POLARIZATION OF INTERNAL-CONVERSION ELECTRONS AND POSITRONS EMITTED AFTER BETA DECAY OF THE NUCLEUS

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We consider the correlation between the direction of emission of internal-conversion electrons and positrons and the direction of emission of β electrons from the preceding β-decaying nucleus. The calculation is conducted for allowed β transitions without taking into account the Coulomb field of the nucleus (in the Born approximation) under some very general assumptions regarding the β-interaction Hamiltonian. The expressions obtained refer to arbitrary 2^j-pole electric and magnetic types of nuclear conversion transitions. The case of VA types of β coupling (with conservation of time parity) is considered as well as the case of β transitions involving a change by unity of the nuclear spin, $\Delta I = \pm 1$ ($\Delta T = \pm 1$). A numerical computation is carried out on the polarization of internal conversion electrons emitted after β decay of the Na^{24} nucleus.

PARITY nonconservation in β decay processes makes the nucleus obtained as a result of β decay polarized in the direction of emitted β electron. It is assumed that the initial nucleus is unpolarized and the direction of emission of the neutrino is not registered. Consequently, if the β decay is followed by conversion with production of electron-positron pairs, the pair particles should be polarized in a definite manner. Let us consider the polarization of the internal-conversion electrons and positrons.

Let I_1 be the angular momentum of the parent nucleus, I_2 and m_2 the momentum and the projection of the momentum of the nucleus produced as a result of the β decay, with I_3 and m_3 the same for the final nucleus.

We are thus considering a process comprised of the $I_1 \to I_2$ (β decay) transition followed by $I_2, m_2 \to I_3, m_3$ (internal conversion with pair production). The β-decay stage of this process is described by a density matrix that characterizes the polarization state of the nucleus due to the β decay.

An expression for the density matrix was obtained by Berestetskiĭ and Rudik.[1] For an allowed β transition in the case of the S, T, A, and V variants of the interaction without allowance for the Coulomb field of the nucleus, the matrix has the form

$$\rho_{m_2 m_2'} = \frac{1}{2I_2+1}\left\{\delta_{m_2 m_2'} + \left(\frac{I_2+1}{I_2}\right)^{1/2} \xi \sum_{\mu} C^{I_2 m_2}_{I_2 m_2'; 1\mu} v^{\mu}\right\}, \qquad (1)$$

where v^{μ} are the spherical components of the β-electron velocity vector

$$v^0 = v_z, \qquad v^{\pm 1} = \mp (v_x \mp i v_y)/\sqrt{2};$$

$C^{a\alpha}_{b\beta;d\delta}$ is the Clebsch-Gordan coefficient; ξ is a constant that determines the angular distribution of the β electrons when a polarized nucleus with momentum I_2 and with an average momentum projection $\langle I_{2z}\rangle$ decays to a state with momentum I_1:

$$\xi = 2\,\mathrm{Re}\,\{(c_T c_S'^* + c_T' c_S^* - c_A c_V'^* - c_A' c_V^*)[I_2/(I_2+1)]^{1/2}\delta_{I_1 I_2} M_F M_{GT}^* + (c_T c_T'^* - c_A c_A'^*)|M_{GT}|^2 \Lambda_{I_1 I_2}\}\{(|c_S|^2 + |c_S'|^2 + |c_V|^2 + |c_V'|^2)|M_F|^2 + (|c_T|^2 + |c_T'|^2 + |c_A|^2 + |c_A'|^2)|M_{GT}|^2\}^{-1}, \qquad (2)$$

$$\Lambda_{I_1 I_2} = [I_2(I_2+1) - I_1(I_1+1) + 2]/2I_2(I_2+1),$$

$$M_F = \left(\int 1\right), \qquad M_{GT} = \left(\int \sigma\right).$$

The probability of conversion with production of electron-positron pairs, for a nucleus which has previously experienced a β decay, is[1]

$$W = \sum_{\substack{m_2 m_2' m_3 \\ MM'}} \rho_{m_2 m_2'} (I_2 m_2 | Q^{(\lambda)}_{jM} | I_3 m_3)^* (I_2 m_2' | Q^{(\lambda)}_{jM'} | I_3 m_3) \times (B^{(\lambda)}_{jM})_{21} (B^{(\lambda)}_{jM'})^*_{21}. \qquad (3)$$

Here $(I_2 m_2 | Q^{(\lambda)}_{jM} | I_3 m_3)$ is the nuclear matrix element of the conversion transition; $Q^{(\lambda)}_{jM}$ or the operator of the 2^j-pole electric ($\lambda = 1$) are magnetic ($\lambda = 0$) moments of the nucleus, corresponding to the given type of conversion transition; $B^{(\lambda)}_{jM}$ is the operator of the interaction between the electron and positron of internal conversion with the field of the multipole;

$$(B^{(\lambda)}_{jM})_{21} = \int \psi_2^* B^{(\lambda)}_{jM}\psi_1\, dr$$

is the matrix element of this operator while ψ_1, and ψ_2 are the wave functions of the positron and electron respectively.

The matrix element of the multipole moment of the nucleus can be represented in the form

$$(I_2 m_2 \,|\, Q^{(\lambda)}_{jM} \,|\, I_3 m_3)^* = Q^{(\lambda)} C^{I_2 m_2}_{I_3 m_3;\, jM}, \tag{4}$$

where $Q^{(\lambda)}$ is independent of the quantum numbers m_2, m_3, and M. Substituting (4) in (3) and taking into account the properties of the Clebsch-Gordan coefficients

$$\sum_{m_2 m_2' m_3} \delta_{m_2 m_2'} C^{I_2 m_2}_{I_3 m_3;\, jM} C^{I_2 m_2'}_{I_3 m_3;\, jM'} = \frac{2I_2+1}{2j+1}\delta_{MM'},$$

$$\sum_{m_2 m_2' m_3} C^{I_2 m_2}_{I_3 m_3;\, jM} C^{I_2 m_2'}_{I_2 m_2;\, 1\mu} C^{I_2 m_2'}_{I_3 m_3;\, jM'}$$

$$= \frac{2I_2+1}{2j+1}\,\frac{j(j+1)+I_2(I_2+1)-I_3(I_3+1)}{2\sqrt{j(j+1)I_2(I_2+1)}}\, C^{jM}_{jM';\,1\mu},$$

we transform the expression into

$$W = \sum_{MM'}\left\{\delta_{MM'} + \frac{j(j+1)+I_2(I_2+1)-I_3(I_3+1)}{2I_2\sqrt{j(j+1)}}\,\xi\sum_{\mu} C^{jM}_{jM';\,1\mu}\, v^{\mu}\right\}$$

$$\times (B^{(\lambda)}_{jM})_{21}(B^{(\lambda)}_{jM'})^*_{21} = \sum_{MM'} A_{MM'}(B^{(\lambda)}_{jM})_{21}(B^{(\lambda)}_{jM'})^*_{21}. \tag{5}$$

The wave functions of the internal-conversion electron and positron will be taken in the form of plane waves

$$\psi_1 = v\exp(i\mathbf{p}_+\mathbf{r}), \qquad \psi_2 = u\exp(-i\mathbf{p}_-\mathbf{r}),$$

where $\mathbf{p}_+$ and $\mathbf{p}_-$ are the momenta of the positron and the electron, while v and u are unit Dirac bispinors for the positron and electron respectively. With these wave functions, the expression for the probability of pair conversion following β decay has the form (accurate to inessential multipliers)

$$W = \sum_{MM'} A_{MM'}\,\mathrm{Sp}\,\{(i\hat{p}_- - m)V^{(\lambda)}_{jM}(i\hat{p}_+ + m)\overline{V^{(\lambda)}_{jM'}}\},$$

where $\hat{p}_\pm = p^i_\pm\gamma_i$; the form of the expressions $V^{(\lambda)}_{jM}$, $\overline{V^{(\lambda)}_{jM'}} = \gamma_4 V^{(\lambda)+}_{jM'}\gamma_4$ will be given below.

The states of the polarization of the conversion electrons and positrons will be described with the aid of density matrices, which can be introduced in the usual manner. The normalized density matrix that determines the polarization of the conversion electrons in this process has the form

$$P_{(-)} = -\frac{1}{2\varepsilon_- W}\left\{\sum_{MM'} A_{MM'}(i\hat{p}_- - m)V^{(\lambda)}_{jM}(i\hat{p}_+ + m)\overline{V^{(\lambda)}_{jM'}}(i\hat{p}_- - m)\gamma_4\right\}, \tag{6}$$

and analogously for positrons

$$P_{(+)} = -\frac{1}{2\varepsilon_+ W}\left\{\sum_{MM'} A_{MM'}(i\hat{p}_+ + m)\overline{V^{(\lambda)}_{jM'}}(i\hat{p}_- - m)V^{(\lambda)}_{jM}(i\hat{p}_+ + m)\gamma_4\right\}. \tag{7}$$

If we do not register the angle θ between the momenta of the internal-conversion electron and positron, then the numerator and the denominator of the expressions (6) and (7) must be integrated with respect to θ.

The electron (positron) polarization 4-vector $\zeta_\mu = \{\boldsymbol{\zeta};\, \zeta_0\}$ satisfies the condition $\zeta_\mu p_\mu = 0$. Therefore, in a reference frame where the electron (positron) is at rest, we have $\mathbf{p} = 0$ and $\zeta_0 = 0$, i.e., $\zeta^0_\mu = \{\boldsymbol{\zeta}^0;\, 0\}$.

Consequently, the polarization properties of the electron (positron) can be determined, as is customarily done, with the aid of the three-dimensional vector $\boldsymbol{\zeta}^0$, connected with $\boldsymbol{\zeta}$,

$$\zeta^0 = \zeta_\perp + m\varepsilon^{-1}\zeta_\parallel, \tag{8}$$

where $\boldsymbol{\zeta}_\perp$ and $\boldsymbol{\zeta}_\parallel$ are the transverse and longitudinal components of the vector $\boldsymbol{\zeta}$, expressed in terms of the electron (positron) density matrix in the following manner:

$$\zeta = i\varepsilon m^{-1}\,\mathrm{Sp}\,\{P\gamma_4\gamma_5\boldsymbol{\gamma}\}. \tag{9}$$

Magnetic type of transition ($\lambda = 0$). The matrix element of the interaction operator of the internal-conversion electron and positron with the multipole field has the form[2]

$$(B^{(0)}_{jM})_{21} = u^* V^{(0)}_{jM} v = q^j(\omega^2 - q^2)^{-1} u^*\boldsymbol{\alpha}\mathbf{Y}_{jjM}(\mathbf{q}/q)\, v,$$

where $\mathbf{q} = \mathbf{p}_+ + \mathbf{p}_-$ and $\omega = \epsilon_+ + \epsilon_-$ are the momentum and the energy of the conversion transition, α is the Dirac velocity matrix, and $Y_{jjM}(\mathbf{q}/q)$ is a transverse spherical vector.

Substituting the expressions for $V^{(0)}_{jM}$ and $V^{(0)}_{jM'}$ in (6), then (6) in (9), and then calculating the trace, summing over the magnetic quantum numbers M and M′ (a method for summing similar expressions is developed in papers by Berestetskiĭ et al.[1,3]), and integrating over the angle θ, we obtain for the polarization vector of the internal-conversion electron the expression

$$\zeta^0_{(-)} = N_0\{m^2\omega J_{2j-2}\,\mathbf{q}(\mathbf{vq}) + \tfrac{1}{2}(\varepsilon_+ - \varepsilon_-)(J_{2j} - \omega^2 J_{2j-2})\,\mathbf{p}_-(\mathbf{vq})\}$$
$$\times\{\varepsilon_-[(\varepsilon_+\varepsilon_- + m^2)J_{2j} - \tfrac{1}{4}J_{2j+2} - \tfrac{1}{4}\omega^2(\varepsilon_+ - \varepsilon_-)^2 J_{2j-2}]\}^{-1},$$
$$N_0 = \xi[j(j+1) + I_2(I_2+1) - I_3(I_3+1)]/2I_2 j(j+1). \tag{10}$$

The form of the expressions J_{2j+k} is given in the appendix.

It is obvious that all the results obtained are symmetrical under a substitution of the electrons symbols (−) for the positron symbols (+) and vice versa, inasmuch as the calculation did not account for the Coulomb field of the nucleus. Therefore the corresponding expression $\zeta^0_{(+)}$ for positrons can be obtained directly from (10), by making everywhere in (10) the interchange $\epsilon_+ \rightleftarrows \epsilon_-$, $\mathbf{p}_+ \rightleftarrows \mathbf{p}_-$.

Electric type transition ($\lambda = 1$). In this case the matrix element of the operator $B^{(1)}_{jM}$ has the form[2]

$$(B^{(1)}_{jM})_{21} = u^* V^{(1)}_{jM} v = \frac{q^j}{\omega^2 - q^2} u^* \Big\{ Y_{jM}(\mathbf{q}/q) - \sqrt{\frac{2j+1}{j}} \frac{\omega}{q} \mathbf{Y}_{j,\,j-1,\,M}(\mathbf{q}/q)\, \boldsymbol{\alpha} \Big\} v,$$

where $Y_{jM}(\mathbf{q}/q)$ is the Laplace spherical function and $\mathbf{Y}_{j,\,j-1,\,M}(\mathbf{q}/q)$ is a spherical vector. We use everywhere the system of mutually-orthogonal spherical vectors described in detail in reference 2.

After performing the same operations with $V^{(1)}_{jM}$ as in the case of magnetic transition, we obtain for the polarization vector of the internal-conversion electron

$$\zeta^0_{(-)} = N_1 \frac{\omega}{m} \Big\{ m^2 J_{2j} (\mathbf{v} - \mathbf{n}(\mathbf{vn})) - \frac{m}{\varepsilon_-} J_{2j} \mathbf{p}_- (\mathbf{v}(\mathbf{p}_+ - \mathbf{p}_-)) - \frac{m^3 \omega^2}{j \varepsilon_-} J_{2j-2} \mathbf{n}(\mathbf{vn}) + \frac{m\omega(\varepsilon_+ - \varepsilon_-)}{2j\varepsilon_-} (\omega^2 J_{2j-4} + (2j-1) J_{2j-2}) \mathbf{p}_-(\mathbf{vq}) \Big\} \times \Big\{ \frac{1}{2} J_{2j+2} - \frac{1}{2} J_{2j} \Big[\omega^2 \frac{5j+1}{2j} + (\varepsilon_+ - \varepsilon_-)^2 \Big] + \omega^2 J_{2j-2} \times \Big(\frac{\omega^2}{2} + 2\varepsilon_+\varepsilon_- + 2m^2 + (\varepsilon_- - \varepsilon_+)^2 \Big) + \frac{j-1}{4j} \omega^2 (\varepsilon_+ - \varepsilon_-)^2 J_{2j-4} \Big\}^{-1}, \qquad (11)$$

where $N_1 = N_0 (j+1)$ and $\mathbf{n} = \mathbf{q}/q$ is a unit vector in the direction of the total momentum of the pair. By corresponding substitution, we can obtain directly from (11) an expression for the polarization vector of the positron $\zeta^0_{(+)}$.

In the case of the V − A variants of the β interaction (with conservation of time parity) the constant ξ, which enters into the expression of the density matrix $\rho_{m_1 m_2'}$, assumes the following form.

$$\xi = \{ c_A^2 |M_{GT}|^2 \Lambda_{I_1 I_2} - 2 c_V c_A [I_2/(I_2+1)]^{1/2} \delta_{I_1 I_2} M_F M^*_{GT} \} \times \{ c_V^2 |M_F|^2 + c_A^2 |M_{GT}|^2 \}^{-1}.$$

In the case of a Gamow-Teller transition where the spin (or isotopic spin) changes by unity, $\Delta I = \pm 1$ ($\Delta T = \pm 1$), this constant is independent of the nuclear matrix elements and of the constants of the β interaction

$$\xi = \Lambda_{I_1 I_2} = [I_2(I_2+1) - I_1(I_1+1) + 2] / 2I_2(I_2+1),$$

and depends only on the spins of the initial and final states of the nucleus (I_1 and I_2).

The expression (11) obtained in the present paper for $\zeta^0_{(-)}$ has been used for a numerical calculation of the polarization of the internal-conversion electrons that follow the β decay of the nucleus $Na^{24}[4^+ (\beta^-) 4^+ (E2) 2^+ (E2) 0^+]$. The transition $4^+ (\beta^-) 4^+$ is a Gamow-Teller transition in the isotopic spin.[4] Since, disregarding the Coulomb field of the nucleus, emission of conversion electrons and positrons with equal momenta is the most probable, we assume in the calculations $\epsilon_+ = \epsilon_- = \omega/2$ [for the conversion transition $4^+ \to 2^+$ (E2)].

The calculation for the polarization of the pair-conversion electrons yielded in this transition

$$\zeta^0_{(-)} = 10^{-2} \{ 3(\mathbf{n}_\beta - \mathbf{n}(\mathbf{n}_\beta \mathbf{n})) - 0{,}7 (\mathbf{n}_\beta \mathbf{n}) \mathbf{n} \}.$$

It is clear therefore that in the conversion transition considered here the transverse polarization is one order of magnitude greater than the longitudinal polarization.

It must be noted that an account of the Coulomb field of the nucleus is apparently inessential in the conversion parts of the calculations, since the internal conversion is the most effective for nuclei with small charge Z, for which the Born approximation gives good results. On the other hand, the Coulomb field of the nucleus, for the β-decay stage of the considered process, can be readily accounted for by using the density matrix $\rho_{m_2 m_2'}$, obtained by Geshkenbeĭn.[5]

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APPENDIX

The quantities J_{2j+k} ($k = 0, \pm 2, -4$) which arise upon integration over the angle of emission of the pair, are determined in the following manner

$$J_{2j-2} = p_+ p_- \int_0^\pi \frac{q^{2j-2}}{(q^2 - \omega^2)^2} \sin\theta \, d\theta = \frac{1}{4(j-2)} \{ (p_+ + p_-)^{2(j-2)} (m^2 - p_+ p_- - \varepsilon_+ \varepsilon_-) - (p_+ - p_-)^{2(j-2)} (m^2 + p_+ p_- - \varepsilon_+ \varepsilon_-) + 4\omega^2 (j-1) J_{2j-4} \} \qquad (j \neq 2). \qquad (A1)$$

When j = 2, we obtain by direct integration

$$J_2 = \ln \frac{m\omega}{p_+ p_- + \varepsilon_+ \varepsilon_- + m^2} + \frac{p_+ p_-}{2m^2}. \qquad (A2)$$

Analogously,

$$J_{2j} = p_+p_- \int_0^\pi \frac{q^{2j}}{(q^2-\omega^2)^2} \sin\theta\, d\theta$$

$$= \frac{1}{4(j-1)} \{(p_+ + p_-)^{2(j-1)} (m^2 - p_+p_- - \varepsilon_+\varepsilon_-)$$

$$- (p_+ - p_-)^{2(j-1)} (m^2 + p_+p_- - \varepsilon_+\varepsilon_-) + 4\omega^2 j J_{2j-2}\} (j \neq 1); \quad \text{(A3)}$$

when j = 1 we have the already obtained expression (A2).

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[3] Berestetskiĭ, Dolginov, and Ter-Martirosyan, JETP **20**, 527 (1950).

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