the plates.

The proposed experiment can also be made with a nonmonochromatic beam of K_2^0 mesons, if from the kinematics of the decay $\theta_1^0 \to \pi^+ + \pi^-$ we determine the momentum of the incident K_1^0 meson and thus determine the time t_0 for each case of decay.

The experiment considered above is of course not the only possible one. For example, instead of observing decays $\theta_1^0 \to \pi^+ + \pi^-$ to the right of plate b, one can register the production of hyperons in this plate. The number of hyperons produced in plate b is proportional to the density of $\overline{K}^0$ mesons in the plate, and this quantity is in turn proportional to

$$\tfrac{1}{2} + \exp\{-\lambda_1 t_0/2\}\, r_a \sin(\varphi_a - \Delta m t_0) \quad (14)$$

(in this formula we have neglected terms of order r^2 and terms of order $\lambda_2 t_0$).

This version of the experiment has the important disadvantage that the effect in which we are interested is in this case a small correction of order r added to the term ½ (the presence of the plate a causes a slight change of the number of hyperons that are produced in plate b). Therefore we think that the version of the experiment considered before is better.

We note that the sign of Δm can be obtained if we study the interference effects in the decay $K_{1,2}^0 \to \pi^+ + \pi^- + \pi^0$ that have been considered recently by Weinberg and Treiman.[7] The experimental study of these effects is, however, evidently much more complicated than the experiment we are suggesting.

The writers are grateful to V. I. Veksler, Ya. B. Zel'dovich, I. Ya. Pomeranchuk, and B. M. Pontecorvo for their interest in this work and for helpful discussions.

Note added in proof (July 19, 1960). In the actual experiment it is advantageous to use thick plates, since this increases the yield of K_1^0 mesons. The calculation for the case of thick plates has been made by S. G. Matinyan. For ~1 cm of copper the yield of K_1^0 mesons is 10^{-4}.

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Translated by W. H. Furry
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