RELAXATION OF THE MAGNETIC MOMENT IN AN ANTIFERROMAGNETIC DIELECTRIC

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The relaxation time of the magnetic moment of an antiferromagnetic dielectric is calculated in the case in which the nonequilibrium magnetic moment is perpendicular to the axis of the crystal. It is shown that at temperatures satisfying condition (17), the relaxation time is inversely proportional to the first power of the temperature.

THE present work deals with the problem of relaxation of the magnetic moment in an antiferromagnetic dielectric in the case in which the external magnetic field and the magnetic moment of the body are perpendicular to the crystal axis z. After switching off of the magnetic field, the magnetic moments of the sublattices begin to relax, turning toward the crystal axis; that is, the magnetic moment induced by application of the field disappears. Since the nonequilibrium value of this magnetic moment is determined by the number of spin waves with momentum k = 0, the magnetic-moment relaxation time found here determines, in order of magnitude, the line width in uniform antiferromagnetic resonance.

As is known, the exchange-interaction Hamiltonian commutes with the total magnetic moment of the body, and therefore it cannot change the previously induced nonequilibrium magnetic moment. The change of the magnetic moment of the body will occur because of weak relativistic interaction.

1. We write the Hamiltonian of an antiferromagnetic dielectric in the following form:

$$\mathcal{H} = \int dV \left[\frac{\alpha}{2}\left(\frac{\partial \mathbf{M}_1}{\partial x_i}\right)^2 + \frac{\alpha}{2}\left(\frac{\partial \mathbf{M}_2}{\partial x_i}\right)^2 + \alpha_{12}\frac{\partial M_{1k}}{\partial x_i}\frac{\partial M_{2k}}{\partial x_i} + \gamma \mathbf{M}_1\mathbf{M}_2 + \frac{\beta}{2}(M_{1x}^2 + M_{1y}^2 + M_{2x}^2 + M_{2y}^2) + \frac{\mathbf{h}^2}{8\pi}\right], \quad (1)$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetic moments of the sublattices; $\mathbf{h}$ is the magnetic field produced by the oscillations of the magnetic moment; α, α_{12}, and γ are constants related to the exchange interaction; and β is the magnetic anisotropy constant.

By analogy with the work of Holstein and Primakoff,[1] we introduce the spin-wave creation and annihilation operators a_j^+ and a_j (j = 1, 2), connected with the sublattice magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$ by the relations

$$m_j^- = M_{jx} - iM_{jy} \approx (2\mu M)^{1/2}[a_j - (\mu/4M)a_j^+a_ja_j],$$
$$m_j^+ = M_{jx} + iM_{jy} \approx (2\mu M)^{1/2}[a_j^+ - (\mu/4M)a_j^+a_j^+a_j],\quad (2)$$
$$m_j^z = M_{jz} - M = -\mu a_j^+a_j.$$

Here μ is the Bohr magneton and M is the saturation magnetic moment of a sublattice. The operators a_j^+ and a_j are subject to the usual commutation rule

$$[a_i(\mathbf{r}), a_j^+(\mathbf{r}')] = \delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \quad (3)$$

By using formulas (2) and expanding the operators $a_j^+(\mathbf{r})$ and $a_j(\mathbf{r})$ in Fourier series, one can write the Hamiltonian (1) in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}; \quad (4)$$

$$\mathcal{H}_0 = \sum_{\mathbf{k}}\{\tfrac{1}{2}A_{\mathbf{k}}a_{1\mathbf{k}}^+a_{1\mathbf{k}} + \tfrac{1}{2}A_{\mathbf{k}}a_{2\mathbf{k}}^+a_{2\mathbf{k}} + B_{\mathbf{k}}a_{1\mathbf{k}}a_{2-\mathbf{k}} + C_{\mathbf{k}}a_{1\mathbf{k}}^+a_{1-\mathbf{k}}^+ + C_{\mathbf{k}}a_{2\mathbf{k}}a_{2-\mathbf{k}} + 2C_{\mathbf{k}}a_{1\mathbf{k}}^+a_{2\mathbf{k}}\} + \text{Herm. adj.} \quad (5)$$

$$\mathcal{H}_{int} = -\frac{\beta\mu^2}{2V}\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4}(a_{1\mathbf{k}_1}^+a_{1\mathbf{k}_2}^+a_{1\mathbf{k}_3}a_{1\mathbf{k}_4} + a_{2\mathbf{k}_1}^+a_{2\mathbf{k}_2}^+a_{2\mathbf{k}_3}a_{2\mathbf{k}_4})\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4). \quad (6)$$

Here the following symbols have been introduced:

$$A_{\mathbf{k}} = \mu M(\alpha k^2 + 2\pi\sin^2\theta_{\mathbf{k}} + \gamma + \beta),$$
$$B_{\mathbf{k}} = \mu M(\alpha_{12}k^2 + 2\pi\sin^2\theta_{\mathbf{k}} + \gamma),$$
$$C_{\mathbf{k}} = \pi\mu M\sin^2\theta_{\mathbf{k}}\exp(-2i\varphi_{\mathbf{k}}), \quad (7)$$

where θ_k and φ_k are the azimuthal and polar angles of the wave vector k; $\Delta(\mathbf{k}) = 1$ for k = 0, = 0 for k ≠ 0. In $\mathcal{H}_{int}$ we have not included terms containing products of three operators $a_{jk}^+a_{jk}$; as will become clear later, these will not interest us.

2. To find the spin-wave spectrum of the antiferromagnetic dielectric, we diagonalize the Hamiltonian (4). To this end, we go over from the operators a_{jk} and a_{jk}^+ to the operators c_{jk} and c_{jk}^+[2,3]:

$$a_{1k} = U_{11}c_{1k}e^{-i\varepsilon_1 t/\hbar} + U_{12}c_{2k}e^{-i\varepsilon_2 t/\hbar} + V_{11}^+ c_{1-k}^+ e^{i\varepsilon_1 t/\hbar} + V_{12}^+ c_{2-k}^+ e^{i\varepsilon_2 t/\hbar},$$
$$a_{2k} = U_{22}c_{2k}e^{-i\varepsilon_2 t/\hbar} + U_{21}c_{1k}e^{-i\varepsilon_1 t/\hbar} + V_{22}^+ c_{2-k}^+ e^{i\varepsilon_2 t/\hbar} + V_{21}^+ c_{1-k}^+ e^{i\varepsilon_1 t/\hbar}. \quad (8)$$

The amplitudes U_{ij} and V_{ij} in (8) are found with the aid of the equation of motion of the operators a_{jk},

$$\dot{a}_{jk} = (i/\hbar)[\mathcal{H}_0, a_{jk}]. \quad (9)$$

On substituting formula (8) in Eq. (9) and equating coefficients of c_{jk} and c_{jk}^+, we get the system of linear homogeneous equations

$$\begin{aligned} &(A_k - \varepsilon_k)U_{11} + B_k V_{21} + 2C_k V_{11} + 2C_k^+ U_{21} = 0,\\ &2C_k U_{11} + 2C_k V_{21} + (A_k + \varepsilon_k)V_{11} + B_k U_{21} = 0,\\ &B_k U_{11} + (A_k + \varepsilon_k)V_{21} + 2C_k^+ V_{11} + 2C_k^+ U_{21} = 0,\\ &2C_k U_{11} + 2C_k V_{21} + B_k V_{11} + (A_k - \varepsilon_k)U_{21} = 0, \end{aligned} \quad (10)$$

and similar equations in which, instead of the quantities U_{11}, U_{21}, V_{11}, and V_{21}, there enter the quantities U_{22}^+, U_{12}^+, V_{22}^+, and V_{12}^+.

On solving (10), we find the dispersion law for spin waves in an antiferromagnetic dielectric:

$$\varepsilon_{1,2}(k) = \mu M\sqrt{2\gamma[\beta + (\alpha - \alpha_{12})k^2]}\,[1 \pm (\pi/\gamma)\sin^2\theta_k]. \quad (11)$$

Here θ_k is the angle between the z axis and the direction of the wave vector **k**. The upper and lower signs correspond to two energy branches, the difference between which is $\Delta\epsilon \sim \mu M$.

For the amplitudes, from (8), we get the following expressions:

$$\begin{aligned} U_{11} = U_{22} = U_{12} = U_{21} \equiv U_k &\approx -\frac{B_k}{2\varepsilon_k}\left(\frac{2\varepsilon_k}{\varepsilon_k + A_k}\right)^{1/2},\\ V_{11} = V_{22} = V_{12} = V_{21} \equiv V_k &\approx -\left(\frac{\varepsilon_k + A_k}{2\varepsilon_k}\right)^{1/2}. \end{aligned} \quad (12)$$

By use of formulas (8) and (12), one can express the Hamiltonian for the interaction of spin waves with one another in terms of the operators c_{ik} and c_{ik}^+:

$$\mathcal{H}_{int} = \mathcal{H}_{int}^{(1)} + \mathcal{H}_{int}^{(2)};$$
$$\begin{aligned} \mathcal{H}_{int}^{(1)} = -\frac{\beta\mu^2}{2V}\sum_{k_1k_2k_3k_4} \{ &[\Phi^{(1)}_{k_1k_2k_3k_4} c_{1k_1}^+ c_{1k_2}^+ c_{1k_3} c_{1k_4} + \Phi^{(2)}_{k_1k_2k_3k_4} c_{1k_1}^+ c_{2k_2}^+ c_{1k_3} c_{2k_4}]\,\Delta(k_1 + k_2 - k_3 - k_4)\\ &+ [\Psi^{(1)}_{k_1k_2k_3k_4} c_{1k_1}^+ c_{1k_2}^+ c_{1k_3}^+ c_{1k_4} + \Psi^{(2)}_{k_1k_2k_3k_4} c_{1k_1}^+ c_{2k_2} c_{2k_3} c_{2k_4}]\\ &\times \Delta(k_1 + k_2 + k_3 - k_4) + \text{Herm. adj.}\}, \end{aligned} \quad (13)$$

where

$$\Phi^{(1)}_{k_1k_2k_3k_4} = \tfrac{1}{2}(U_{k_1}U_{k_2}U_{k_3}U_{k_4} + V_{k_1}V_{k_2}V_{k_3}V_{k_4}),$$
$$\Phi^{(2)}_{k_1k_2k_3k_4} = 2U_{k_1}V_{k_2}U_{k_3}V_{k_4},$$
$$\Psi^{(1)}_{k_1k_2k_3k_4} = 2U_{k_1}V_{k_2}U_{k_3}V_{k_4}, \quad \Psi^{(2)}_{k_1k_2k_3k_4} = 2V_{k_1}V_{k_2}U_{k_3}V_{k_4}.$$

$\mathcal{H}_{int}^{(2)}$ is obtained from $\mathcal{H}_{int}^{(1)}$ by the substitution $c_{1k} \leftrightarrow c_{2k}$.

We now calculate the mean value of the square of the magnetic moment and of the square of its component perpendicular to the crystal axis:

$$\langle\mathfrak{M}^2\rangle = \left\langle\left[\int(\mathbf{M}_1 + \mathbf{M}_2)\,dV\right]^2\right\rangle,$$
$$\langle\mathfrak{M}_\perp^2\rangle = \left\langle\left[\int(M_{1\perp} + M_{2\perp})\,dV\right]^2\right\rangle. \quad (14)$$

With the aid of formulas (2), (8), and (11), the quantities $\langle\mathfrak{M}^2\rangle$ and $\langle\mathfrak{M}_\perp^2\rangle$ can be expressed in terms of the occupancy numbers n_{10} and n_{20} of spin waves with momentum $\mathbf{k} = 0$.

The averaged quantities $\mathfrak{M}^2$ and $\mathfrak{M}_\perp^2$ have the following form:

$$\langle\mathfrak{M}^2\rangle = \langle\mathfrak{M}_\perp^2\rangle = 2\mu_{eff} MV[(1 + \cos 2\varphi_0)n_{10} + (1 - \cos 2\varphi_0)n_{20}],$$

where φ_0 is the polar angle of the spin wave vector with $\mathbf{k} = 0$, and

$$\mu_{eff} = \mu\frac{(\varepsilon_0 + A_0)^2 + B_0^2}{(\varepsilon_0 + A_0)^2 - B_0^2} \approx \mu\left(\frac{\gamma}{2\beta}\right)^{1/2}.$$

To determine φ_0, it is necessary to take into account the boundary conditions on the vectors **M** and **H**. In the case in which the antiferromagnet fills the half-space $x > 0$ and the crystal axis is directed parallel to its surface, the value of φ_0 is zero. Suppressing the index 1 on n_{10}, we get

$$\langle\mathfrak{M}^2\rangle = \langle\mathfrak{M}_\perp^2\rangle = 4\mu_{eff} MVn_0. \quad (15)$$

Thus we see that the relaxation of the magnetic moment of an antiferromagnetic dielectric is determined by the number of spin waves with momentum $\mathbf{k} = 0$. From knowledge of the interaction Hamiltonian $\mathcal{H}_{int}$, one can find the change of the number of spin waves with momentum $\mathbf{k} = 0$. We remark that a change of n_0 cannot cause processes of union of two spin waves into one, or of splitting of one spin wave into two, since in these processes it is impossible to satisfy simultaneously the laws of conservation of energy and of momentum. Therefore for determination of the change of n_0 with time, as has already been indicated, it is necessary to take into account, in the Hamiltonian $\mathcal{H}_{int}$, the later terms of the expansion, describing processes that involve participation of four spin waves.

On using expression (13) for the interaction Hamiltonian that describes these processes, we get the following kinetic equation:

$$\dot{n}_0 = \mathcal{L}_0\{n\},$$

$$\mathcal{L}_0\{n\} = \frac{32\pi}{\hbar}\frac{\beta^2\mu^4}{V^2}U_0^2 \sum_{1,2,3} \{5U_1^2U_2^2U_3^2[(n_0+1)(n_1+1)n_2n_3 - n_0n_1(n_2+1)(n_3+1)]\,\Delta(\mathbf{k}_1-\mathbf{k}_2 - \mathbf{k}_3)\,\delta(\varepsilon_0+\varepsilon_1-\varepsilon_2-\varepsilon_3) + 2U_1^2U_2^2U_3^2[(n_0+1)(n_1+1)(n_2+1)n_3 - n_0n_1n_2(n_3+1)]\,\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)\,\delta(\varepsilon_0+\varepsilon_1+\varepsilon_2-\varepsilon_3)\}.$$

In writing $\mathcal{L}_0\{n\}$ we have assumed that $\epsilon_{1\mathbf{k}} \approx \epsilon_{2\mathbf{k}}$ and $U_\mathbf{k} \approx V_\mathbf{k}$; this is correct at sufficiently low temperatures, $\mu M \ll T \ll \Theta_C$. Since the occupation numbers n_0 are large ($n_0 \gg 1$), the collision operator $\mathcal{L}_0$ can be expressed approximately in the form

$$\mathcal{L}_0\{n\} = -n_0/\tau_0.$$

The kinetic equation then takes the form

$$\dot{n}_0 = -n_0/\tau_0,$$

where the relaxation time τ_0 is determined by the formula

$$\frac{1}{\tau_0} = \frac{\beta^2U_0^2\mu^4}{4\pi^5\gamma^3\sigma^6\hbar(\alpha-\alpha_{12})^3T}(e^\xi-1)\,J(T),$$

$$J(T) = \int\frac{dx_1\,dx_2\,dx_3}{x_1x_2x_3}[(x_1^2-\xi^2)(x_2^2-\xi^2)(x_3^2-\xi^2)]^{1/2} \times U_1^2U_2^2U_3^2n_1^0n_2^0n_3^0e^{x_2}[5\delta(\xi+x_1-x_2-x_3) + 2\delta(\xi+x_1+x_2-x_3)]\,\delta[(x_1^2-\xi^2)^{1/2}\mathbf{n}_1 + (x_2^2-\xi^2)^{1/2}\mathbf{n}_2 - (x_3^2-\xi^2)^{1/2}\mathbf{n}_3]. \qquad (16)$$

Here $n_\mathbf{k}^0 = (e^{\epsilon_\mathbf{k}/T}-1)^{-1}$ is the Bose equilibrium distribution function, $\xi = \epsilon_0/T$, $\sigma = \mu M/T$, and $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ are unit vectors in the directions $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$.

At temperatures

$$T \gg (\beta\mu M\Theta_c)^{1/2} \qquad (17)$$

(which corresponds to $\xi \ll 1$), the expression for $J(T)$ simplifies considerably, and except for a numerical factor of order unity we get the following expression for $1/\tau_0$:

$$\frac{1}{\tau_0} \sim \beta^2\frac{\mu M}{\hbar}\frac{\mu M}{\Theta_c}\frac{T}{\Theta_c}. \qquad (18)$$

In conclusion, the author considers it a pleasant duty to express his profound gratitude to A. I. Akhiezer and V. G. Bar'yakhtar for proposing the problem and for discussion.

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