THEORY OF RELAXATION OF THE MAGNETIC MOMENT IN FERROMAGNETIC MATERIALS

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The relaxation of the magnetic moment of a ferromagnetic material is considered. It is shown that because of sd-exchange interaction between spin waves and conduction electrons, there is first established a quasi-equilibrium distribution of spin waves and conduction electrons with a definite nonequilibrium value of the projection of the magnetic moment along the axis of easiest magnetization. Then because of weak relativistic spin-orbit interaction, there is gradually established an equilibrium value of this quantity. The relaxation time of the projection of the magnetic moment along the axis of easiest magnetization is independent of temperature and of order of magnitude 10^{-8} to 10^{-9} sec.

1. Kinetic and relaxation phenomena in ferromagnetic materials are determined by various processes of interaction of spin waves with conduction electrons and with one another. In the temperature range

$$\Theta_c \gg T \gg 4\Theta_c (Ja/\hbar v_0)^2$$

(Θ_c is a quantity of the order of magnitude of the Curie temperature, J is the sd-exchange integral, a is the lattice constant, and v_0 is the limiting Fermi velocity), the strongest interaction is the sd-exchange interaction of spin waves and conduction electrons; because of it, there is established a quasi-equilibrium distribution of conduction electrons and spin waves, corresponding to a definite value of the magnetic moment of the body.

The transition to the equilibrium value of the magnetic moment is caused by a relativistic spin-orbit interaction of spin waves and conduction electrons. This interaction is weak in comparison with the sd-exchange interaction, and therefore the relaxation of the magnetic moment proceeds slowly in comparison with the process of establishment of the quasi-equilibrium distribution functions.

2. We shall use a model of a ferromagnet that starts with the concept of two groups of electrons — the conduction electrons (s electrons) and the ferromagnetic d electrons.[1] The s and d electron interaction energy operator is the sum of a Hamiltonian $\mathcal{H}_1$, which describes the exchange interaction between s and d electrons, and a Hamiltonian $\mathcal{H}_2$, which describes the interaction of the magnetic moment $\mathbf{M}(\mathbf{r}, t)$ of the d electrons with the conduction-electron current $\mathbf{j}(\mathbf{r}, t)$.

The Hamiltonian $\mathcal{H}_1$ has the form

$$\mathcal{H}_1 = \mu \int \varphi^+ (\mathbf{r}, t)\, \sigma\, \varphi (\mathbf{r}, t)\, \mathbf{H}^{(e)} (\mathbf{r}, t) d\mathbf{r}, \quad (1)$$

where μ is the Bohr magneton, σ is the spin operator of a conduction electron, φ^+ and φ are the creation and absorption operators of a conduction electron, and $\mathbf{H}^{(e)}$ is the exchange magnetic field, equal to

$$\mathbf{H}^{(e)} (\mathbf{r}, t) = \int J (\mathbf{r} - \mathbf{r}')\, \mathbf{M} (\mathbf{r}', t)\, d\mathbf{r}'.$$

The Hamiltonian $\mathcal{H}_2$ has the form

$$\mathcal{H}_2 = \int \mathbf{M} (\mathbf{r}, t)\, \mathbf{H} (\mathbf{r}, t)\, d\mathbf{r}, \quad (2)$$

where $\mathbf{H}(\mathbf{r}, t)$ is the magnetic field produced by the conduction-electron current:[2]

$$\mathbf{H} (\mathbf{r}, t) = \frac{1}{c} \int \frac{[\mathbf{j} (\mathbf{r}', t), \mathbf{r} - \mathbf{r}']}{|\mathbf{r} - \mathbf{r}'|^3} e^{-q|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}',$$

$$q = \left(\frac{4\pi n e^2}{mc^2}\right)^{1/2}$$

(q^{-1} is the shielding radius; e, m, and n are respectively the charge, mass, and density of the electrons; c is the speed of light).

The current operator $\mathbf{j}$ is connected with φ^+ and φ by the relation

$$\mathbf{j} = (ie\hbar/2m) \{\varphi \nabla \varphi^+ - (\nabla \varphi) \varphi^+\}.$$

The operators φ^+ and φ can be expanded as series of Bloch wave functions $u_{\mathbf{k}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$:

$$\varphi_\sigma (\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} c_{\mathbf{k}\sigma} u_{\mathbf{k}} (\mathbf{r}) e^{i\mathbf{k}\mathbf{r}}, \quad \varphi^+_\sigma (\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} c^+_{k\sigma} u^*_{\mathbf{k}} (\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}},$$

where $c^+_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are the creation and annihilation operators of an electron with wave vector $\mathbf{k}$ and spin projection σ.

On further expressing $\mathbf{M}(\mathbf{r}, t)$ in terms of the creation and annihilation operators $a^+_{\mathbf{f}}$, $a_{\mathbf{f}}$ of the spin waves, we finally express the Hamiltonian of the ferromagnet in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{1\,int} + \mathcal{H}_{2\,int}; \tag{3}$$

$$\mathcal{H}_0 = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}\sigma} c^+_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{f}} \varepsilon_{\mathbf{f}} a^+_{\mathbf{f}} a_{\mathbf{f}}, \tag{4}$$

$$\mathcal{H}_{1\,int} = \mu\left(\frac{4\mu M_0}{V}\right)^{1/2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{f}} J_{\mathbf{f}} \Lambda_{\mathbf{k}\mathbf{k}'} \{a^+_{\mathbf{f}} c^+_{\mathbf{k}'_-} c_{\mathbf{k}_+} + a_{-\mathbf{f}} c^+_{\mathbf{k}'_+} c_{\mathbf{k}_-}\} \Delta(\mathbf{k}' - \mathbf{k} + \mathbf{f}), \tag{5}$$

$$\mathcal{H}_{2\,int} = \frac{\pi i e}{c}\left(\frac{4\mu M_0}{V}\right)^{1/2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{f},\sigma} \Lambda_{\mathbf{k}\mathbf{k}'} f^{-2}\left(1 - \frac{q}{f}\tan^{-1}\frac{f}{q}\right) \times c_{\mathbf{k}\sigma} c^+_{\mathbf{k}'\sigma} \{a^+_{\mathbf{f}}\, \mathbf{f} \times [\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{k}'}]^- + a_{-\mathbf{f}}\, \mathbf{f} \times [\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{k}'}]^+\} \Delta(\mathbf{k}' - \mathbf{k} + \mathbf{f}). \tag{6}$$

Here $\epsilon_{\mathbf{f}} = \Theta_c(af)^2 + \epsilon_0$ is the energy of a spin wave; $E_{\mathbf{k}\sigma} = E_{\mathbf{k}} + 2\mu M_0 J_0 \sigma$ is the energy of an electron with wave vector $\mathbf{k}$ and spin projection σ ($\sigma = \pm\frac{1}{2}$); $v_{\mathbf{k}} = \hbar^{-1}\partial E_{\mathbf{k}}/\partial\mathbf{k}$ is the velocity of the conduction electrons; $A^\pm = A_x \pm iA_y$;

$$\Lambda_{\mathbf{k}\mathbf{k}'} = \frac{1}{\Omega}\int_\Omega u^*_{\mathbf{k}} u_{\mathbf{k}'}\, d\mathbf{r}, \quad J_{\mathbf{f}} = \int J(\mathbf{r}) e^{i\mathbf{f}\mathbf{r}} d\mathbf{r}, \quad \Delta(\mathbf{k}) = \begin{cases} 1, & \mathbf{k} = 0 \\ 0, & \mathbf{k} \neq 0 \end{cases} \tag{7}$$

(Ω is the volume of the elementary cell).

The operator $\mathcal{H}_{1\,int}$ describes the creation and annihilation of a spin wave with change of the projection of the electron spin (sd-exchange interaction), and the operator $\mathcal{H}_{2\,int}$ describes the creation and annihilation of a spin wave without change of the projection of the electron spin.

3. The change in unit time of the number of spin waves with wave vector $\mathbf{f}$, caused by the sd-exchange interaction $\mathcal{H}_{1\,int}$ and the spin-orbit interaction $\mathcal{H}_{2\,int}$, is determined by the formulas

$$n_{\mathbf{f}} = \dot{n}_{\mathbf{f}}^{\,col} \equiv \mathcal{L}_{\mathbf{f}}\{n, N\},$$
$$\mathcal{L}_{\mathbf{f}}\{n, N\} = \mathcal{L}_{\mathbf{f}}^{(e)}\{n, N\} + \mathcal{L}_{\mathbf{f}}^{(r)}\{n, N\}, \tag{8}$$

where the collision integrals $\mathcal{L}_{\mathbf{f}}^{(e)}$ and $\mathcal{L}_{\mathbf{f}}^{(r)}$, connected respectively with the Hamiltonians $\mathcal{H}_{1\,int}$ and $\mathcal{H}_{2\,int}$, are equal to

$$\mathcal{L}_{\mathbf{f}}^{(e)}\{n, N\} = \frac{2\pi}{\hbar}\frac{4\mu^3 M_0}{V} \sum_{\mathbf{k},\mathbf{k}'} |J_{\mathbf{f}}\Lambda_{\mathbf{k}'\mathbf{k}}|^2 \{(n_{\mathbf{f}} + 1) N_{\mathbf{k}_+}(1 - N_{\mathbf{k}'_-}) - n_{\mathbf{f}} N_{\mathbf{k}'_-}(1 - N_{\mathbf{k}_+})\} \Delta(\mathbf{k}' - \mathbf{k} + \mathbf{f})\, \delta(\varepsilon_{\mathbf{f}} - E_{\mathbf{k}_+} + E_{\mathbf{k}'_-}). \tag{9}$$

$$\mathcal{L}_{\mathbf{f}}^{(r)}\{n, N\} = \frac{2\pi}{\hbar}\frac{\pi^2 e^2}{c^2}\frac{4\mu M_0}{V} \sum_{\mathbf{k},\mathbf{k}',\sigma} \left|\frac{\Lambda_{\mathbf{k}\mathbf{k}'}}{f^2}\right|^2 \left(1 - \frac{q}{f}\tan^{-1}\frac{f}{q}\right)^2 \times |\mathbf{f} \times [\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{k}'}]^+|^2 \{(n_{\mathbf{f}} + 1)(1 - N_{\mathbf{k}'\sigma}) N_{\mathbf{k}\sigma} - n_{\mathbf{f}}(1 - N_{\mathbf{k}\sigma}) N_{\mathbf{k}'\sigma}\} \Delta(\mathbf{k} - \mathbf{k}' - \mathbf{f})\, \delta(\varepsilon_{\mathbf{f}} + E_{\mathbf{k}} - E_{\mathbf{k}}). \tag{10}$$

Here $n_{\mathbf{f}}$ and $N_{\mathbf{k}\sigma}$ are the distribution functions of the spin waves and of the electrons, respectively.

The change in unit time of the number of electrons in the state $\mathbf{k}$, σ is determined by the following kinetic equation:

$$\dot{N}_{\mathbf{k}\sigma} = (\dot{N}_{\mathbf{k}\sigma})^{col} \equiv L^{(e)}_{\mathbf{k}\sigma}\{n, N\} + L^{(r)}_{\mathbf{k}\sigma}\{n, N\}, \tag{11}$$

where the collision integrals $L^{(e)}_{\mathbf{k}\sigma}$ and $L^{(r)}_{\mathbf{k}\sigma}$ are connected with the Hamiltonians $\mathcal{H}_{1\,int}$ and $\mathcal{H}_{2\,int}$ and are equal to

$$L^{(e)}_{\mathbf{k}_+}\{n, N\} = \frac{8\pi\mu^3 M_0}{\hbar V} \sum_{\mathbf{f},\mathbf{k}'} |J_{\mathbf{f}}\Lambda_{\mathbf{k}\mathbf{k}'}|^2 \{n_{\mathbf{f}}(1 - N_{\mathbf{k}_+}) N_{\mathbf{k}'_-} - (n_{\mathbf{f}} + 1)(1 - N_{\mathbf{k}'_-}) N_{\mathbf{k}_+}\} \Delta(\mathbf{k}' - \mathbf{k} + \mathbf{f})\, \delta(E_{\mathbf{k}'_-} - E_{\mathbf{k}_+} + \varepsilon_{\mathbf{f}}), \tag{12}$$

$$L^{(r)}_{\mathbf{k}_+}\{n, N\} = \frac{8\pi^2 e^2}{\hbar c^2}\frac{\mu M_0}{V} \sum_{\mathbf{f},\mathbf{k}'} \left|\frac{\Lambda_{\mathbf{k}\mathbf{k}'}}{f^2}\right|^2 \left(1 - \frac{q}{f}\tan^{-1}\frac{f}{q}\right)^2 \times |\mathbf{f} \times [\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{k}'}]^+|^2 \{[(n_{\mathbf{f}} + 1)(1 - N_{\mathbf{k}_+}) N_{\mathbf{k}'_+} - n_{\mathbf{f}} N_{\mathbf{k}_+}(1 - N_{\mathbf{k}'_+})] \Delta(\mathbf{k} - \mathbf{k}' + \mathbf{f}) \times \delta(E^+_{\mathbf{k}'} - E^+_{\mathbf{k}} - \varepsilon_{\mathbf{f}}) - [(n_{\mathbf{f}} + 1) N_{\mathbf{k}_+}(1 - N_{\mathbf{k}'_+}) - n_{\mathbf{f}}(1 - N_{\mathbf{k}_+}) N_{\mathbf{k}'_+}] \Delta(\mathbf{k} - \mathbf{k}' - \mathbf{f})\, \delta(E^+_{\mathbf{k}'} - E^+_{\mathbf{k}} + \varepsilon_{\mathbf{f}})\}. \tag{13}$$

The collision operators $L^{(e)}_{\mathbf{k}_-}$ and $L^{(r)}_{\mathbf{k}_-}$ have a similar form.

By use of the expressions for the collision operators, one can find the mean rate for the various processes of collision of spin waves with electrons. This rate is determined in accordance with the formula

$$W = \frac{1}{\tau} = -\sum\left(\frac{\delta\mathcal{L}}{\delta n}\right)_0 n^0 \Big/ \sum n^0, \tag{14}$$

where $(\delta\mathcal{L}/\delta n)_0$ is the variational derivative of the collision integral with respect to the distribution function, evaluated at the equilibrium values of the distribution functions of the spin waves, $n^0_{\mathbf{f}}$, and of the electrons $N^0_{\mathbf{k}\sigma}$.

By use of the expression (9) for $\mathcal{L}^{(e)}_{\mathbf{f}}$ and of formula (14), one can calculate the mean rate of emission or absorption of a spin wave by an electron, as a result of exchange interaction:

$$\frac{1}{\tau_s^{(e)}} = \frac{(ak_0)^2}{\pi^{1/2}\zeta(3/2)}\frac{\Theta_0}{\hbar}\left(\frac{T}{\Theta_c}\right)^{1/2}\Psi(e^{-\Theta_0/T}), \tag{15}$$

where T is the temperature; $\Theta_0 = 4\Theta_c(J/E^*)^2$; $E^* = hv_0/a$; v_0 and $\hbar k_0$ are the limiting Fermi velocity and momentum of an electron; and

$$\Psi(x) = \frac{1}{2}\ln(1 - x)\ln\frac{x^2}{1 - x} + \int_0^{x/(1-x)} \frac{\ln(1 + t)}{t}\, dt.$$

The expression for $1/\tau_s^{(e)}$ simplifies considerably in the limiting cases of high and low temperatures:

$$\frac{1}{\tau_s^{(e)}} \approx \frac{\sqrt{\pi}}{6\zeta(^3/_2)}(ak_0)^2\frac{\Theta_0}{\hbar}\left(\frac{T}{\Theta_c}\right)^{1/2} \quad \text{for } \Theta_c \gg T \gg \Theta_0,$$

$$\frac{1}{\tau_s^{(e)}} \approx \frac{(ak_0)^2}{\pi^{3/2}\zeta(^3/_2)}\frac{\Theta_0}{\hbar}\left(\frac{\Theta_0}{\Theta_c}\right)^{1/2}\left(\frac{\Theta_0}{T}\right)^{1/2}e^{-\Theta_0/T} \text{ for } \Theta_0 \gg T. \quad (16)$$

We note that for $T = \Theta_0$, both limiting expressions in formula (16) have the same order of magnitude.

In a similar way one can calculate the mean rate of emission or absorption of a spin wave by an electron, as a result of the relativistic interaction $\mathscr{H}_{2\,\text{int}}$:

$$\frac{1}{\tau_s^{(r)}} \approx \frac{512\sqrt{\pi}}{3\zeta(^3/_2)}\frac{\Theta_c}{\hbar}\left(\frac{\Theta_c}{T}\right)^{1/2}(ak_0)^2\left(\frac{\mu M_0 m a^2}{\hbar^2}\right)^2 \ln y_0^{-1},$$

$$y_0 = (\Theta_c/T)^{1/2} aq \qquad (\Theta_c \gg T \gg \Theta_c(aq)^2). \quad (17)$$

On putting $\Theta_c = 10^3$ °K, $ak_0 \sim 1$, $a \sim 10^{-8}$ cm, $m \sim 10^{-27}$ g, and $T \sim \Theta_0 \sim 10$°K, we get $\tau_s^{(e)} \approx 10^{-11}$ sec, $\tau_s^{(2)} \approx 10^{-8}$ sec; that is, $\tau_s^{(e)} \ll \tau_s^{(r)}$ for $T \geq \Theta_0 \approx 10$°K. This means that in the temperature range $T \gtrsim \Theta_0$, the inequality $\mathscr{L}^{(e)} \gg \mathscr{L}^{(r)}$ holds.

The mean rate of scattering of electrons by a spin wave, as a result of exchange interaction, can be calculated by use of the expression (12) for $L_{\mathbf{k}}^{(e)}$. We give here the final answer for $1/\tau_e^{(e)}$ in the limiting cases of low and high temperatures:[3]

$$\frac{1}{\tau_e^{(e)}} \approx \frac{3}{8\pi}\frac{E^*}{\hbar}\left(\frac{T}{\Theta_c}\right)^2 e^{-\Theta_0/T} \text{ for } T \ll \Theta_0,$$

$$\frac{1}{\tau_e^{(e)}} \approx \frac{3}{8\pi}\frac{E^*}{\hbar}\frac{\Theta_0}{\Theta_c}\frac{T}{\Theta_c}\left(\ln\frac{T}{\Theta_c} - 1\right) \text{for } \Theta_c \gg T \gg \Theta_0. \quad (18)$$

By use of the expression (13) for $L^{(r)}$, one can calculate the mean rate of scattering of an electron by a spin wave, as a result of the relativistic interaction $\mathscr{H}_{2\,\text{int}}$:

$$\frac{1}{\tau_e^{(r)}} \approx \frac{\mu M_0}{\hbar}\left(\frac{e^2 k_0}{mc^2}\right)\left(\frac{1}{aq}\right)^2\frac{T}{\Theta_c}. \quad (19)$$

By comparison of the expressions (18) and (19), it is easily verified that in the temperature range $T \gtrsim \Theta_0$, the inequality $\tau_e^{(e)} \ll \tau_e^{(r)}$ holds.

4. Thus, in the temperature range $T \gtrsim \Theta_0$, the strongest interaction is the exchange. Consequently, in determining the quasistationary distribution functions n and N one can start from the equations[4]

$$L^{(e)}\{n, N\} = 0, \qquad \mathscr{L}^{(e)}\{n, N\} = 0. \quad (20)$$

It is easily seen that the general solution of these equations has the form

$$n_{\mathbf{f}} = \left[\exp\left(\frac{\varepsilon_{\mathbf{f}} - \gamma}{T}\right) - 1\right]^{-1}, \; N_{\mathbf{k}\pm} = \left[\exp\left(\frac{E_{\mathbf{k}\pm} - \zeta_\pm}{T}\right) + 1\right]^{-1}, \quad (21)$$

where $\zeta_+ = \gamma + \zeta_-$.

By use of the conservation law for the number of electrons,

$$\sum_{\mathbf{k}\sigma} N_{\mathbf{k}\sigma} = \text{const},$$

we get

$$\zeta_+ = E_0 + \gamma/2, \qquad \zeta_- = E_0 - \gamma/2.$$

The arbitrary γ can be related to the size of the projection of the magnetic moment along the axis of easiest magnetization (the z axis):

$$\mathfrak{M}_z = -\mu\int\varphi^+\sigma_z\varphi\,dr + \int M_z dr$$
$$= M_0 V - 2\mu\sum_{\mathbf{f}} n_{\mathbf{f}} + \mu\sum_{\mathbf{k}}(N_{\mathbf{k}-} - N_{\mathbf{k}+}). \quad (22)$$

The possibility of the existence of solutions of equations (20) with an arbitrary value of the chemical potential γ is connected with the fact that the magnetic moment of the body commutes with the exchange-interaction Hamiltonian $\mathscr{H}_{1\,\text{int}}$. We note that in formula (22) we have not included the contribution of the orbital magnetic moment of the s electrons; this is permissible if the length of the free path of the electrons is much smaller than the Larmor radius in a field M_0.

We now take into consideration the relativistic interaction $\mathscr{H}_{2\,\text{int}}$. Then the distribution (21), since it satisfies (20), will no longer satisfy the equations

$$\mathscr{L}^{(e)}\{n, N\} + \mathscr{L}^{(r)}\{n, N\} = 0, \quad L^{(e)}\{n, N\} + L^{(r)}\{n, N\} = 0.$$

Since, however, $L^{(e)} \gg L^{(r)}$ and $\mathscr{L}^{(e)} \gg \mathscr{L}^{(r)}$, the distribution (21) with a slowly varying parameter γ can satisfy approximately the kinetic equations

$$\dot{n}_{\mathbf{f}} = \mathscr{L}_{\mathbf{f}}^{(e)} + \mathscr{L}_{\mathbf{f}}^{(r)}, \qquad \dot{N}_{\mathbf{k}\sigma} = L_{\mathbf{k}\sigma}^{(e)} + L_{\mathbf{k}\sigma}^{(r)}.$$

Since the size of the projection of the magnetic moment, $\mathfrak{M}_z$, is determined by the occupancy numbers of the spin waves and of the electrons, it is possible, by use of the kinetic equations (8) and (11) and of the quasi-equilibrium distribution functions (21), to determine the change of magnetic moment with time caused by the relativistic spin-orbit interaction. On differentiating equation (22) for $\mathfrak{M}_z$ with respect to time, we get

$$\dot{M}_z \equiv \frac{\dot{\mathfrak{M}}_z}{V} = -\frac{\mu}{2\pi^2}\left\{\frac{k_0^2}{\hbar v_0} + \frac{\pi T}{2a^3\Theta_c}\frac{1}{\sqrt{\varepsilon_0\Theta_c}}\right\}\dot{\gamma}$$
$$= \frac{\mu}{V}\sum_{\mathbf{k}}\{(\dot{N}_{\mathbf{k}-})^{\text{col}} - (\dot{N}_{\mathbf{k}+})^{\text{col}}\} - \frac{2\mu}{V}\sum_{\mathbf{f}}\dot{n}_{\mathbf{f}}^{\text{col}}.$$

Since the relativistic interaction $\mathscr{H}_{2\,\text{int}}$ does not change the number of electrons with a given spin projection,

$$\sum_{\mathbf{k}}(\dot{N}_{\mathbf{k}\pm})^{\text{col}} = 0.$$

By using the expression (10) for $\mathscr{L}_{\mathbf{k}}^{(\mathrm{r})}$, one easily gets the following equation for the change of the quantity γ with time:

$$\dot{\gamma} = -\gamma/\tau, \tag{23}$$

$$\frac{1}{\tau} \approx \frac{8(\pi^2 - 8)}{3\cdot 137}\left(\frac{v_0 k_0}{cq}\right)^2 \frac{c}{v_0}\left(\frac{\varepsilon_0 \Theta_c}{E^{*2}}\right)^{1/2} \frac{\mu M_0}{\hbar}. \tag{24}$$

The change of the projection of the magnetic moment, $\mathfrak{M}_z$, with time is determined by the formula

$$(\mathfrak{M}_z - \overline{\mathfrak{M}}_z) = (\mathfrak{M}_{z0} - \overline{\mathfrak{M}}_z)\, e^{-t/\tau}, \tag{25}$$

where $\overline{\mathfrak{M}}_z$ is the equilibrium value of the magnetic moment at the given temperature. On setting $v_0 \sim 10^8$ cm/sec, $\epsilon_0 \sim 1°$K, $n \sim 10^{22}$ cm^{-3}, $M_0 \sim 10^3$ gauss, $a \sim 10^{-8}$ cm, and $\Theta_c \sim 10^{3}$°K, we get $1/\tau \sim 10^8$ to 10^9 sec^{-1}. We emphasize that the relaxation time of the magnetic moment is independent of temperature.

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